Modelling current-voltage characteristics of practical superconductors

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Outline

1. **Statement of the problem**
   - Motivation
   - The underlying physical problem
   - Macroscopic *material law*

2. **(Numerical) modelling**
   - Thermodynamic model: general framework
   - The power-law-like $\mathcal{F}(\mathbf{J})$ formulation
   - The power-law-like $\mathbf{E}(\mathbf{J})$ formulation

3. **Application**
   - Approximations to the *helical cable* geometry
   - Fingerprints of the $\mathbf{E}(\mathbf{J})$ law

4. **Conclusions**

1. Statement of the problem

1.1 Motivation

The Macroscopic Maxwell Equations must be supplied with a SOUND and PRACTICAL expression of the superconducting material law

In quasistatic conditions:

$$\mathbf{E}(\mathbf{J}) = \rho(\mathbf{J})\mathbf{J}$$

$$\Downarrow$$

$$\left(\mu_0 \frac{\partial}{\partial t} - \rho(\mathbf{J})\nabla^2\right)\mathbf{H} = (\nabla \times \mathbf{H}) \times \nabla\rho(\mathbf{J})$$

⋆ A number of particular choices exist for $\rho(\mathbf{J})$, but FE codes lack an implementation for general purpose

⋆ $\rho(\mathbf{J})$ is not always a scalar, neither a tensor !!

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1.2. The underlying physical problem

Local Geometry of Ampère's law ($J_{\parallel}$, $J_{\perp}$)

$$\mathbf{1} \equiv \mathbf{H}/H \quad ; \quad \mathbf{2} \equiv \nabla H/\|\nabla H\| \quad ; \quad \mathbf{3} \equiv \mathbf{1} \times \mathbf{2}$$

$$\Rightarrow \quad \mathbf{J} = H(-\partial_2\theta + \partial_3\phi)\mathbf{1} + (H\partial_1\theta)\mathbf{2} + (H\partial_1\phi - \partial_2 H)\mathbf{3}$$

Example 1: uniform current density + axial field

$$\mathbf{1} = (-y, x, 1)/\sqrt{1+\rho^2}$$

$$\mathbf{2} = (x, y, 0)/\rho$$

$$\mathbf{3} = (-y, x, -\rho^2)/\rho\sqrt{1+\rho^2}$$

$$J_1 = J_0/\sqrt{1+\rho^2} = -H\partial_2\theta$$

$$J_2 = 0$$

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1.3. The underlying physical problem

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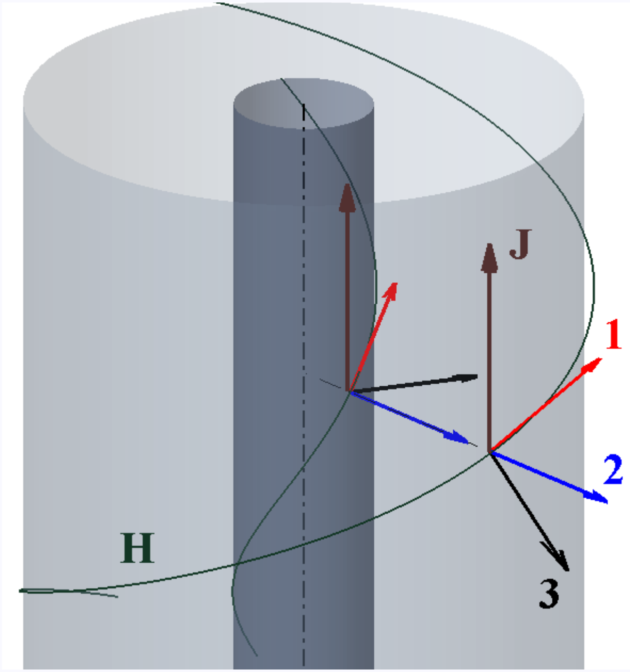
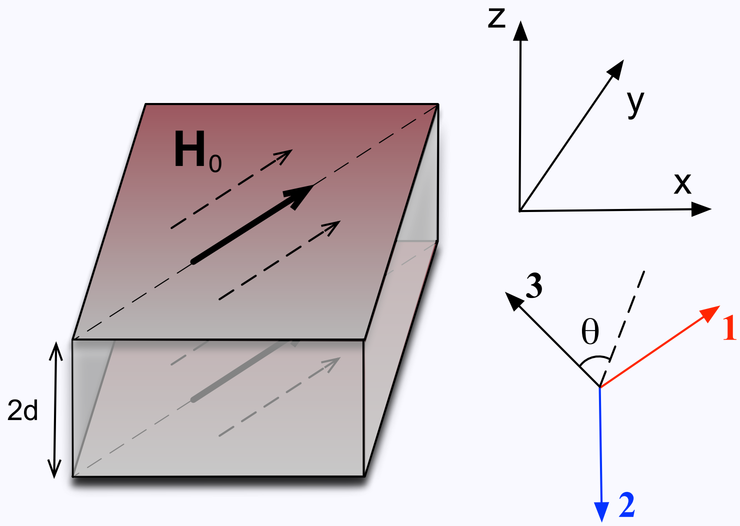


Example 2: planar sample in rotating field



$J_{\parallel}$ only comes from the tilt between adjacent layers $(-\partial_2\theta)$

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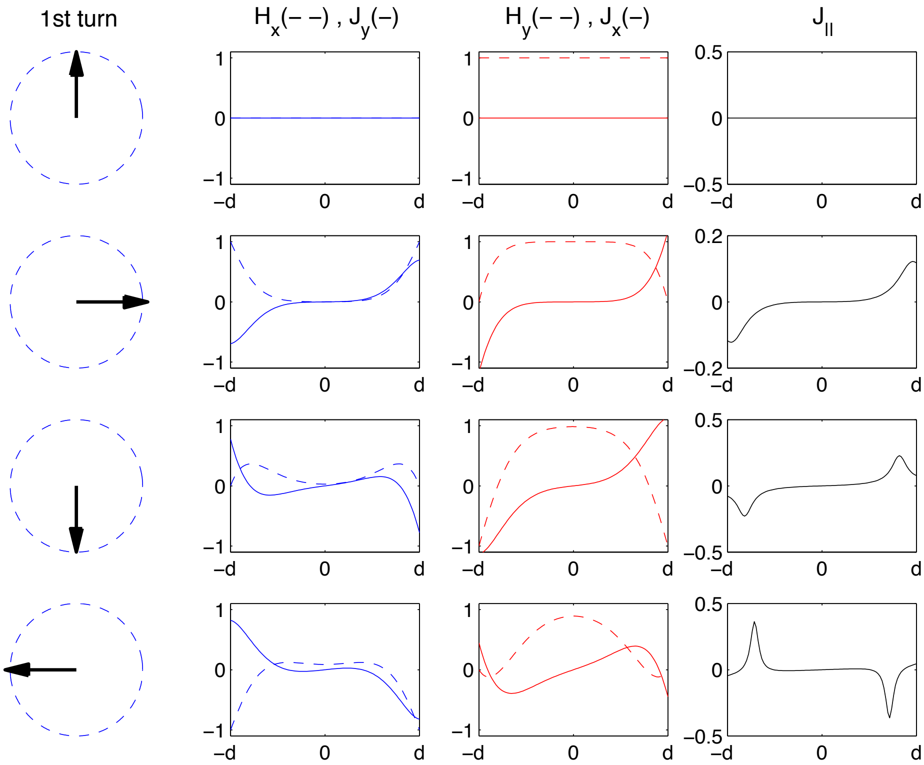
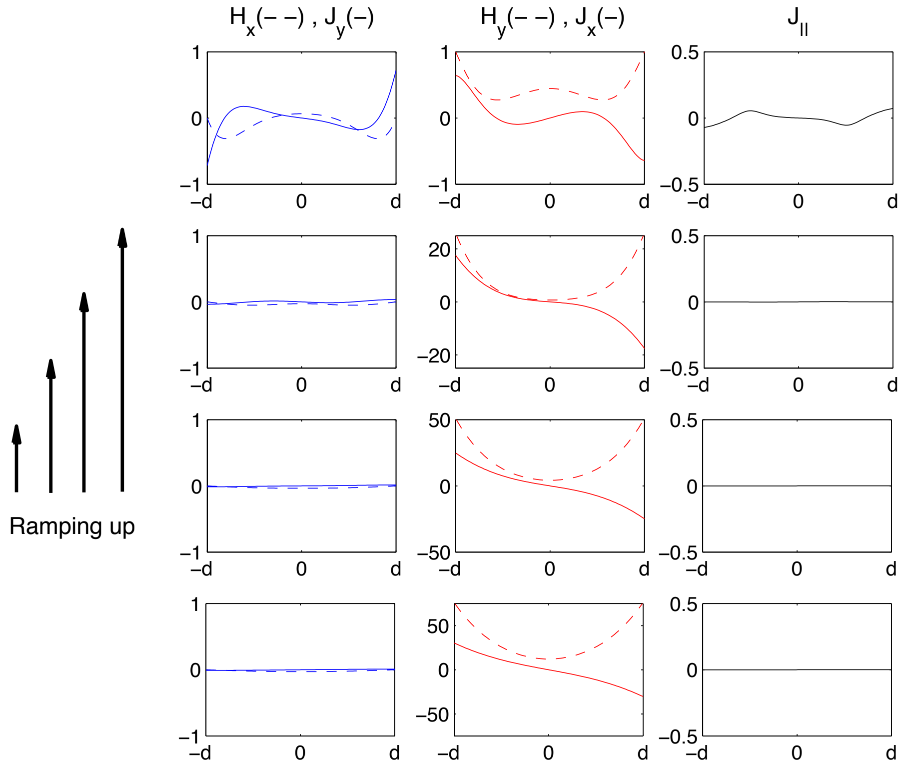
The appearance of $J_{\parallel}$ . . .



The disappearance of $J_{\|}$ ...



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Partial conclusions

A) ⋆ Rotations of the magnetic field are shielded by $J_{\parallel}$

B) ⋆ In MQS, when rotation ceases $J_{\parallel}$ disappears

Here, we have solved: $$\nabla^2\mathbf{H} = (\mu_0/\rho_0)\frac{\partial \mathbf{H}}{\partial t}$$

then

$$\mathbf{J}\cdot\mathbf{H} = 0 \Rightarrow \frac{\partial (H_x/H_y)}{\partial t} = 0$$

In a superconductor

A) is true

B) both $J_{parallel}$ and $J_{\perp}$ persist in MQS regime

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1.3. Material law in type-II superconductors

⋆ Electromagnetic energy of the Vortex Lattice

$$W_{\rm SC} = \frac{1}{\mu_0}\int_\Omega \mathbf{V}\cdot\left(\mathbf{b}_1 + \frac{1}{2}\mathbf{b}_2 - \mu_0\mathbf{H}\right)$$

$\mathbf{V} = \sum_i \Phi_0\delta^2(\mathbf{r}-\mathbf{r}_i)\mathbf{n}_i$: vorticity

$\mathbf{b}_2$ flux density of the equilibrium Vortex Lattice

$\mathbf{b}_1$ flux related to other sources

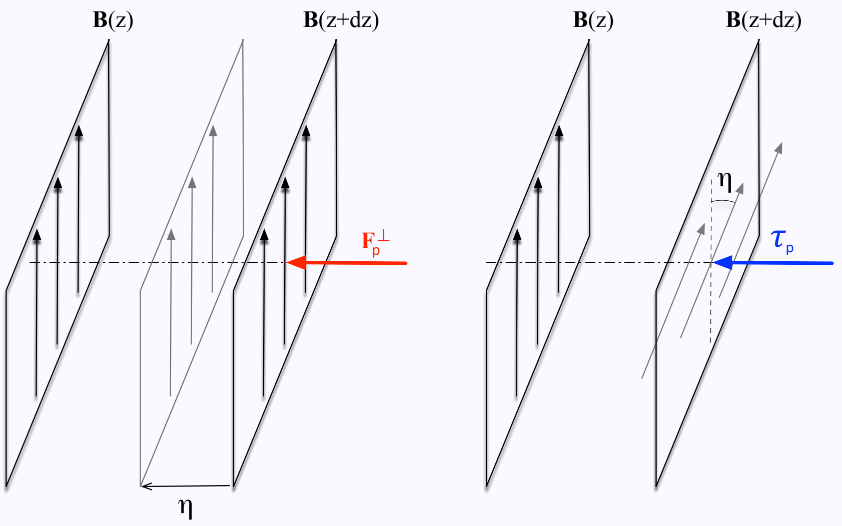
$\mathbf{H}$ field intensity: $\nabla\times\mathbf{H} = \mathbf{J}_0$

⋆ The equilibrium ($\partial_\eta W_{\rm SC} = 0$) is given by a triangular vortex lattice with a uniform macroscopic field $\mathbf{B}$ parallel to $\mathbf{H}$. Then $\mathbf{B} = \mu_0\mathbf{H}$ and $\nabla\times\mathbf{B} = 0$ well within the sample

In non-ideal (practical) superconductors B may vary in intensity ($J_\perp$) and orientation ($J_\parallel$)

Then: $W_{Full} = W_{SC} + W_{Pinning}$

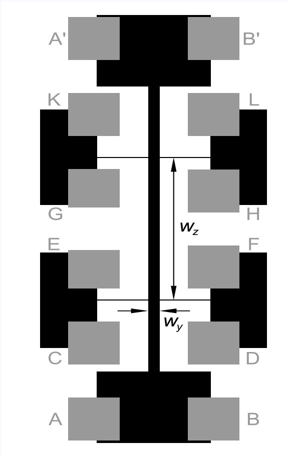
Equilibrium: $\partial_\eta W_{SC} + \partial_\eta W_{Pinning}$ (forces + *constraints* = 0)

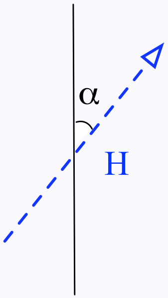


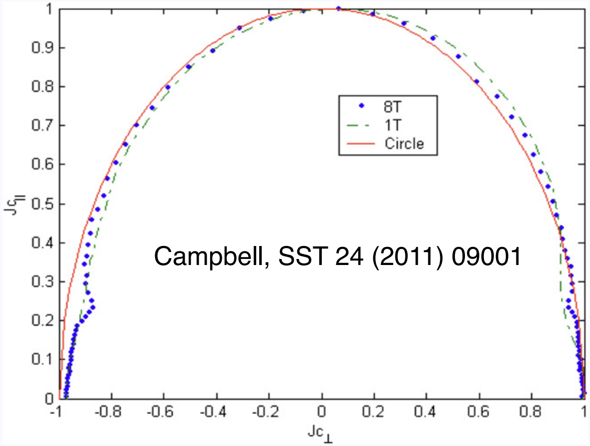
$$J_\perp \propto F_p^\perp = \underset{\sim}{F_p} \cos\alpha \, ; \; J_\parallel \propto \tau_p \propto F_p^\parallel = \underset{\sim}{F_p} \sin\alpha \Rightarrow \frac{J_\perp^2}{a^2} + \frac{J_\parallel^2}{b^2} = 1$$

Clarifying E(J): CWDC experiment

Supercond. Sci. Technol. 24 (2011) 062002: Clem, Weigand, Durrell, Campbell







An elliptic $J_{\|}(J_{\perp})$ law has been reported

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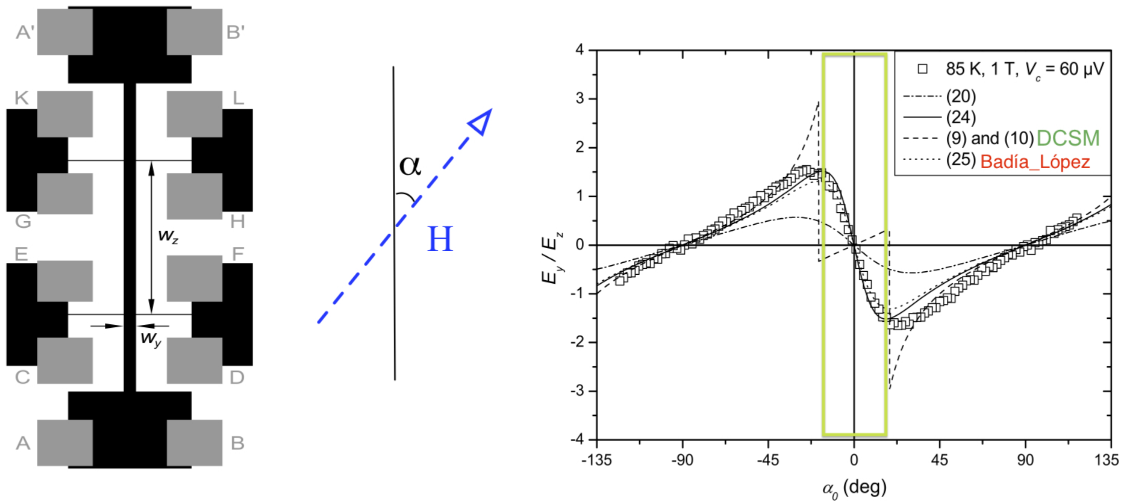

Eq.(25) corresponds to the Critical State Theory ...

that postulates a *non-functional* relation $\{\mathbf{E},\mathbf{J}\} \Rightarrow \mathbf{J} \in \Delta$

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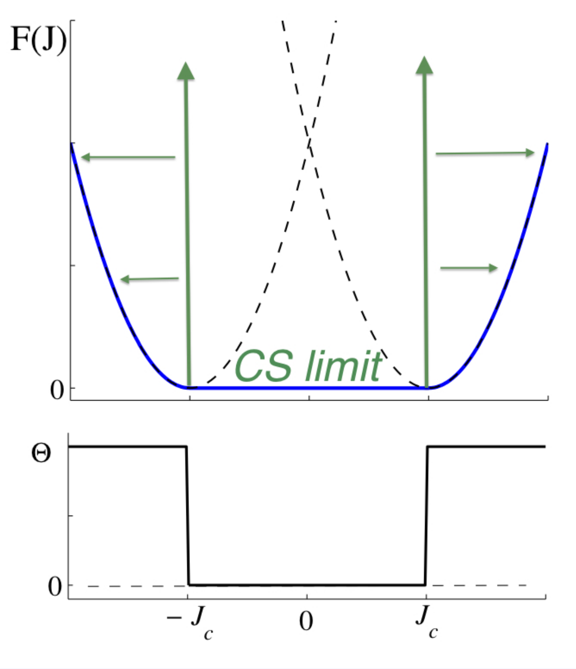
2. Numerical Modelling $\hookrightarrow$ $\mathbf{E(J)}$

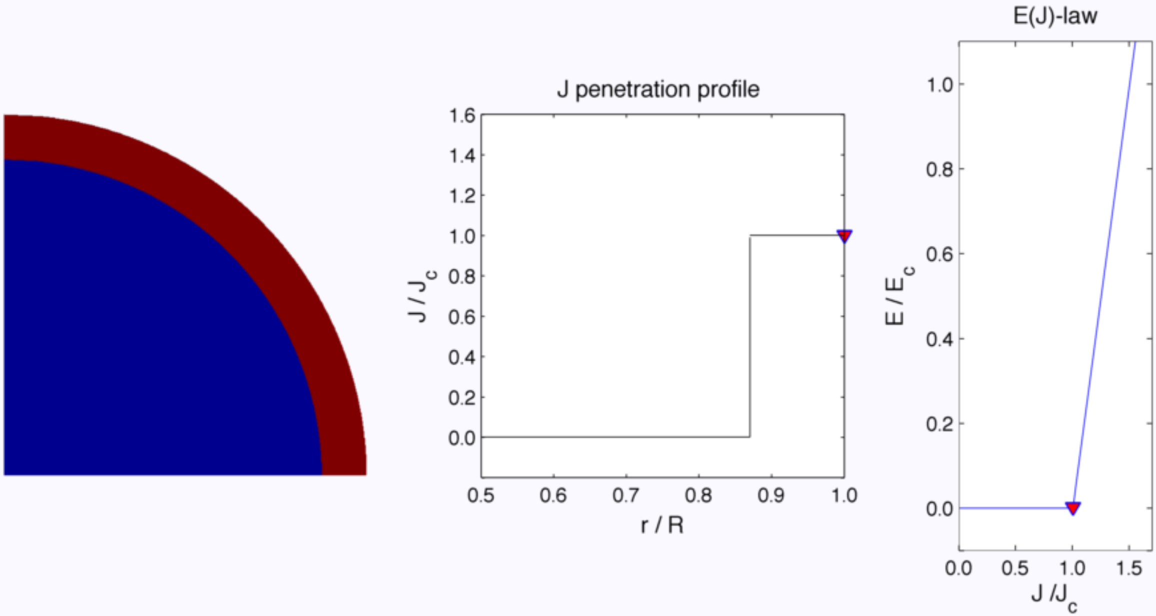
2.1. Thermodynamic model (SST 2012)

$$\texttt{Minimize}\quad \mathcal{C} \equiv \frac{\mu_0}{2}\int_{\mathbb{R}^3} \|\mathbf{H}_{n+1} - \mathbf{H}_n\|^2 + \Delta t \int_\Omega \mathcal{F}[J]$$



$$\mathbf{E} = \nabla_{\mathbf{J}}\mathcal{F}$$

Academic 1D example: transport along type-II cylinder with *quasi-linear* E(J)



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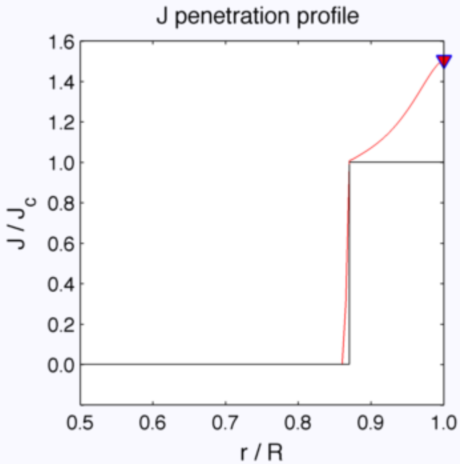

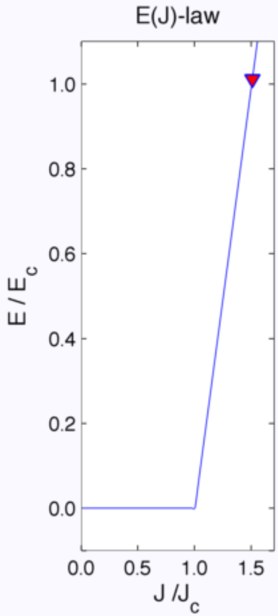

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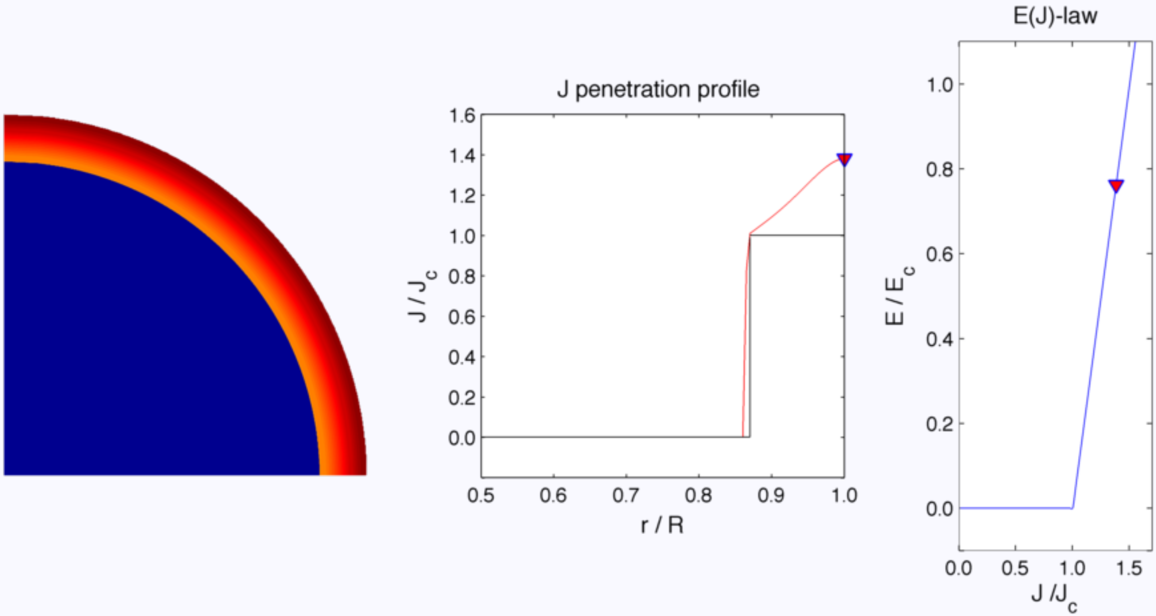

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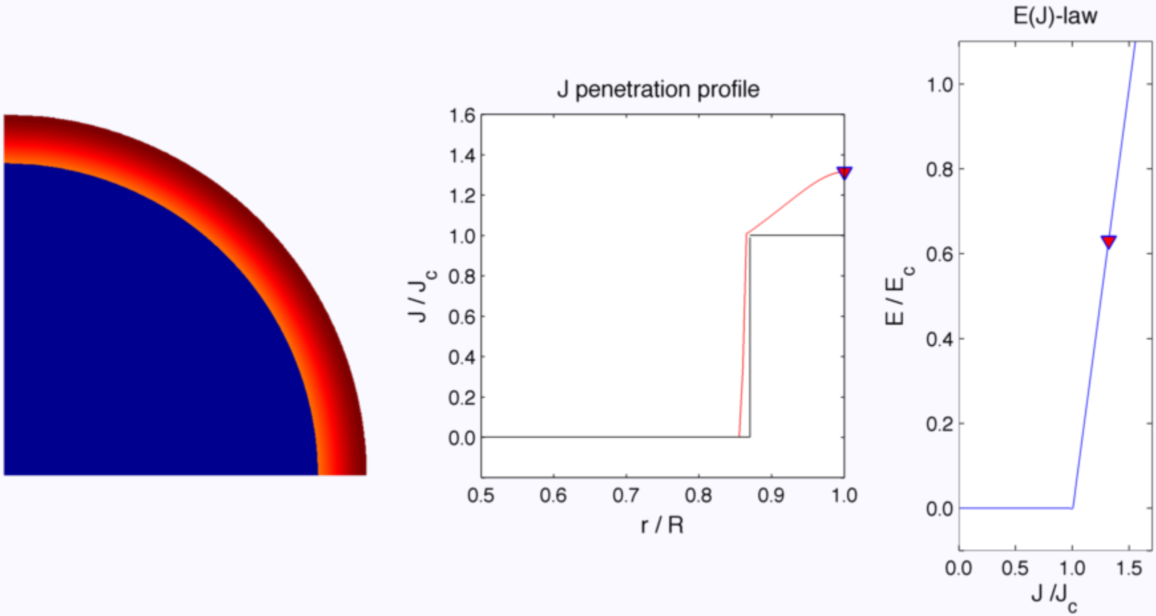

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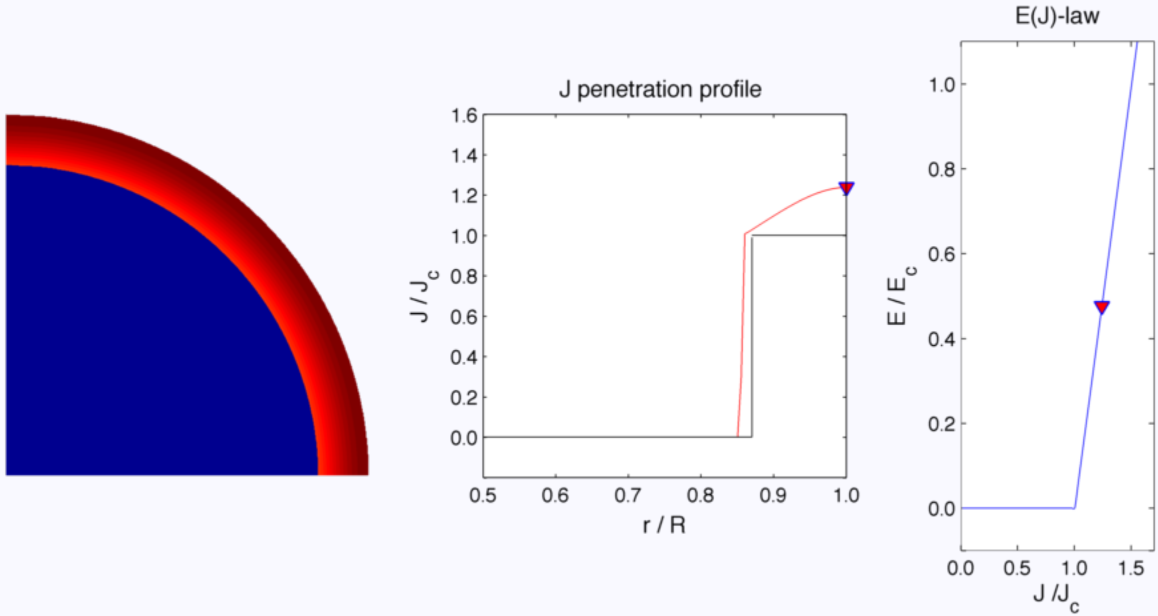

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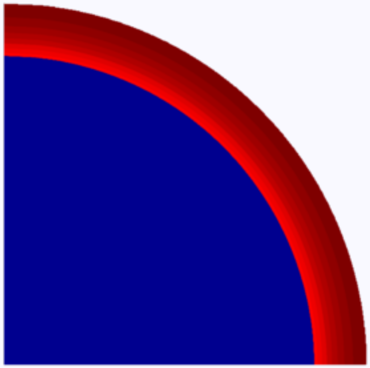

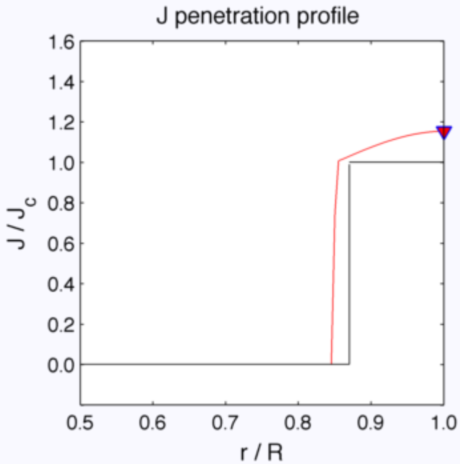

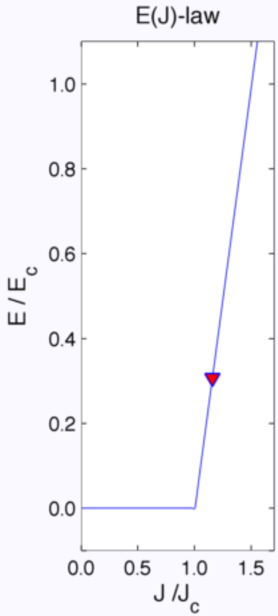

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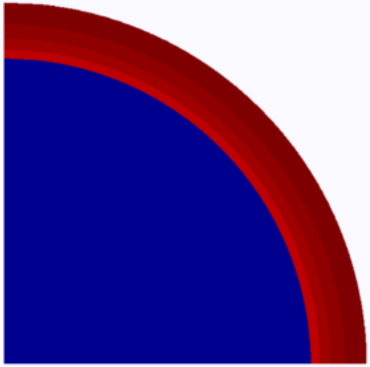

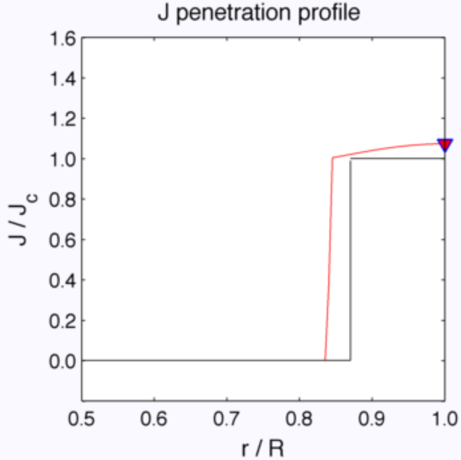

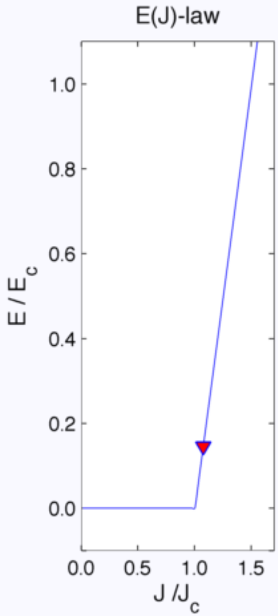

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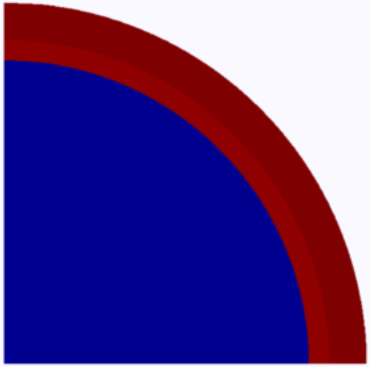

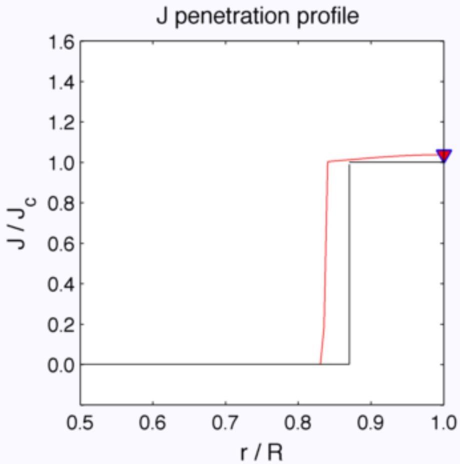

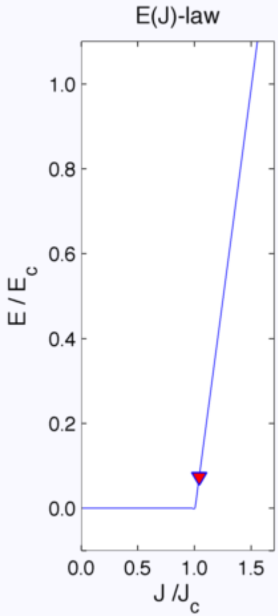

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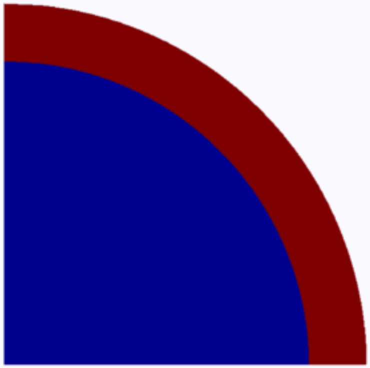

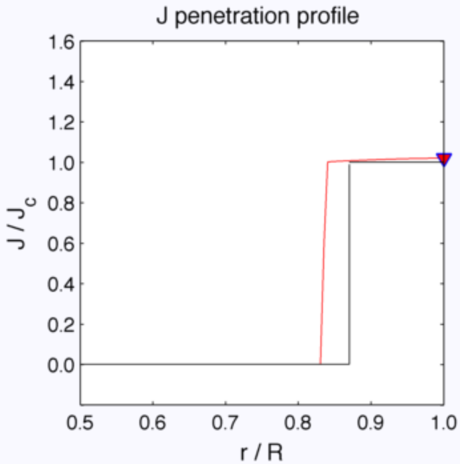

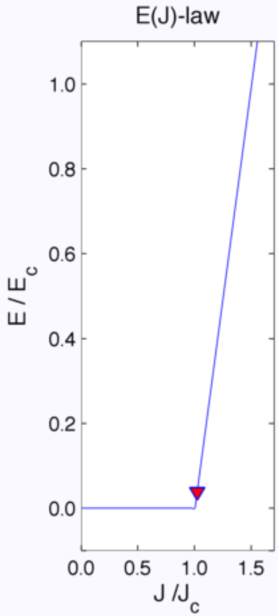

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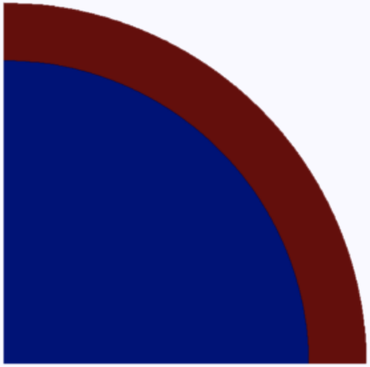
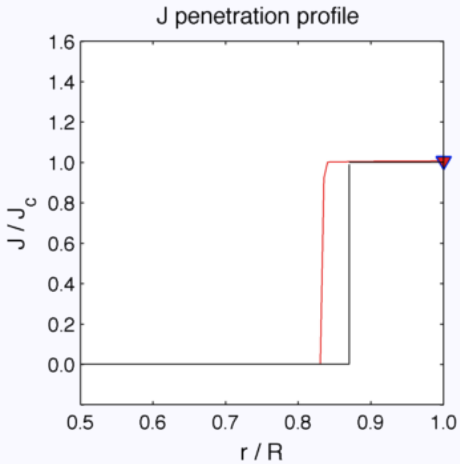
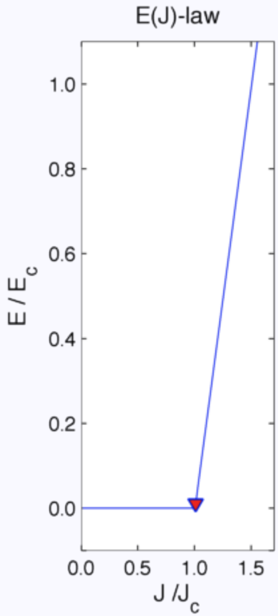
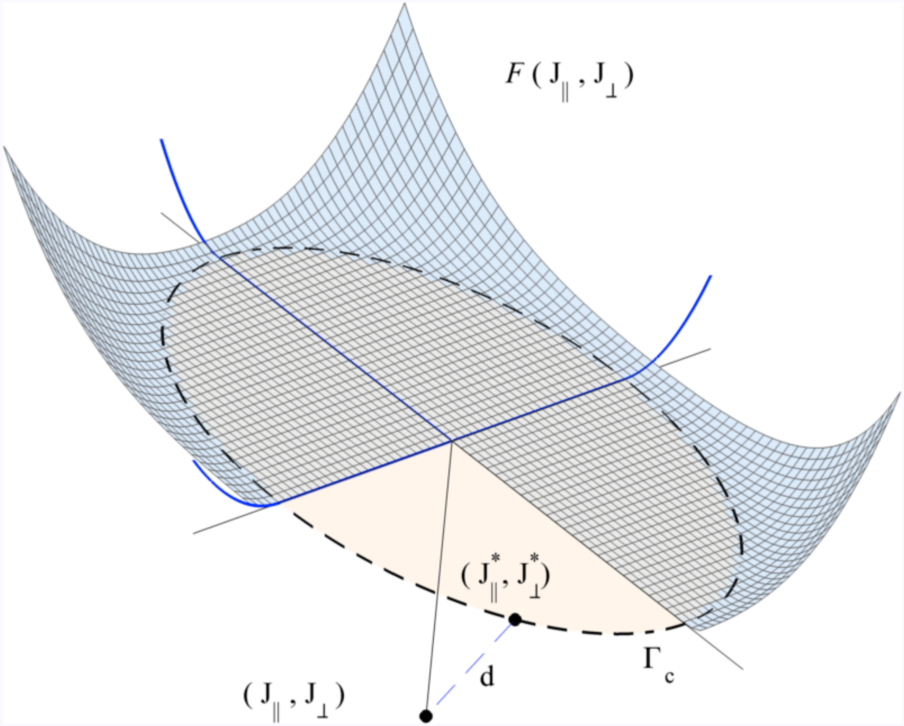






Towards 3D modelling: expanding the yield region



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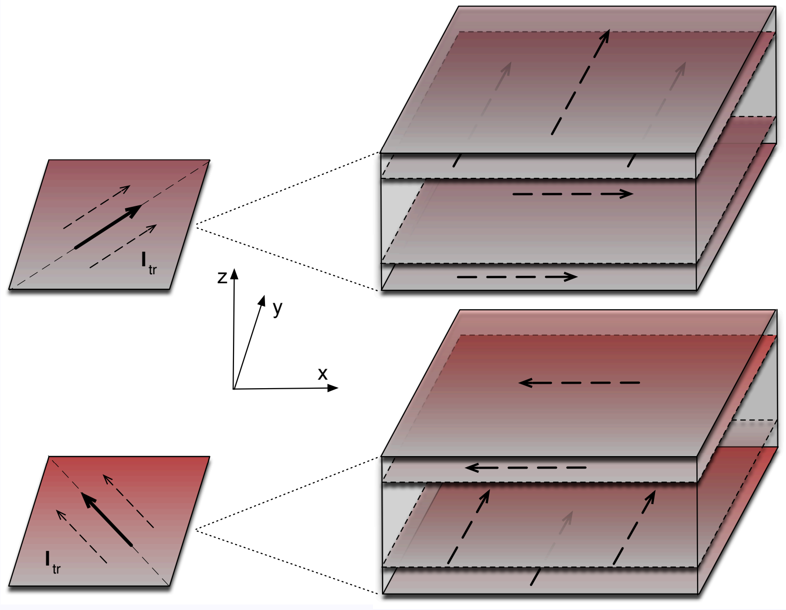
Expanded yield region: first example

Transport along crossed tapes



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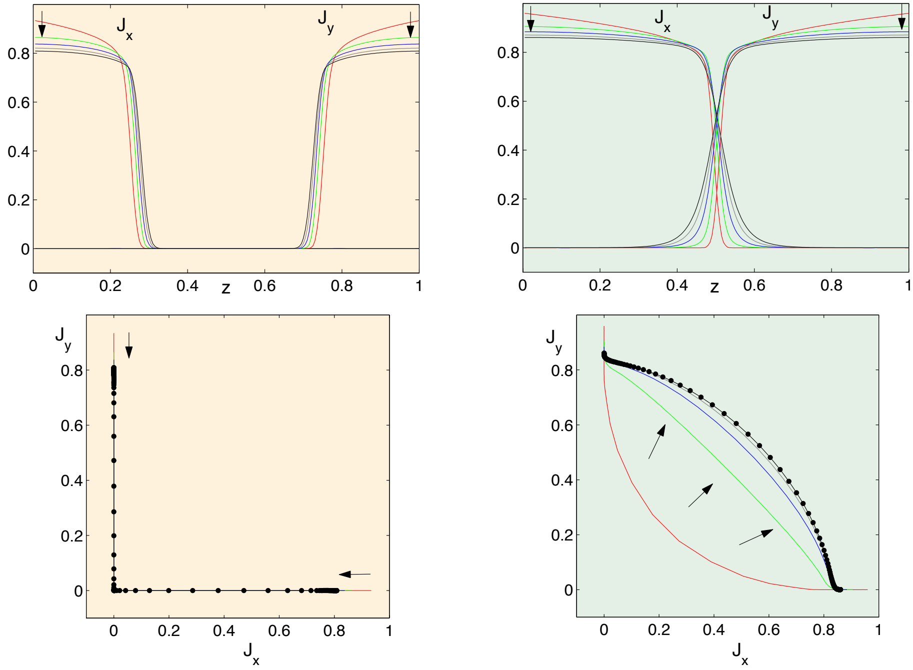

2.2. The power-law-like $\mathcal{F}(\mathbf{J}\ )$ formulation

In SST 2012 it was shown that

$$\mathcal{F}_{\mathrm{QLL}}(J) = \frac{1}{2}\,\rho\,\Theta_{\Gamma}(\mathbf{J})\,(J \pm J_{c\perp})^{2} \qquad \text{(Quasi-linear-law)}$$

&

$$\mathcal{F}_{\mathrm{PL}}(J) = F_0 \left(\frac{J}{J_{c\perp}}\right)^{\mathrm{M}}, \; M \gg 1 \qquad \text{(Power-law)}$$

are equivalent in 1D

⋆ Here, we generalize $\mathcal{F}_{\mathrm{PL}}$ to 3D

$$\mathcal{F}_{\mathrm{PL}}(\mathbf{J}) = F_0 \left[\left(\frac{J_{\parallel}}{J_{c\parallel}}\right)^{2} + \left(\frac{J_{\perp}}{J_{c\perp}}\right)^{2}\right]^{\mathrm{M}}$$

2.3. The power-law-like E(J) formulation

$$\mathbf{E}_{\mathrm{PL}}(\mathbf{J}) = \nabla_{\mathbf{J}}\,\mathcal{F}_{\mathrm{PL}}(\mathbf{J})$$

$$\Downarrow$$

$$\mathbf{e}(\,\mathbf{j}\,) = \left(j^2 + \gamma j_{\|}^2\right)^{\mathrm{M}-1}(\,\mathbf{j} + \gamma\mathbf{j}_{\|}\,)$$

with the definitions:

$$\gamma \equiv J_{c\perp}^2/J_{c\|}^2 - 1 \equiv \Gamma^2 - 1$$

$$\mathbf{j} \equiv \mathbf{J}/J_{c\perp}$$

$$\mathbf{e} \equiv \mathbf{E}/(2MF_0J_{c\perp})$$

⋆ Applied to CWDC experiment:

$$\frac{e_y}{e_z} = \frac{\gamma\sin\alpha\,\cos\alpha}{1+\gamma\cos^2\alpha} = \frac{(\Gamma^2-1)\tan\alpha}{\Gamma^2+\tan^2\alpha} \quad \checkmark$$

2.3. The power-law-like E(J) formulation

$$\mathbf{E}_{\rm PL}(\mathbf{J}) = \nabla_{\mathbf{J}}\,\mathcal{F}_{\rm PL}(\mathbf{J})$$

$$\Downarrow$$

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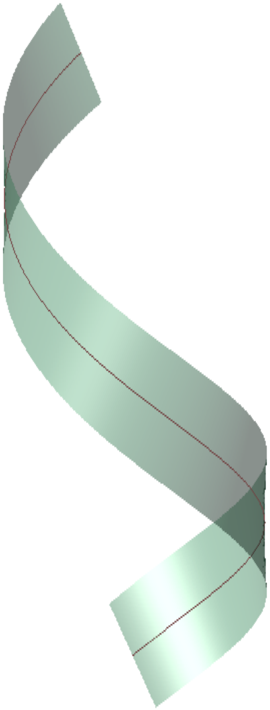
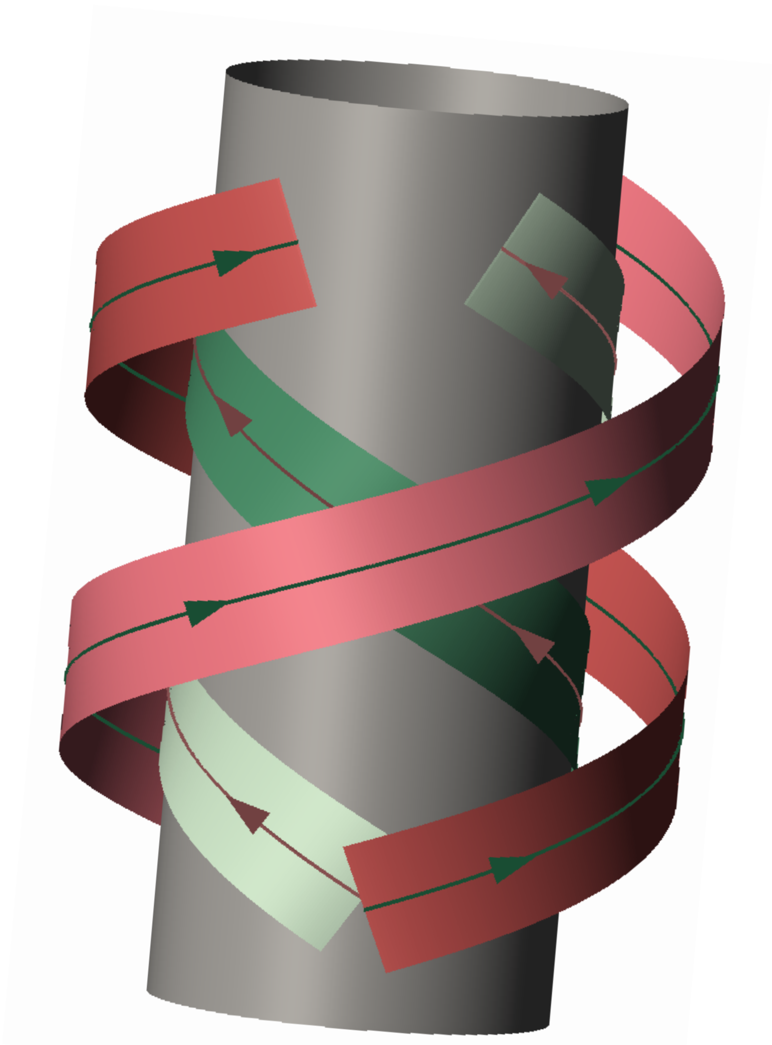
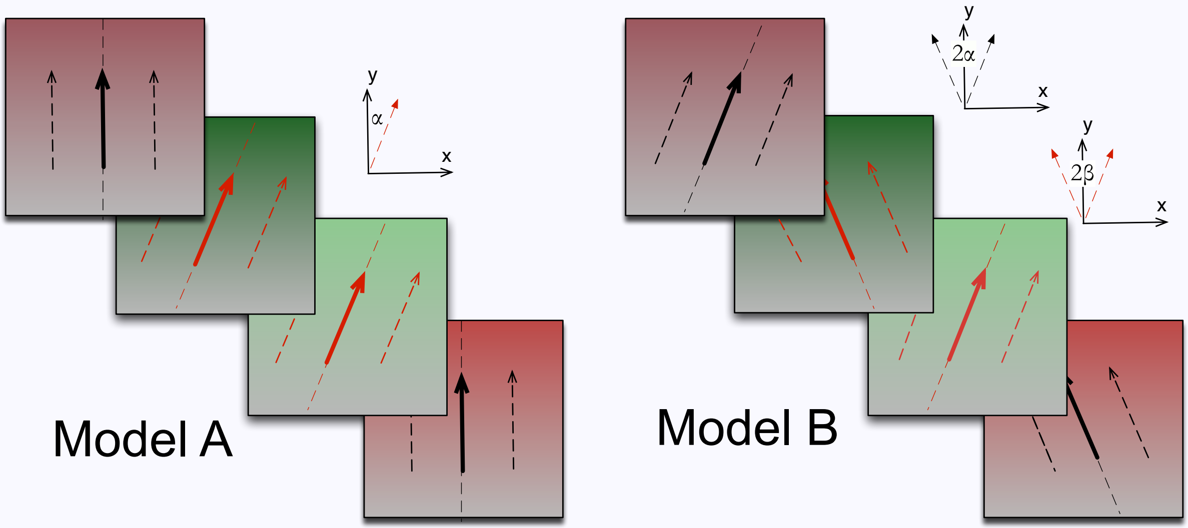
3. Application





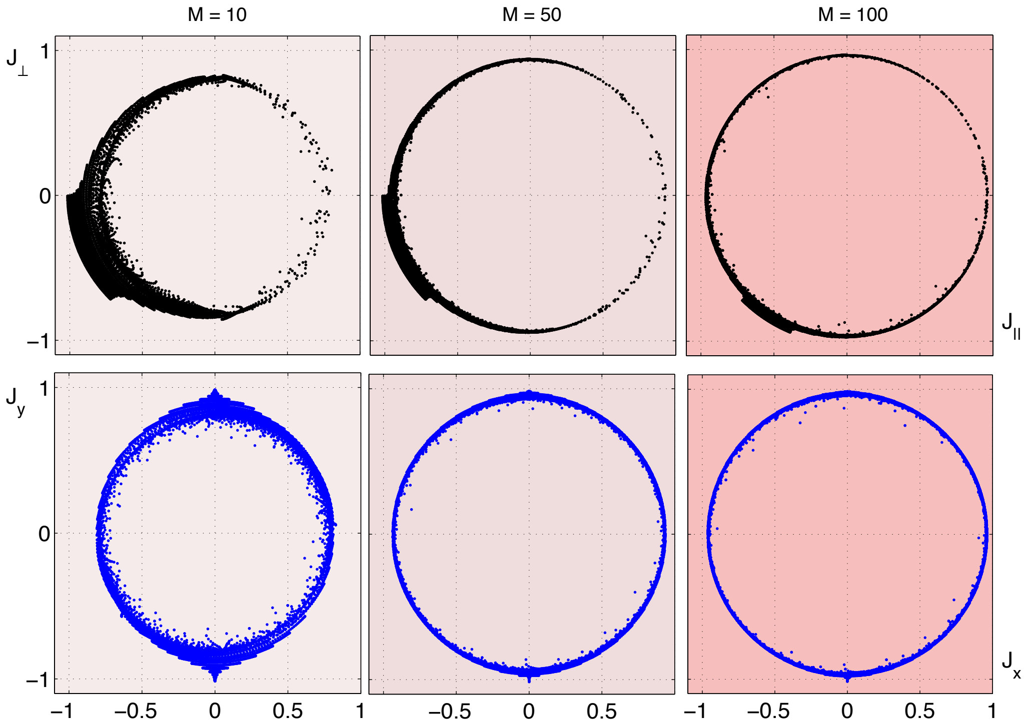
3.1. Approximations to the helical problem



In all cases $I_{\text{tr}}(t) = I_0 \sin \omega t$ along each layer

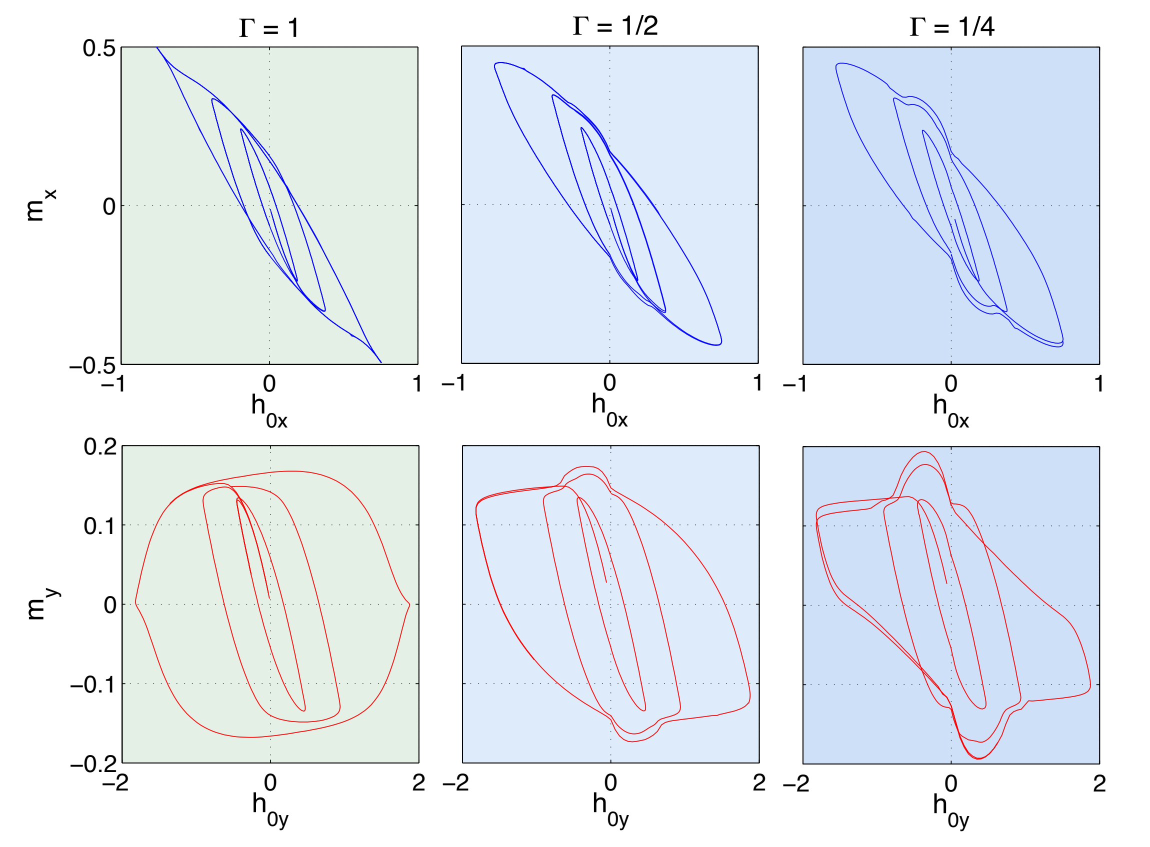
and we obtain $\mathbf{j}(z,t)$ across the layers

Model A: influence of the power-law exponent



In this case $\Gamma = 1$

Model A: influence of the anisotropy ratio



In this case $M = 10$ & $\alpha = 67.5^\circ$

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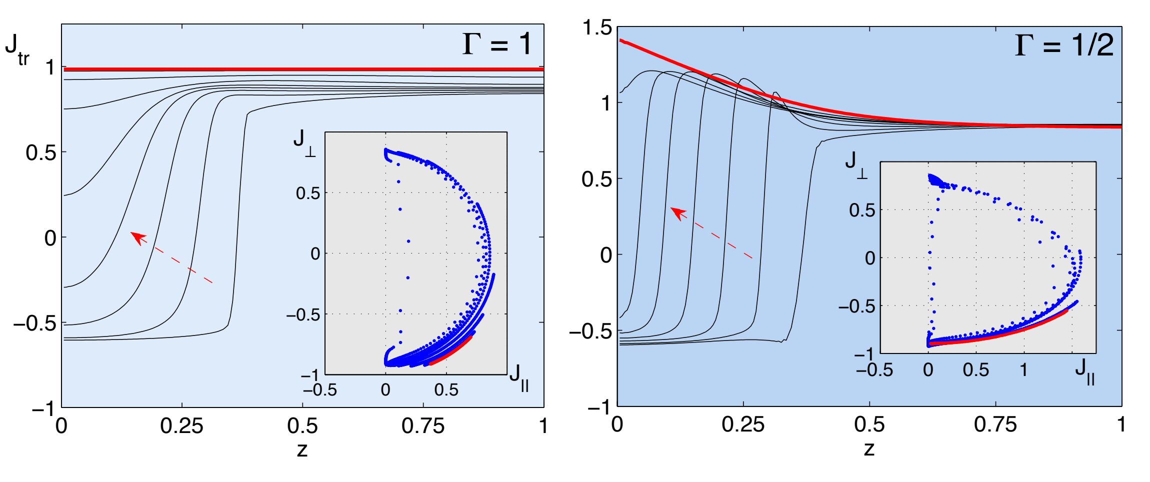
Model B: the current flow ($2\alpha = 2\beta = 45°$)



Anisotropic ⇒ inhomogeneous

4. Conclusions

⋆ **Elliptic yield region of current density $J_{\perp}(J_{\|})$**

Experimental evidence (CWDC)

The minimal physical model (unique $\mathbf{F}_p$)

⋆ **Numerical modelling: the "power-law" $\mathbf{E}(\mathbf{J})$**

Equivalent $\mathcal{F}(\mathbf{J})$ formulation fully tested

A feasible and sound form of $\mathbf{E}(\mathbf{J})$ given

Next: implementation of $\mathbf{E}(\mathbf{J})$ in FE codes . . .

Many thanks for your attention !

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