Complex Critical States in type-II Superconductors

A. Badía–Majós[1], C. López[2], and H. S. Ruiz[1]

[1] Departamento de Física de la Materia Condensada–I.C.M.A.,
C.S.I.C., Universidad de Zaragoza, SPAIN
[2] Departamento de Matemáticas, Universidad de Alcalá de Henares, SPAIN

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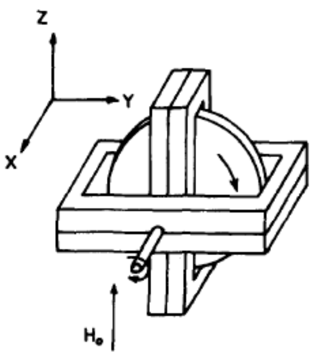
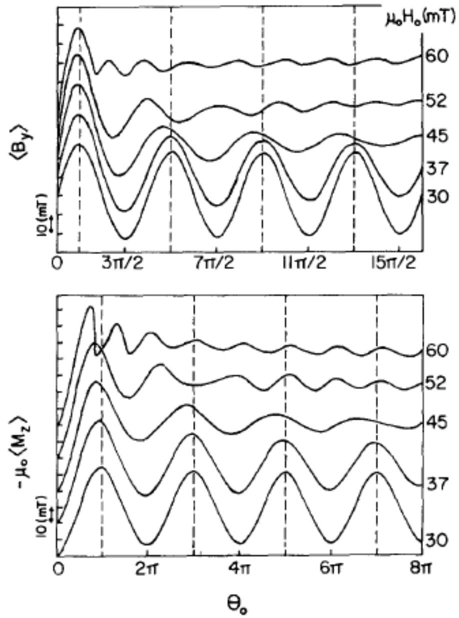
1. *Motivation* *(beyond 1D Bean's model)*

1.1. Experimental scenario 70's . . . 90's: rotation

Hysteresis losses and magnetic phenomena in rotating disks of type-II superconductors

R. Boyer, G. Fillion, and M.A.R. LeBlanc
Physics Department, University of Ottawa, Ottawa, Canada K1N 6N5





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- Variational statement
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- Conclusions

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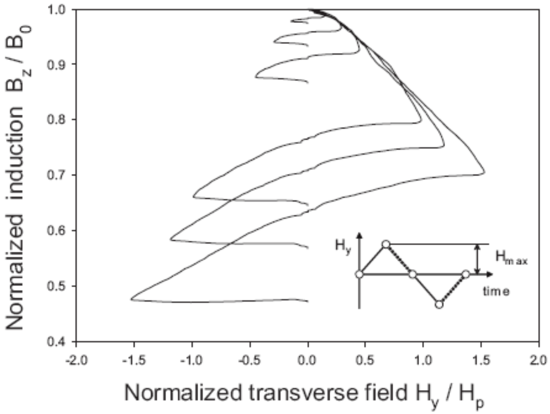
1.2. Recent experimental issues: more on crossing

PHYSICAL REVIEW B **75**, 174515 (2007)

Behavior of bulk high-temperature superconductors of finite thickness subjected to crossed magnetic fields: Experiment and model

Ph. Vanderbemden,[1] Z. Hong,[2] T. A. Coombs,[2] S. Denis,[3] M. Ausloos,[4] J. Schwartz,[5] I. B. Rutel,[5] N. Hari Babu,[6] D. A. Cardwell,[6] and A. M. Campbell[6]



2. *Contents*

- Variational statement of the critical state
  $\longrightarrow$ MQS approximation for conductors
  $\longrightarrow$ Critical state conduction law
- Results: double critical state model
  $\longrightarrow$ T *(transverse)* vortex shaking
  $\longrightarrow$ T states vs CT states
  $\longrightarrow$ smooth DCSM
- Conclusions

3. Variational statement

3.1. MQS approximation for conductors

$$\texttt{Minimize} \quad \mathcal{C} \equiv \frac{\mu_0}{2} \int_{\mathbb{R}^3} \|\vec{H}_{\mathrm{n+1}} - \vec{H}_{\mathrm{n}}\|^2 + \frac{\Delta t}{2} \int_\Omega \int_0^J E(J)$$

$$\Updownarrow$$

$$\texttt{Min} \quad \int_\Omega \int_\Omega \left[\frac{\vec{J}_{\mathrm{n+1}}(\vec{x}) \cdot \vec{J}_{\mathrm{n+1}}(\vec{x}\,')}{\|\vec{x} - \vec{x}\,'\|} - 2 \frac{\vec{J}_{\mathrm{n}}(\vec{x}) \cdot \vec{J}_{\mathrm{n+1}}(\vec{x}\,')}{\|\vec{x} - \vec{x}\,'\|} \right] + \frac{8\pi}{\mu_0} \int_\Omega \left(\vec{A}_{\mathrm{e,n+1}} - \vec{A}_{rme,n} \right) \cdot \vec{J}_{\mathrm{n+1}} + \frac{4\pi \Delta t}{\mu_0} \int_\Omega \int_0^J E(J)$$

3.2. *Critical state conduction law*

$$\texttt{Min}\quad \int_\Omega\int_\Omega \left[\frac{\vec{J}_{\rm n+1}(\vec{x})\cdot\vec{J}_{\rm n+1}(\vec{x}\,')}{\|\vec{x}-\vec{x}\,'\|} - 2\frac{\vec{J}_{\rm n}(\vec{x})\cdot\vec{J}_{\rm n+1}(\vec{x}\,')}{\|\vec{x}-\vec{x}\,'\|}\right]$$

$$+\frac{8\pi}{\mu_0}\int_\Omega \left(\vec{A}_{\rm e,n+1}-\vec{A}_{\rm e,n}\right)\cdot\vec{J}_{\rm n+1}$$

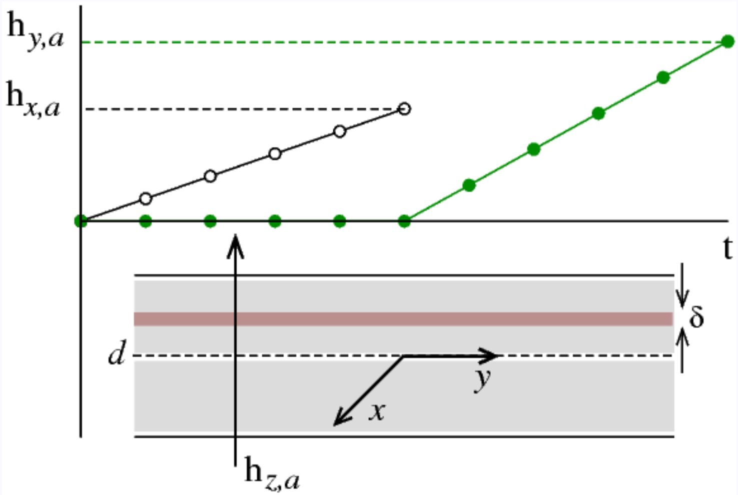
$$\texttt{for}\quad \|\vec{J}\| \le J_c \quad (1D)$$

$$\Downarrow$$

$$\vec{J}\in\partial\Delta$$

GENERAL CRITICAL state

3.3. Numerical statement



Discretize by current layers

$$\boxed{\begin{aligned} \xi_\ell &\equiv J_{x,\ell}\,\delta \\ \psi_\ell &\equiv J_{y,\ell}\,\delta \end{aligned}} \quad \Rightarrow \quad \begin{aligned} h_{x,\ell} &= \delta\left(\sum_{m>\ell}\psi_m + \psi_\ell/2\right) \\ h_{y,\ell} &= \delta\left(\sum_{m>\ell}\xi_m + \xi_\ell/2\right) \end{aligned}$$

Discretized variational principle

$$\text{Min} \quad \frac{1}{2}\sum_{i,j}\xi_{i,\mathrm{n}+1}M^x_{ij}\,\xi_{j,\mathrm{n}+1} - \sum_{i,j}\xi_{i,\mathrm{n}}M^x_{ij}\,\xi_{j,\mathrm{n}+1}$$

$$+\frac{1}{2}\sum_{i,j}\psi_{i,\mathrm{n}+1}M^y_{ij}\,\psi_{j,\mathrm{n}+1} - \sum_{i,j}\psi_{i,\mathrm{n}}M^y_{ij}\,\psi_{j,\mathrm{n}+1}$$

$$+\mu_0\sum_i \xi_{i,\mathrm{n}+1}(h_{x0,\mathrm{n}+1}-h_{x0,\mathrm{n}})$$

$$+\mu_0\sum_i \psi_{i,\mathrm{n}+1}(h_{y0,\mathrm{n}+1}-h_{y0,\mathrm{n}})$$

$$\text{for} \quad (1-h^2_{x,i})\xi_i^2 + (1-h^2_{y,i})\psi_i^2 - 2h_{x,i}h_{y,i}\,\xi_i\psi_i \le j^2_{c\perp}$$

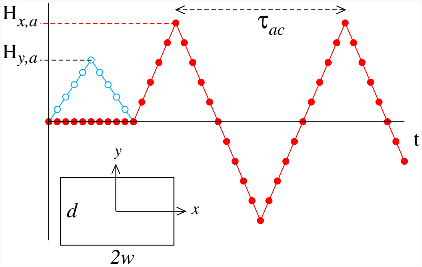
$$\text{and} \quad h^2_{x,i}\,\xi_i^2 + h^2_{y,i}\,\psi_i^2 + 2h_{x,i}h_{y,i}\,\xi_i\psi_i \le j^2_{c\parallel}$$

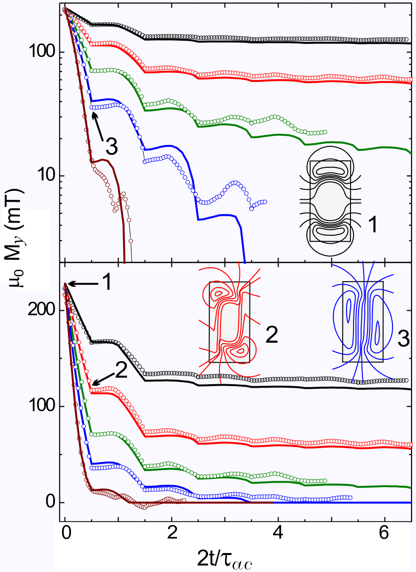
$$M^x_{ij} = M^y_{ij} \equiv 1 + 2\left[\min\{i,j\}\right]$$

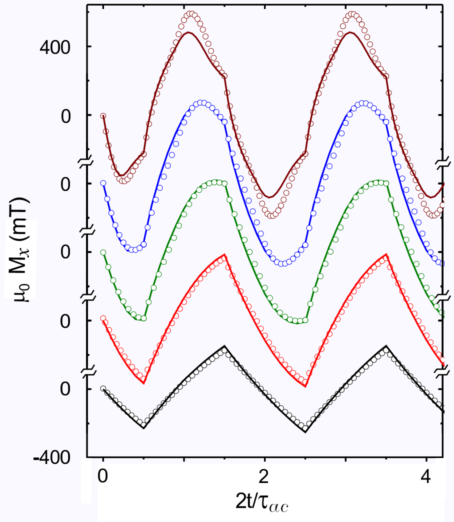
$$M^x_{ii} = M^y_{ii} \equiv 2\left(\frac{1}{4}+i-1\right)$$

4. Results

4.1. "T" vortex shaking in thick strips (Luzuriaga 2009)

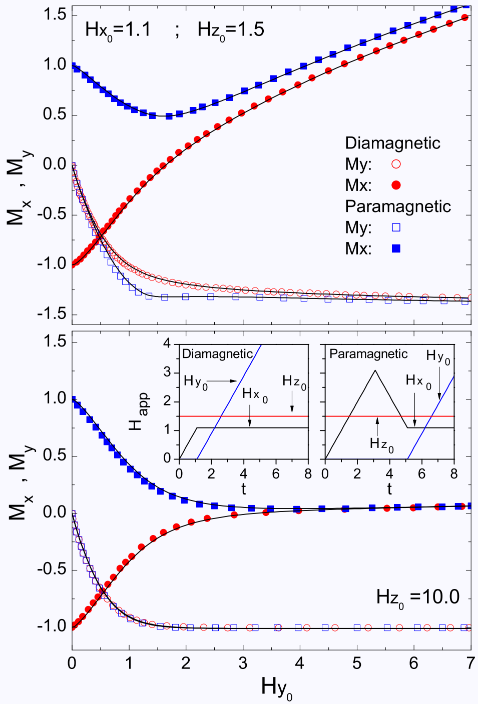




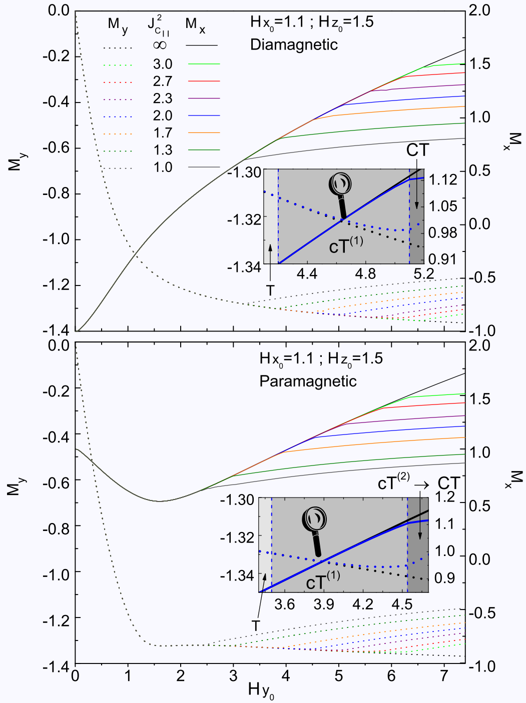


4.2. "T" states vs "CT" states (submitted)

T states (vs Brandt 2007)



T → CT transition



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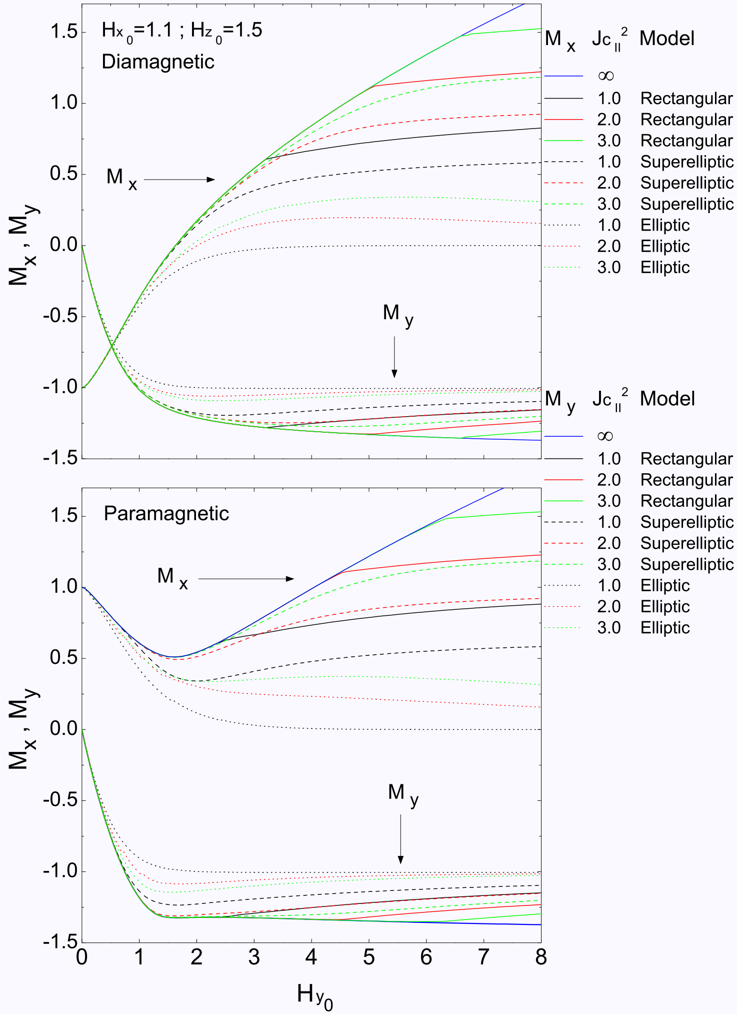
4.3. Smoothing of the CS model (submitted)



5. *Conclusions*

- The *General Critical State* problem in type-II superconductivity may be posed as a quadratic minimization problem.

- The variational statement admits a numerical treatment that can incorporate 3D effects for *realistic* conduction laws $\vec{J} \in \Delta$, related to the vortex interactions (flux pinning and cutting).

- The relative importance and interplay of flux depinning and flux cutting losses is quantified through the influence of the region Δ on the magnetic properties of the superconductor.