Flux transport in superconducting materials

Guide to macroscopic physics

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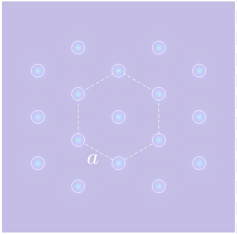
Outline

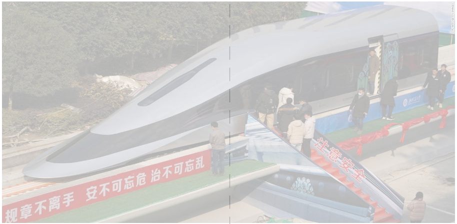
1. A quick tour through superconductivity
2. Superconducting material law (macroscopic)
3. Problem 1: demagnetisation
4. Problem 2: relaxation effects
5. Problem 3: magnetic levitation

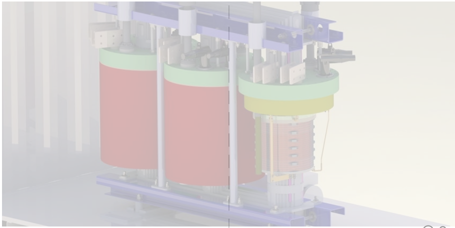
Directions about using this document

$$\mathbf{B} = \langle \mathbf{b} \rangle$$

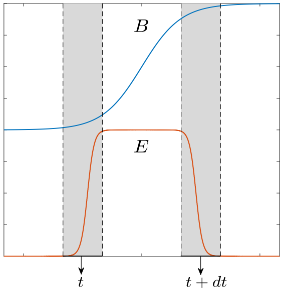


Quick tour through SC





LOW FREQUENCY ELECTRODYNAMICS (MQS)



$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} \qquad \text{(Ampere's law)}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \qquad \text{(Faraday's law)}$$

$$\nabla \cdot \mathbf{B} = 0 \qquad \text{(solenoidality of } \mathbf{B}\text{)}$$

$$\mathbf{B} \approx \mu_0 \mathbf{H} \qquad \text{(. material laws .)}$$

RANGE OF APPLICATION OF THE BULK-MQS-MODELLING

Property	Typical range	YBaCuO
Temperature	$T < 0.8\, T_c$	77 K
Applied magnetic field	$Hc_1 \ll H \ll Hc_2$	$\simeq$1 T
Sample dimensions	$L > 100\mu$ m	$\lambda \simeq 100nm$
Frequency	$\nu < 1$ KHz	$\leftarrow$

Energy storage and energy losses

- Energy stored

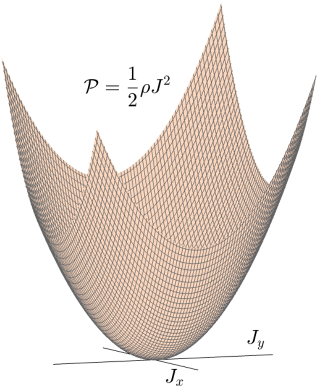
$$U_{\mathrm{M}} = \frac{1}{2\mu_0}\int_{\mathbb{R}^3} B^2 dV$$

$$= \frac{\mu_0}{8\pi}\int_V\int_{V'}\frac{\mathbf{J}(\mathbf{r})\cdot\mathbf{J}(\mathbf{r}')}{\|\mathbf{r}-\mathbf{r}'\|}dVdV'$$

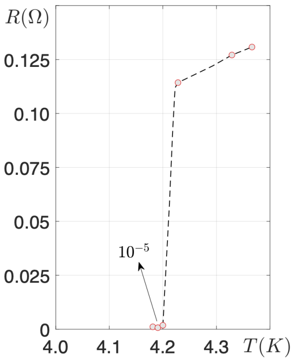
$$= \frac{1}{2}\sum_{ij} I_i M_{ij} I_j$$

- Energy losses.

$$W_{\mathrm{LOSS}} = \int\int 2\mathcal{P}\,dV\,dt$$

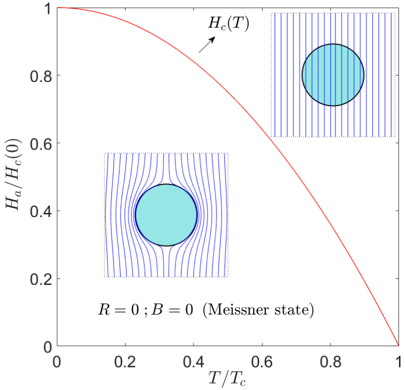


Essential superconductivity (London equations)



Perfect conductivity..

$$\mathbf{E} = \frac{\partial}{\partial t}\left(\Lambda_{\rm L}\mathbf{J}\right)$$



Expulsion of magnetic flux..

$$\boldsymbol{\nabla}\times\Lambda_{\rm L}\mathbf{J} = -\mathbf{B}$$

FROM CLASSICAL TO QUANTUM

- .Unified London equations.

$$\mathbf{J} = -\mathbf{A}/\Lambda_{\mathrm{L}} + \nabla\chi/\Lambda_{\mathrm{L}}$$

vector potential — gauge function

- .Ginzburg-Landau free energy.

$$\mathcal{F}_{\mathrm{S}} = \mathcal{F}_{\mathrm{N0}}(T) + \alpha|\Psi|^2 + \frac{\beta}{2}|\Psi|^4 + \xi^2|\alpha|\left(\nabla\sqrt{|\Psi|}\right)^2 + \frac{\mu_0\lambda_{\mathrm{L}}^2}{2}J^2 + \frac{B^2}{2\mu_0}$$

superconducting condensation — kinetics of carriers

$$\Psi = |\Psi|\mathrm{e}^{i\theta} \quad ; \quad \theta = \frac{q}{\hbar}\chi$$ (.complex order parameter.)

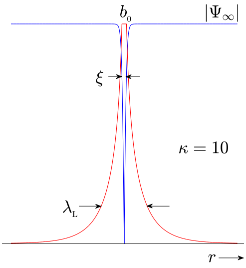
$$\alpha/\beta = -|\Psi_\infty|^2$$

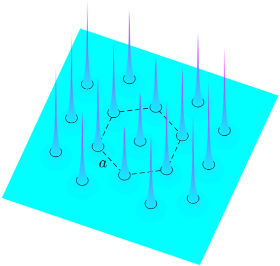
Flux vortices

- Magnetic flux is quantized

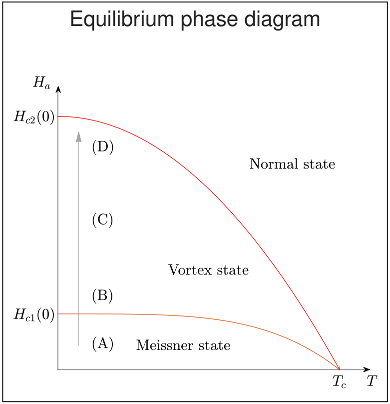
$$\Lambda_{\rm L}\oint_{\mathcal{C}}\mathbf{J}\cdot d\boldsymbol{\ell}+\iint_{\mathcal{S}}\mathbf{B}\cdot d\mathbf{s}=\frac{\hbar}{2e}\oint_{\mathcal{C}}\boldsymbol{\nabla}\theta\cdot d\boldsymbol{\ell}=n\pi\frac{\hbar}{e}\equiv n\,\Phi_0$$

- In type-II materials, this gives way to the Flux Line Lattice..

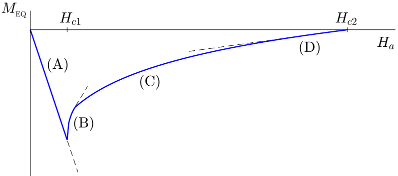




TYPE-II SUPERCONDUCTORS (EQUILIBRIUM)



$$\mathbf{M}_{\mathrm{EQ}} = \frac{\langle \mathbf{B} \rangle}{\mu_0} - \mathbf{H}_a$$



TYPE-II SUPERCONDUCTORS (DISSIPATION/METASTABILITY)

- *Transport* currents imply vortex drift

$$\mathbf{f}_i = \mathbf{J}_{\mathrm{T}} \times \Phi_0 \hat{k}$$

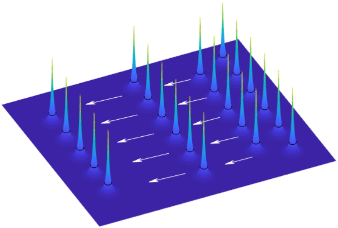
$$\Downarrow$$

$$\frac{d\mathcal{F}_{\mathrm{SN}}}{dt} + \frac{d\mathcal{F}_{\mathrm{N}}}{dt} + \frac{d\mathcal{F}_{\mathrm{EM}}}{dt} = - \; W \; - \; \mathrm{div}\, \mathcal{J}_E$$

irrev. losses

$$W = \sigma_{\mathrm{F}}\, \mathbf{e}^2 + \gamma|(-\mathrm{i}\partial_t - \Phi)\Psi|^2 \, ..$$

Joule



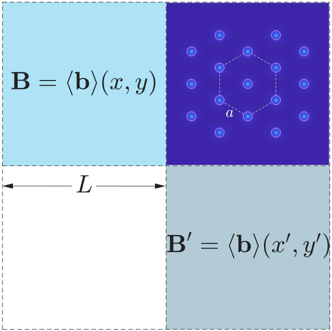
- Sustainable by flux *pinning* forces

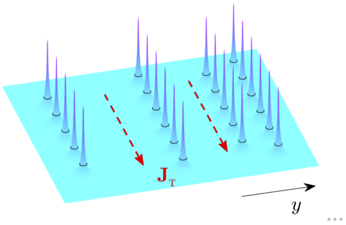
$$\mathbf{f}_i + \mathbf{f}_P = 0$$

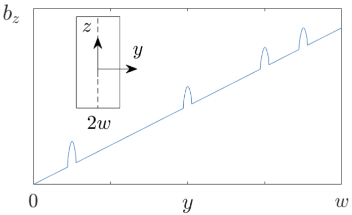
MACROSCOPIC VIEW OF THE FLUX TRANSPORT PROBLEM

$\langle \mathbf{B} \rangle$ implies very different length scales

$L \approx 1\text{mm}$, $a \approx 100$ nm







$$\mathbf{J} \times \mathbf{B} + \mathbf{P} = 0$$

(B.D. Josephson)

Pinning “withholding” forces

Superconducting material law

Min $\frac{1}{2}\sum_{i,j}\xi_{i,n+1}M^x_{ij}\,\xi_{j,n+1} - \sum_{i,j}\xi_{i,n}M^x_{ij}\,\xi_{j,n+1}$

$+\ \frac{1}{2}\sum_{i,j}\psi_{i,n+1}M^y_{ij}\,\psi_{j,n+1} - \sum_{i,j}\psi_{i,n}M^y_{ij}\,\psi_{j,n+1}$

$+\ \mu_0\sum_i \xi_{i,n+1}(h_{x0,n+1} - h_{x0,n})$

$+\ \mu_0\sum_i \psi_{i,n+1}(h_{y0,n+1} - h_{y0,n})$

for $(1-h^2_{x,i})\xi_i^2 + (1-h^2_{y,i})\psi_i^2 - 2h_{x,i}h_{y,i}\,\xi_i\psi_i \le j^2_{c\perp}$

and $h^2_{x,i}\,\xi_i^2 + h^2_{y,i}\,\psi_i^2 + 2h_{x,i}h_{y,i}\,\xi_i\psi_i \le j^2_{c\parallel}$

$$M^x_{ij} = M^y_{ij} \equiv 1 + 2\,[\min\{i,j\}]$$

$$M^x_{ii} = M^y_{ii} \equiv 2\left(\frac{1}{4} + i - 1\right)$$

STATEMENT OF THE PROBLEM

- Maxwell equations

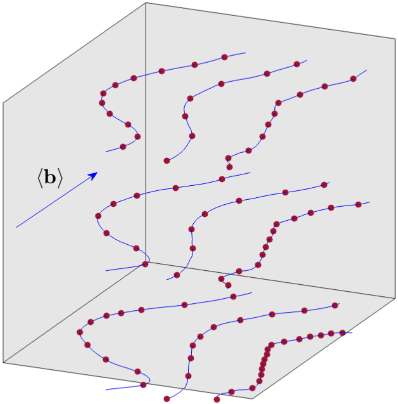
$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \cdot \mathbf{B} = \mathbf{0}$$

- Material law

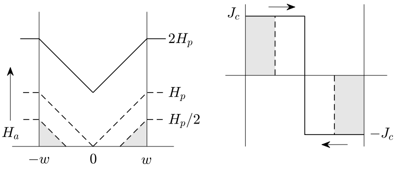
$$\mathbf{E} = \mathbf{E}(\mathbf{J}, \mathbf{B})$$



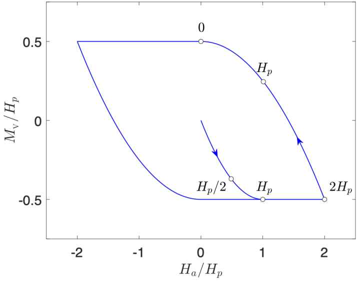
Simplest solution: Bean's model (CSM)

- Infinite slab $|x| \leq w$ in parallel field $(0, 0, \mu_0 H_a)$ ..

$$\frac{dB}{dx} = \pm\mu_0 J_c(B) \quad \text{or} \quad 0$$



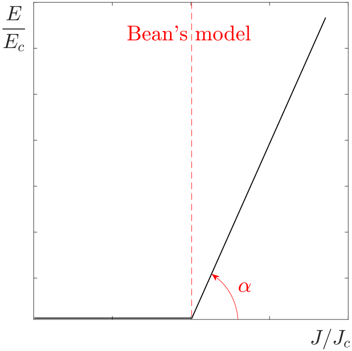
$$M_V = \frac{\langle B \rangle}{\mu_0} - H_a$$



- Provides physical interpretation
- May be used to .characterise. the sample $\Delta M_V = J_c w$

Providing background for the CSM

- A singular $\{\mathbf{E},\mathbf{J}\}$ law

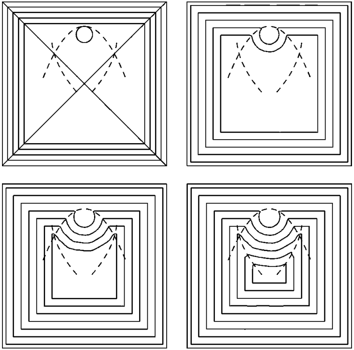


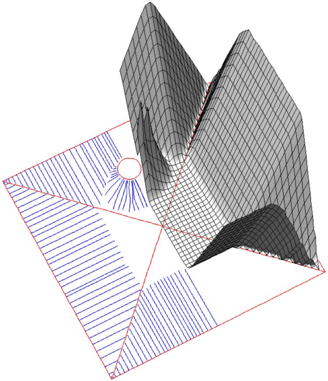
$\star$ Bean's limit : $\alpha \to \pi/2 \iff \begin{cases} \rho \to 0, & J \leq J_c \\ \rho \to \infty, & J > J_c \end{cases}$

SIDE BENEFITS OF GENERALISING THE CSM PICTURE

- Flux penetration in long cylinder of square section with a hole ..

$$\mathbf{H}_a \parallel \hat{k}$$





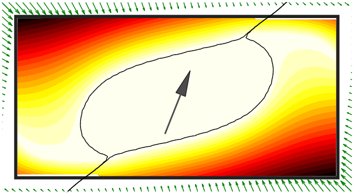
Current density streamlines

Flux paths and .induced. electric field

SIDE BENEFITS OF GENERALISING THE CSM PICTURE

- Flux penetration in long cylinder of rectangular cross section

$$\mathbf{H}_a \perp \hat{k}$$

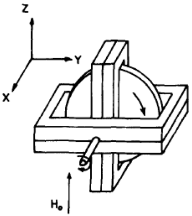


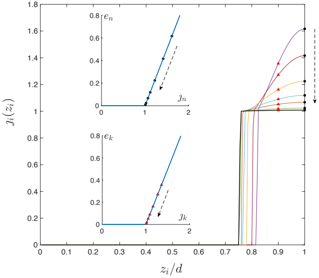
Flux paths and density of generated heat

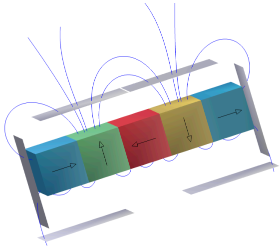
$$W_{\text{LOSS}} = \int \int \mathbf{E} \cdot \mathbf{J} \, dV \, dt$$

SHORTCOMINGS OF BEAN'S MODEL

- New physical phenomena (rotation experiments .., crossed fields 🔗)
- Finite resistivity (time relaxation)
- Finite size effects, non-uniform fields







Generalising Bean's model (I)

- The evolutionary statement

$$\texttt{Minimize} \quad \mathcal{C} \equiv \frac{1}{2\mu_0}\int_{\mathbf{R}^3} \|\mathbf{B}_{n+1} - \mathbf{B}_n\|^2 + \Delta t \int_\Omega \mathcal{P}[J]$$

$$\Updownarrow$$

$$\mathcal{C}[\mathbf{J}_{n+1}] = \int_V d^3\mathbf{r} \int_V d^3\mathbf{r}' \left[\frac{\mathbf{J}_{n+1}(\mathbf{r}) \cdot \mathbf{J}_{n+1}(\mathbf{r}')}{\|\mathbf{r} - \mathbf{r}'\|} - 2\frac{\mathbf{J}_n(\mathbf{r}) \cdot \mathbf{J}_{n+1}(\mathbf{r}')}{\|\mathbf{r} - \mathbf{r}'\|} \right]$$

self-interaction

$$+ \frac{8\pi}{\mu_0} \int_V d^3\mathbf{r}\, (\mathbf{A}_{e,n+1} - \mathbf{A}_{e,n}) \cdot \mathbf{J}_{n+1} + \frac{4\pi\Delta t}{\mu_0} \int_V d^3\mathbf{r}\, \mathcal{P}(J_{\|,n+1}, J_{\perp,n+1})$$

interaction with EM sources

interaction with thermal modes

GENERALISING BEAN'S MODEL (II)

- Re-interpretation of Ohm's law ..

$$\texttt{Minimize} \quad \mathcal{C}$$

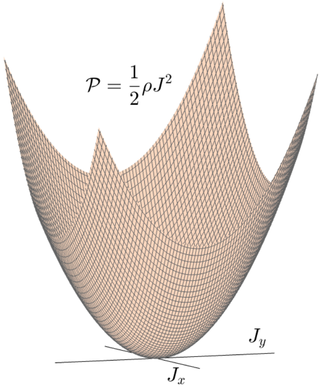
$$\Downarrow$$

$$\mu_0 \frac{\mathbf{B}_{\mathrm{n+1}} - \mathbf{B}_{\mathrm{n}}}{\Delta t} = -\rho \nabla \times \nabla \times \mathbf{B}_{\mathrm{n+1}} = \rho \nabla^2 \mathbf{B}_{\mathrm{n+1}}$$

$$\updownarrow$$

$$\mu_0 \frac{\partial \mathbf{B}}{\partial t} = \rho \nabla^2 \mathbf{B}$$

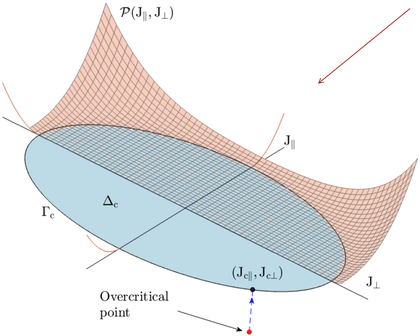
Diffusion equation in normal conductors



Generalising Bean's model (III)

$$\text{Min}\left\{\mathcal{C}[\mathbf{B}_{n+1}] \equiv \frac{1}{2\mu_0}\int_{\mathbb{R}^3} \|\mathbf{B}_{n+1} - \mathbf{B}_n\|^2 + \Delta t \int_{\Omega} \mathcal{P}[J]\right\}$$

Inertial term: Faraday's — Thermal loss



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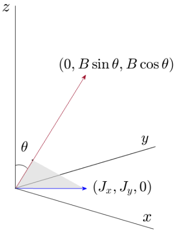
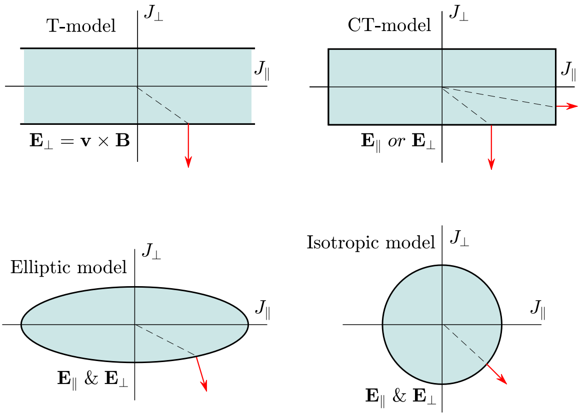


$$\mathbf{J}_\parallel = (\mathbf{J}\cdot\hat{\mathbf{B}})\hat{\mathbf{B}}$$

$$\mathbf{J}_\perp = \mathbf{J} - \mathbf{J}_\parallel$$

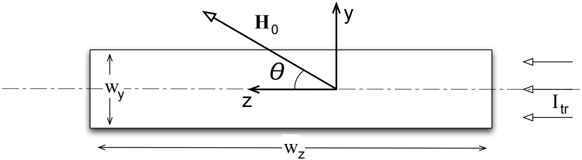
GENERALISING BEAN'S MODEL (IV)

$$\nabla \times \mathbf{B} = \underbrace{\nabla B \times \hat{\mathbf{B}}}_{J_\perp} + \underbrace{B\,\nabla \times \hat{\mathbf{B}}}_{J_\parallel,\,J_\perp}$$

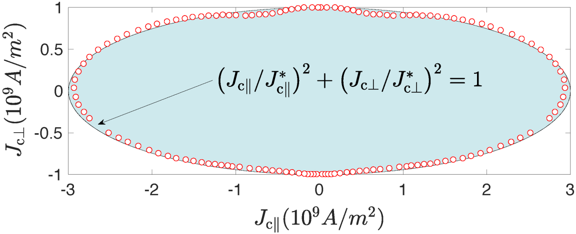


THE EXPERIMENTAL DISSIPATION FUNCTION (YBCO)

- A smart experiment. ($W_z \gg W_y \Rightarrow \theta$ is well defined !)



- Experimental data by courtesy of A. M. Campbell



SUPERCONDUCTING MATERIAL LAW: APPLICATION

- Circuital interpretation (FEM)

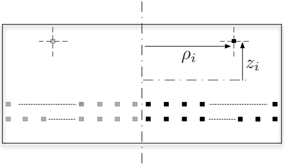
$$\mathcal{C} \equiv \mathcal{C}_{\rm JJ}+\mathcal{C}_{\rm J0}+\mathcal{C}_{\rm JS}+\Delta t\,\mathcal{W}_{\rm JE} = \left[\frac{1}{2}\langle \mathrm{J}|\,\mathrm{m}\,|\mathrm{J}\rangle-\langle \mathrm{J}^{\vee}|\,\mathrm{m}\,|\mathrm{J}\rangle+\langle\Delta\psi_{\rm S}|\mathrm{J}\rangle+\Delta t\,\mathcal{W}_{\rm JE}\right]$$

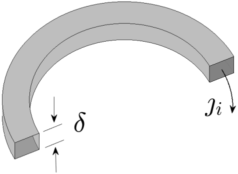
$\mathcal{C}_{\rm JJ}$: self energy of the evolutionary circulating currents

$\mathcal{C}_{\rm J0}$: interaction energy of the evolutionary currents with a “frozen” distribution

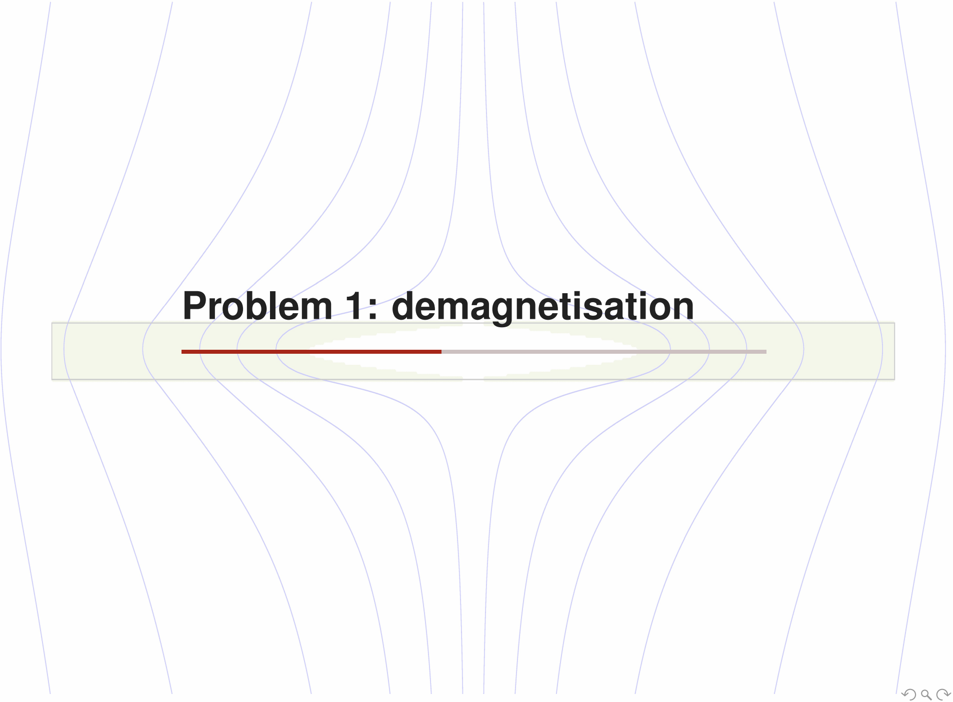
$\mathcal{C}_{\rm JS}$: interaction energy of the evolutionary currents with the magnetic source

$\Delta t\,\mathcal{W}_{\rm JE}$: energy related to the entropy production due to dissipative mechanisms





Problem 1: demagnetisation



Demag in Meissner state (I)

- Ideal susceptibility (χ_{ideal})

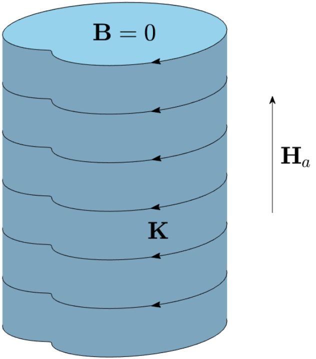
$$\mathbf{B}_{\text{full}} = \mu_0 H_a \hat{\boldsymbol{k}} + \mu_0 K \hat{\boldsymbol{k}} = 0 \implies K = -H_a$$

$$\Downarrow$$

$$M_{\text{V}} = \frac{m}{\text{vol}} = \frac{K \cdot L \cdot A}{A \cdot L} = K = -H_a$$

$$\updownarrow$$

$$\chi_{\text{ideal}} = \frac{M_{\text{V}}}{H_a} = -1$$



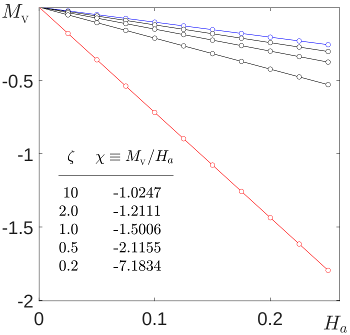
Demag in Meissner state (II)

- Demagnetising factor (ellipsoids in parallel field, $\zeta \equiv c/a$)

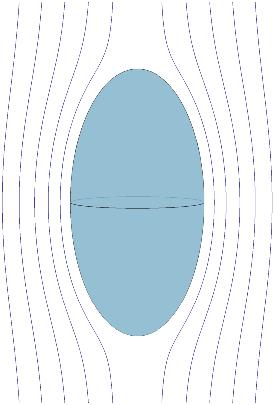
$$M_{\mathrm{V}} = \chi_{\mathrm{ideal}}(H_a + H_{\mathrm{d}}) = \chi_{\mathrm{ideal}}(H_a - N M_{\mathrm{V}})$$

$$\Downarrow$$

$$\chi = \frac{M_{\mathrm{V}}}{H_a} = \frac{\chi_{\mathrm{ideal}}}{1 + N\chi_{\mathrm{ideal}}} = \frac{-1}{1-N}$$

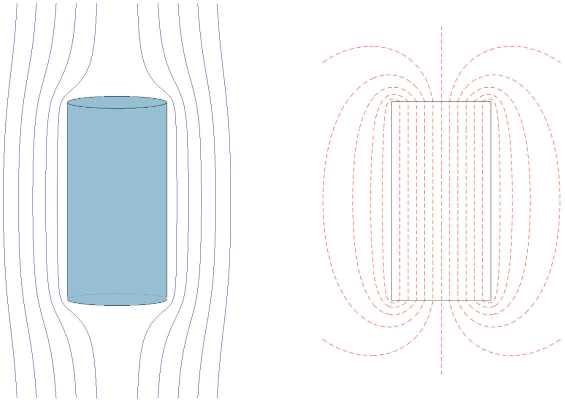


ζ	$\chi \equiv M_{\mathrm{V}}/H_a$
10	-1.0247
2.0	-1.2111
1.0	-1.5006
0.5	-2.1155
0.2	-7.1834



Demag in Meissner state (III)

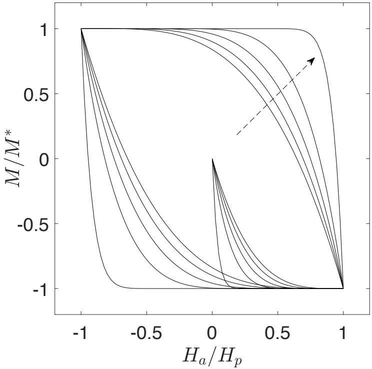
- Cylindrical symmetry

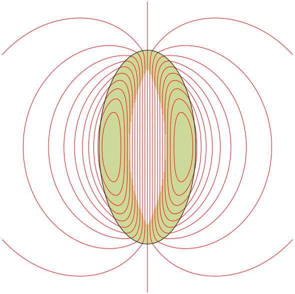


$$\min\ U = \frac{1}{2}\langle K|\, m\, |K\rangle + \langle \psi_{Appl}|K\rangle \Rightarrow |K\rangle = m^{-1}\,|\psi_{Appl}\rangle$$

CRITICAL STATE OF FINITE SAMPLES (I)

- Ellipsoidal symmetry

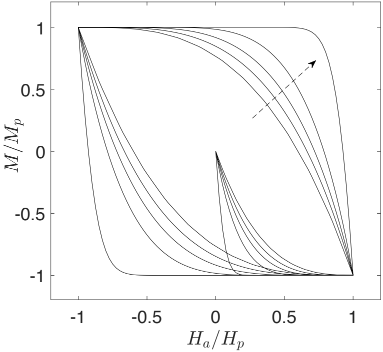


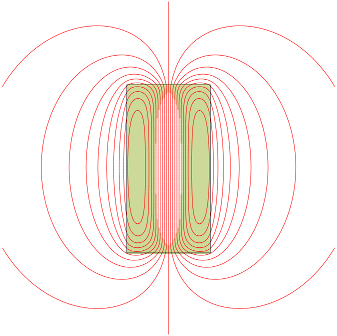


$$\zeta = 10, 2, 5, 0, 5, 0, 1 \quad ; \quad H_p = J_c\, a \quad ; \quad M^* = \frac{3\pi J_c a}{32} \,..$$

Critical state of finite samples (II)

- Cylindrical symmetry

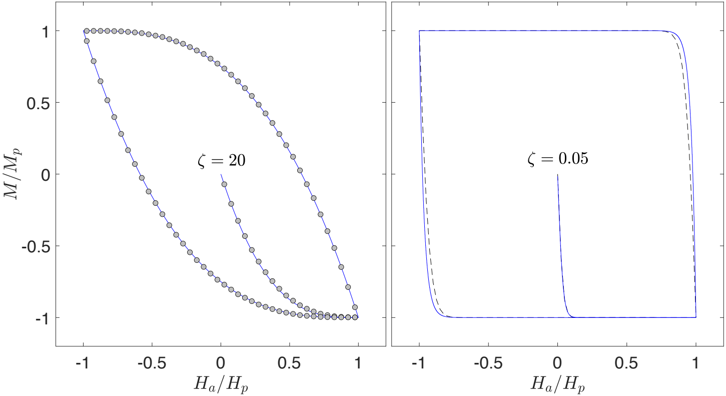




$$\zeta = 10, 2, 5, 0, 5, 0, 1 \quad ; \quad H_p = J_c\, R \quad ; \quad M_p = \frac{J_c R}{3}$$

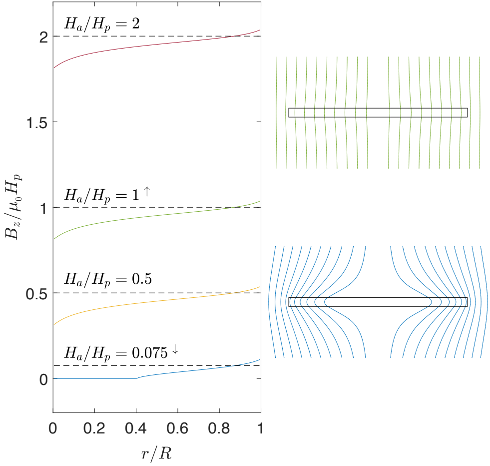
CRITICAL STATE IN EXTREME GEOMETRIES

- Cylindrical symmetry



.Bean. .Mikheenko.

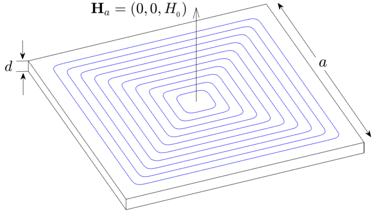
THE INTERNAL MAGNETIC FIELD: $J_c(B)$ VS $J_c(\mu_0 H_a)$

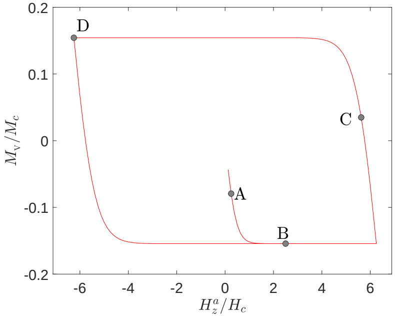


.The thinner the sample the better $\Delta M_V(B) \approx \Delta M_V(\mu_0 H_a)$

Flat samples: forward CSM problem (I)

- The stream function method (σ) ..



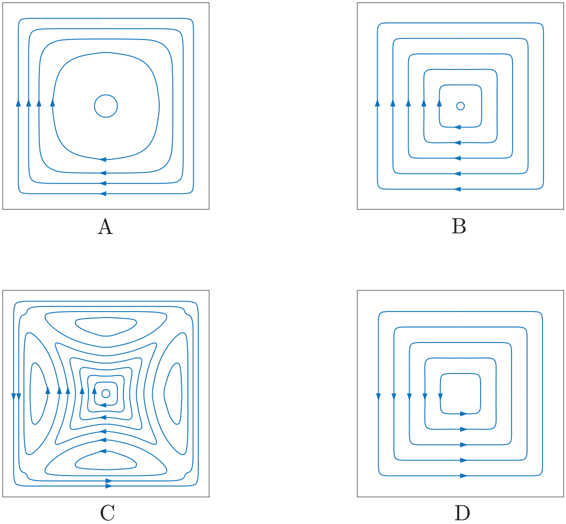


$$\mathbf{K}(x,y) = \int_{-d/2}^{d/2} \mathbf{J}\,(x,y,z)dz \equiv -\hat{z} \times \boldsymbol{\nabla}\sigma$$

...

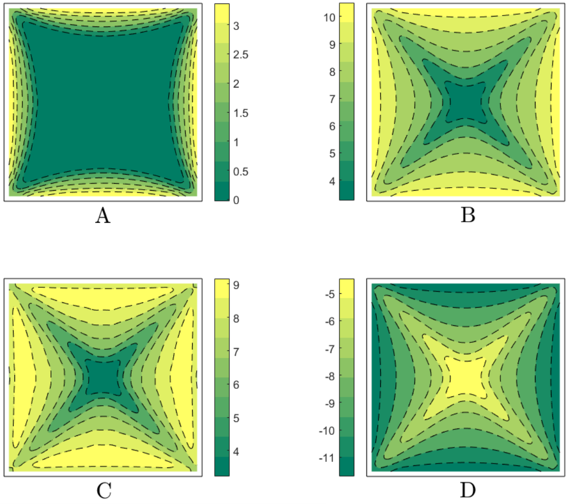
Flat samples: forward CSM problem (II)

- Current density streamlines



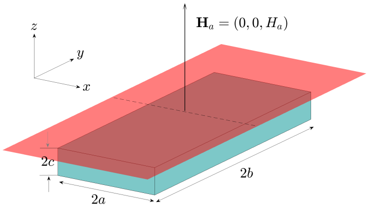
FLAT SAMPLES: FORWARD CSM PROBLEM (III)

- On-surface flux pattern: $B_z(x, y)$ ..

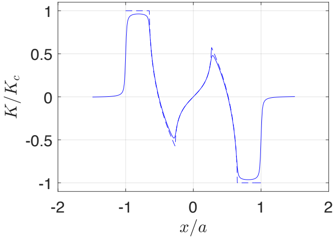


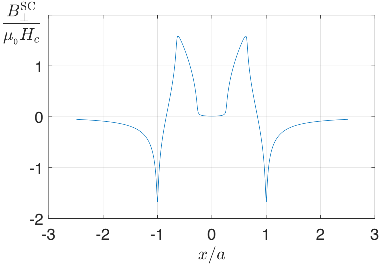
Flat samples: inverse CSM problem

- Reconstruction of the current density profile (magneto-optics)

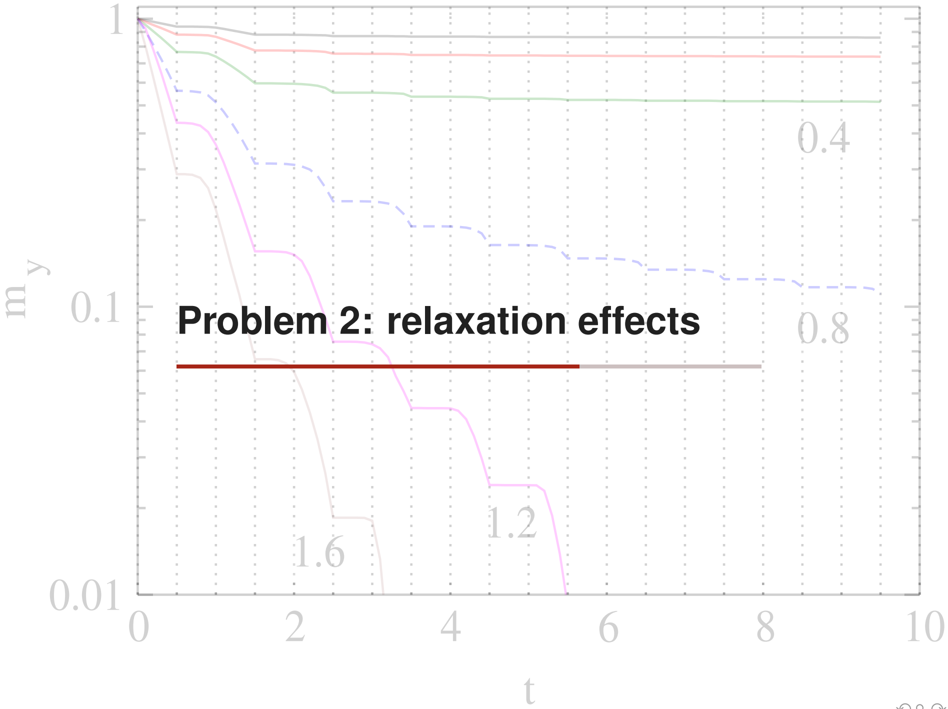


$$|B_z^{sc}\rangle = Z\,|K\rangle \quad \overset{??}{\Longrightarrow} \quad |K\rangle = R\,|B_z^{sc}\rangle$$





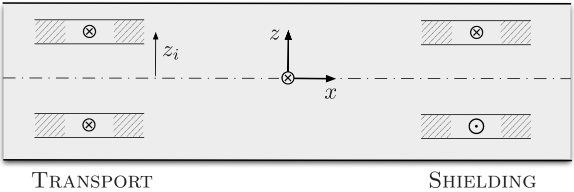
Problem 2: relaxation effects



Simplified transport problem

- Statement: **J** along $y-$axis of a plate

$$|x| < \infty$$
$$|y| < \infty$$
$$|z| < d$$



$$K = \int_{-d}^{d} J_y(z)\, dz \quad \Rightarrow \quad K_c = 2J_c\, d$$

Critical state formulation (quasi-static)

- Bean model (instantaneous response)

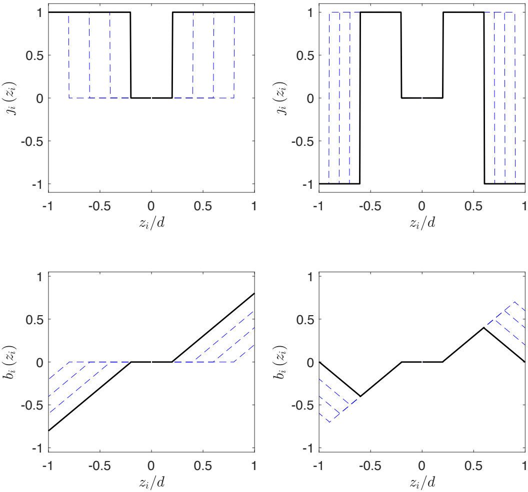
$$\texttt{minimise} \quad \left[\frac{1}{2}\langle \mathtt{J}|\,\mathtt{m}\,|\mathtt{J}\rangle - \langle \mathtt{J}^{\vee}|\,\mathtt{m}\,|\mathtt{J}\rangle + \langle \Delta\psi_{\mathrm{S}}|\mathtt{J}\rangle + \Delta t\, \mathcal{W}_{\mathrm{JE}}\right]$$

$$\downarrow$$

$$\begin{cases} \texttt{minimise} \left[\frac{1}{2}\langle \mathtt{J}|\,\mathtt{m}\,|\mathtt{J}\rangle - \langle \mathtt{J}^{\vee}|\,\mathtt{m}\,|\mathtt{J}\rangle\right] \\ \texttt{for } |\jmath_i| \leq 1 \\ \texttt{and } \sum_{i=1}^{n} \jmath_i = \frac{K}{2J_c d}\, n \end{cases}$$

CRITICAL STATE FORMULATION (QUASI-STATIC)

- Current and flux density penetration profiles (AC cycle)



Relaxation in flux-flow regime (I)

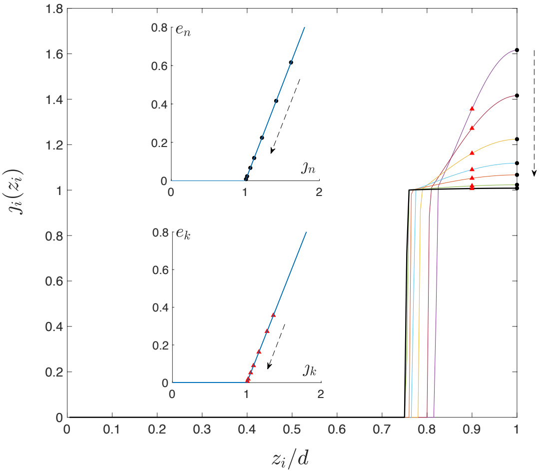
- Piece-wise linear approximation

$$\mathcal{P}(J) = \begin{cases} \rho_{\rm f}(J+J_c)^2/2 & , \quad J < -J_c \\ 0 & , \quad -J_c \le J \le J_c \\ \rho_{\rm f}(J-J_c)^2/2 & , \quad J_c < J \end{cases}$$

$$\Downarrow$$

$$\begin{cases} \texttt{minimise} \left[\frac{1}{2}\langle \mathtt{J}|\,\mathtt{m}\,|\mathtt{J}\rangle - \langle \mathtt{J}^{\vee}|\,\mathtt{m}\,|\mathtt{J}\rangle + \langle \mathtt{J}|\,\mathtt{P}\,|\mathtt{J}\rangle \mp \mathbf{2}\,\langle \mathbf{1}|\,\mathtt{P}\,|\mathtt{J}\rangle\right] \\ \texttt{for} \sum_{i=1}^{n} \jmath_i = \dfrac{K}{2J_c\,d}\,n \end{cases}$$

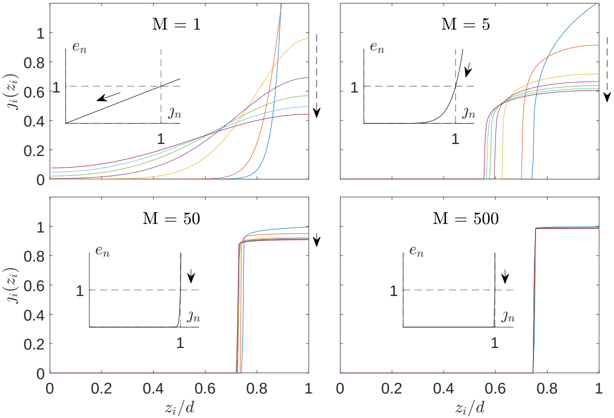
Relaxation in flux-flow regime (II)



Time-scale to establish the Critical State: $\tau = \dfrac{\mu_0 d^2}{\rho_f}$

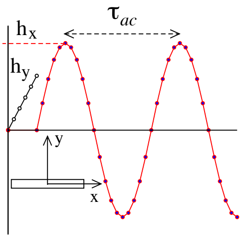
RELAXATION IN FLUX-CREEP REGIME

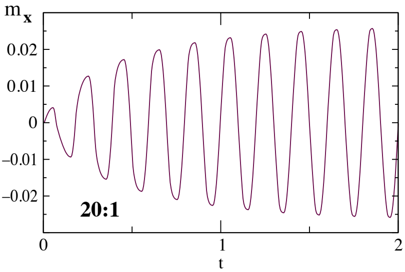
- Power-law approximation ..

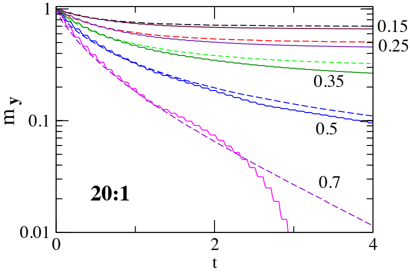


$$\mathbf{E}(\mathbf{J}) = \rho(J)\mathbf{J} \quad , \quad \rho \equiv \rho_0 \left(\frac{J}{J_c}\right)^{2M-2} \quad \Rightarrow \quad \mathcal{P} = \frac{\rho_0 J_c^2}{2M}\left(\frac{J}{J_c}\right)^{2M}$$

Relaxation? in the critical state (flux shaking) ..

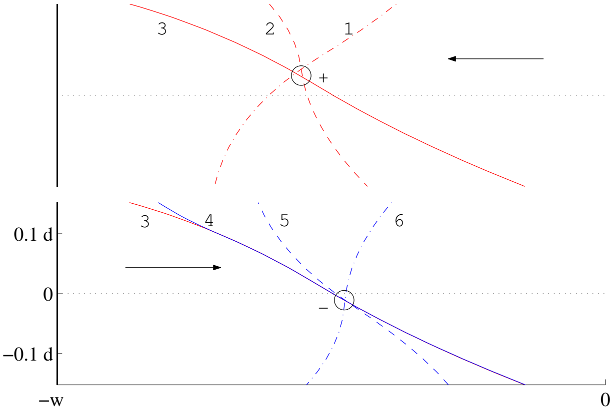




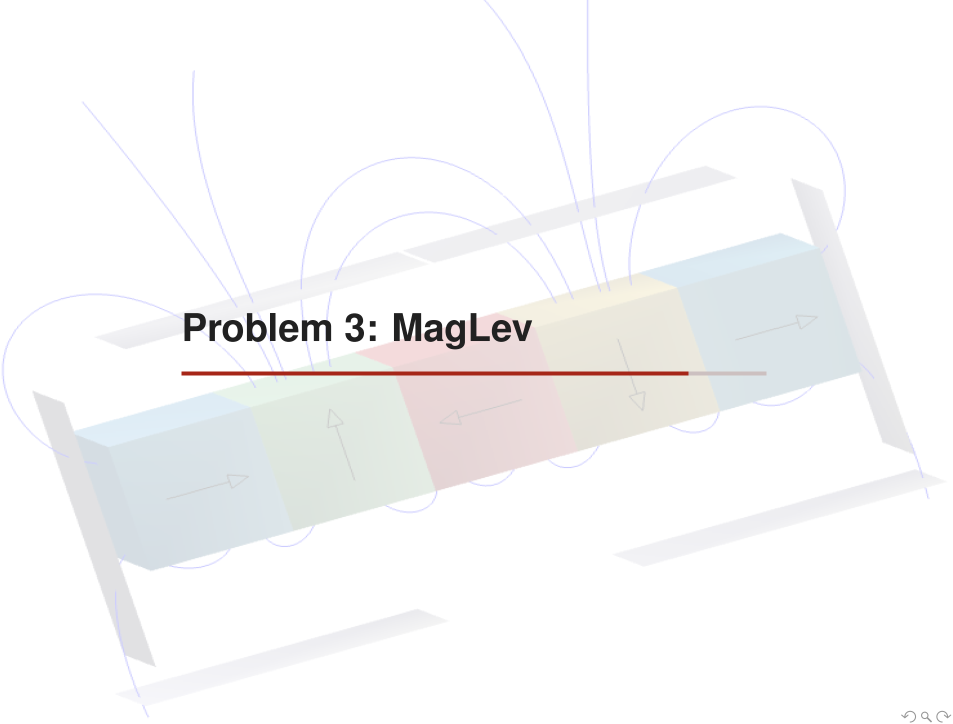


SIDEWINDING DRIFT OF VORTICES

- *Flux lines are not only pictures* (B. D. Josephson)



Problem 3: MagLev



Levitation in Meissner state (I)

- Historical note

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The second point of interest is that, before detailed classifications of rheological systems be postulated, there is still a need for determining rheological constants in different apparatus on the same systems, to ensure that a true constant of the system is obtained. Thus in this example use of the ball viscometer would lead to the conclusion that the 860 p. viscosity of the solution at 20° C. was a constant of the liquid, whereas checking with the tube viscometer shows that 860 p. is, in fact, dependent on the apparatus as well as the liquid.

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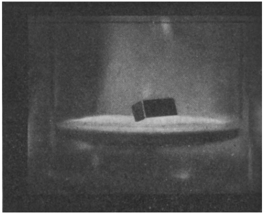
Birmingham, 15.

[1] Wood, G. F., Nissan, A. H., and Garner, F. H., *J. Inst. Petrol.*, **33**, 71 (1946).

A Floating Magnet

BY assuming that diamagnetic bodies are pushed out of a magnetic field, it may be shown that a diamagnetic particle attracted to a magnet by gravitational forces will take up a position in space in the equatorial plane of the straight magnet at a certain distance from the latter. The 'satellite' can vibrate elastically about the point of equilibrium, describing a certain curve. The period of vibration in the radial and meridional directions is close to the period of the Kepler rotation of a magnetically indifferent satellite about a body of the same mass. Several identical particles arrange themselves around the magnet. Such a combination



disk 40 mm. in diameter in a Dewar vessel over liquid helium.

The experimental test of these views was possible through the kindness of Prof. P. L. Kapitza, in the Institute of Physical Problems, Moscow.

The lower the coercive force of the magnet, the smaller the magnet itself must be. Carbon steel magnets, for example, can 'float' when they have the dimensions of 0·5 mm. × 9 mm. By scattering microscopically small magnets over the surface of a body, it is possible to reveal superconductive inclusions directly, since the magnetic particles will roll to the spots where there is no superconductivity.

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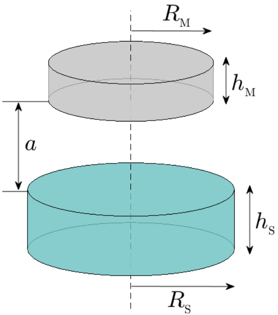
LEVITATION IN MEISSNER STATE (II)

- Analytical approximation (*method of images*)



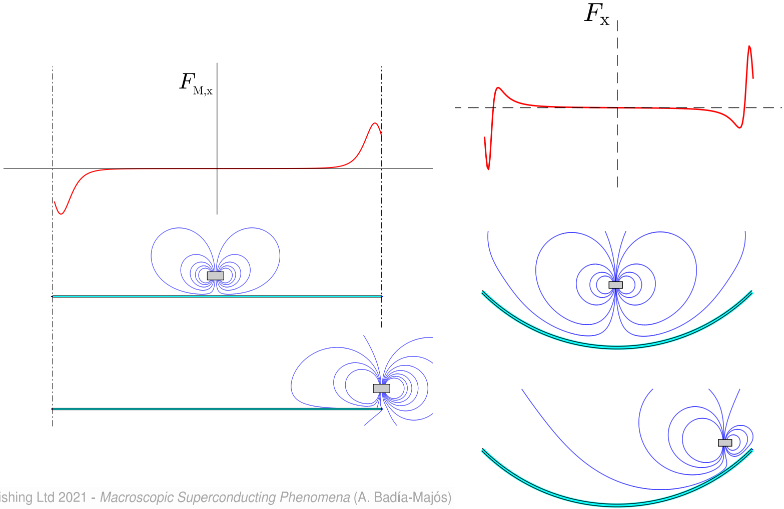
$a \ll R_S \;;\; R_M \ll R_S$; $R_M \approx a$ allowed

$$F(a) = 2\mu_0 M_0^2 R_M \left[2a_2 \frac{K(p_2) - E(p_2)}{p_2} - a_1 \frac{K(p_1) - E(p_1)}{p_1} - a_3 \frac{K(p_3) - E(p_3)}{p_3} \right]$$

....

Levitation in Meissner state (III)

- The issue of *diamagnetic* mechanical stability



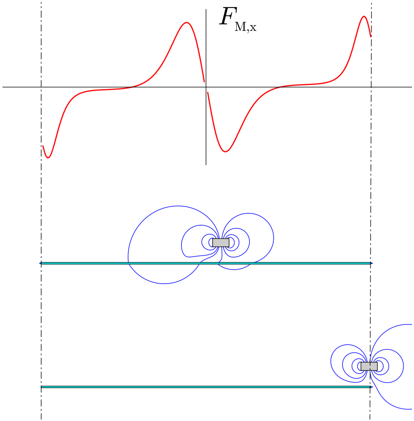
LEVITATION IN CRITICAL STATE (I)

- Self-stabilised structures

Pinned flux lines pin the magnet.

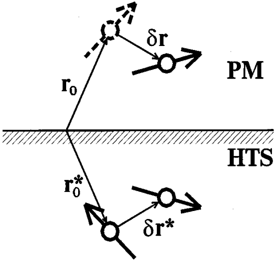
Lateral restoring (guidance) force:

$$\frac{\mathbf{F}_{\alpha j}}{\mu_0 M_0 J_c W^2} = \frac{\jmath_j}{2\pi} \frac{(x_j - x_\alpha, 0, z_j - z_\alpha)}{(x_j - x_\alpha)^2 + (z_j - z_\alpha)^2}$$



LEVITATION IN CRITICAL STATE (II)

- Analytical approximation (the *.frozen image.* method) for dipoles



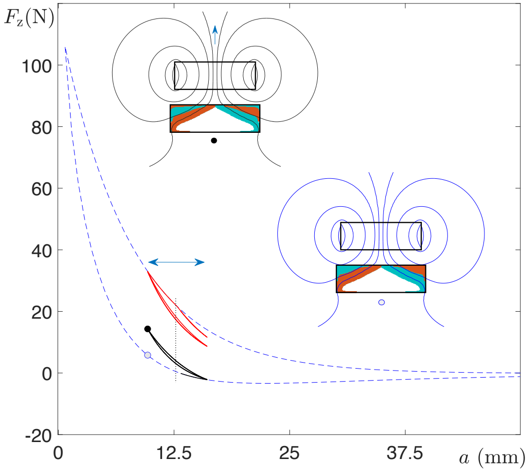
$$\mathbf{B}_{\text{images}} = \mathbf{B}_{\text{frozen}} + \mathbf{B}_{\text{active}}$$

By using $\mathbf{F} = (\mathbf{m}\nabla)\mathbf{B}_{\text{images}}$:

$$\mathbf{F} = \frac{3\mu_0 m^2}{2\pi}\left[\frac{1}{16a^4} - \frac{1}{(a+a_0)^4}\right]\hat{k}$$

Levitation in Critical State (III)

- Numerical solution. Hysteresis ....



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- Navigation and tooltips work well with Acrobat reader, mainly in Full-screen mode, function partially with other readers and with browsers like Chrome, but fail with Preview viewer and Safari.
- Exercising with situations related to Problems 1, 2 and 3 is possible through MATLAB (OCTAVE compatible) codes available as *supplementary material*. See directories *Fig9_6, Fig9_7, Fig9_8* (Problem 1), *Fig8_10* (Problem 2), and *Fig13_7_8* (Problem 3).
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