PART III: PRODUCTION THEORY

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MICROECONOMICS I

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  - Unit 1: Concept and scope of Economics
  - Unit 2: Demand, supply and market
- Part II: Demand theory
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Production theory

BASIC CONCEPTS

•Production: process of transformation from factors (inputs) to products (outputs).

This production process can affect:

- the composition of the goods
- the spatial location of the goods (transportation)
- the temporal location of the goods (storage)

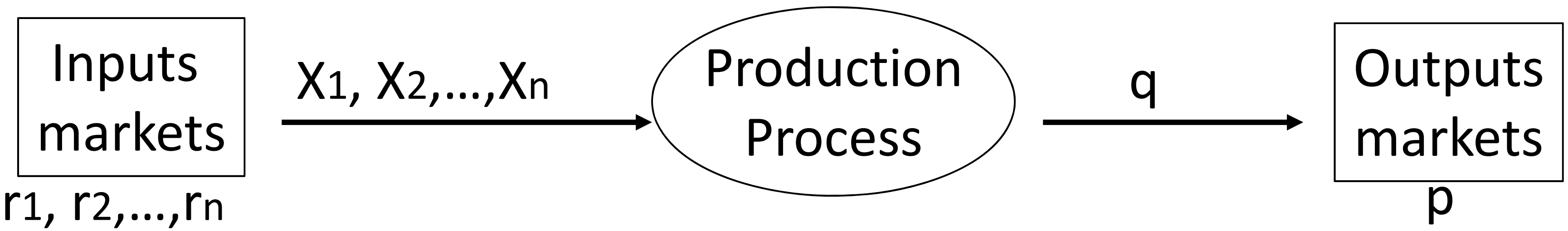
•Producer = firm = company:

Agent who transforms goods and services (production factors) into other goods and services (products), so that these new products satisfy human needs while maximizing profit.

- The final objective of the firm: PROFIT MAXIMIZATION

The producer must decide the quantity of inputs he/she wants to DEMAND (how to produce): This decision depends on:
* The technology
* The price of the inputs

The producer must decide the quantity of outputs he/she wants to SUPPLY (how much to produce): This decision depends on :
* The demand of the market
* The structure of the market (the type of competition)



(We assume that only one output is produced: SIMPLE PRODUCTION)

SHORT RUN AND LONG RUN

- In the production process there are variable inputs (which can vary) and fixed inputs (which cannot vary)

- Long run: Amount of time needed to assume that all production inputs are variable, that is to say, period of time in which the firm can vary the quantity of all inputs.
- Short Run: Period of time during which quantities of one or more production factors are fixed, but quantities of other inputs can vary.

TECHNICAL AND ECONOMIC EFFICIENCY

- A production process is technically efficient if there are no other processes that use less quantity of some inputs and no more of the rest, to obtain the same amount of output.

- A production process is economically efficient if it allows the same quantitiy of output at the lowest cost.

•Example: Let's assume a firm that produces output Q by combining two inputs L (Labour) and K (Capital). The producer knows that the process produces 10 units of output and requires the quantities of L and K according to the following table:

Production process	L	K	q
P. P. 1	3	2	10
P. P. 2	4	3	10
P. P. 3	2	3	10
P. P. 4	2	4	10
P. P. 5	4	1	10

•The firm will initially apply the criteria of technical efficiency.

Technically inefficient, as it needs more L and more K compared to P. P. 1 (P. P. 2)

Technically inefficient, as it needs equal L but more K compared to P. P. 3 (P. P. 4)

- No rational firm will ever use a production process that is technically inefficient.

Production process	L	K	q
P. P. 1	3	2	10
P. P. 2	4	3	10
P. P. 3	2	3	10
P. P. 4	2	4	10
P. P. 5	4	1	10

P. P. 1: Technically efficient, as it needs more L but less K compared to P. P. 3

P. P. 3: Technically efficient, as it needs more K but less L compared to P. P. 1

P. P. 5: Technically efficient, as it needs less K but more L compared to P. P. 1 and P. P. 3

Once the firm has selected the technically efficient production process, how does it decide which one is the most economic efficient?

The firm will apply the criteria of economic efficiency, for which it needs to know the price of the inputs: w (price of factor L) and r (price of factor K)

Production processes technically efficient	L	K	q	Production cost (w.L+ r.K) if wº=4 and rº=2	Production cost (w.L+ r.K) if w'=1 and r'=3
P. P. 1	3	2	10	4.3+2.2=16 m.u.	1.3+3.2=9 m.u.
P. P. 3	2	3	10	**4.2+2.3=14 m.u.**	1.2+3.3=11 m.u..
P. P. 5	4	1	10	4.4+2.1=18 m.u.	**1.4+3.1=8 m.u.**

Technical efficiency $\nRightarrow$ / $\Leftarrow$ **Economic efficiency**

Initial prices: the P.P. 3 is economically efficient

Final prices: the P.P. 5 is economically efficient

- A technically efficient production process will become inefficient if there are technical innovations, that is to say, if there are new production processes that allow the same amount of output with less quantity of inputs.

- A production process that is economically efficient with certain input prices may be economically inefficient with other input prices.

- The concepts of technical and economic efficiency allow us to distinguish between:
  - technical aspects of the production process: analyze the relationship between quantity of inputs and quantity of outputs in order to answer the question **How to produce…? (Unit 6).**
  - economic aspects of the production process: incorporate input prices in the analysis to answer the question **…at the minimum cost? (Unit 7)**

Unit 6: PRODUCTION

1. Technology and the Production Function
2. The Marginal Rate of Technical Substitution (MRTS)
3. Returns to scale
4. Total, Average, and Marginal Product

1. Technology and the Production Function

We first examine the main limitation any firm faces in its decision on how, and how much to produce:

- **Technology:**
  - general definition: set of all available production processes in the economy to produce an output.
  - strict definition: production processes that are technically efficient:
    - They maximize the quantity of output using the same amount of inputs, or,
    - They minimize the quantities of inputs needed to produce the same quantity of output.

1. Technology and the Production Function

Production Function: assigns to each combination of inputs the maximum quantity of output:

$$f: \mathbb{R}_+^n \longrightarrow \mathbb{R}_+$$

$$(x_1, x_2, \ldots, x_n) \longrightarrow q$$

$$\boxed{q = f(x_1, x_2, \ldots, x_n)}$$

Characteristics of the production function:

1. It is technically efficient.
2. It represents the technology in its strict definition.
3. It is cardinal (unlike the utility function, that is ordinal).
4. Technical progress changes the production function (with the same quantities of inputs, more output can now be produced).

1. Technology and the Production Function

Example: consider two inputs: L (labour) and K (capital)

$$\mathbf{f:}\ \mathbb{R}_+^2 \longrightarrow \mathbb{R}_+$$

$$(L, K) \longrightarrow q$$

q= f(L, K)

- Properties of the production function
  1. It is a continous function (output and inputs are perfectly divisible)
  2. It is a twice-differentiable function: That is, **q = f(L, K)** has first partial derivatives:

     Marginal productivity of L: indicates the variation in output when L changes, with K constant.

     $$\frac{\partial f}{\partial L} = f_L(L,K) > 0$$

     Marginal productivity of K: indicates the variation in output when K changes, with L constant.

     $$\frac{\partial f}{\partial K} = f_K(L,K) > 0$$

     and second partial derivatives: $\mathbf{f_{LL}}<0$; $\mathbf{f_{LK}}= \mathbf{f_{KL}}$; $\mathbf{f_{KK}}<0$

1. Technology and the Production Function

3. It can be represented in the input space by using isoquant curves: An isoquant curve (iso=equal, quant=quantity) is the set of all combinations of inputs which allows the production of the same quantity of output.

   In other words: it is the set of combinations (L, K) to which the production function assigns the same value.

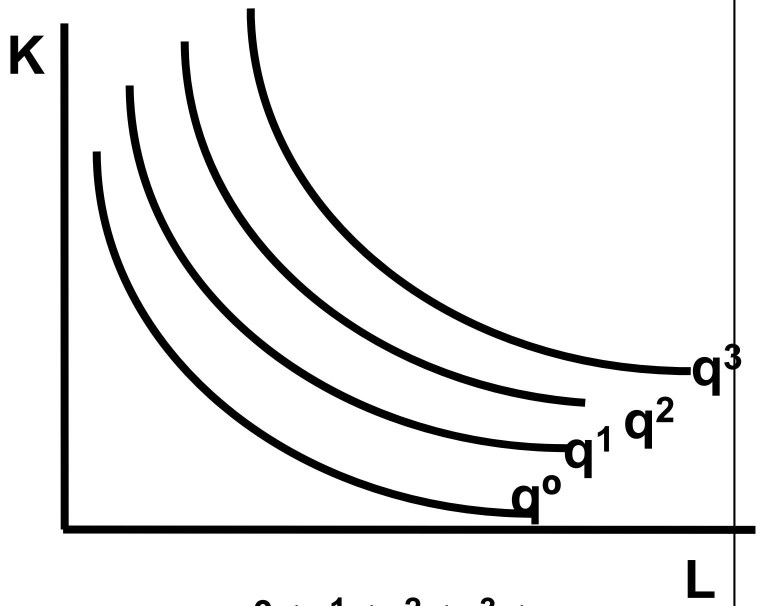
   Analytically:

   given q=f(L, K), the equation of the isoquant of level qº is obtained by resolving K as a function of L from the expression qº=(L, K)→K=K(L)

- Example: Given the production function $q = L^2K$, obtain the expression of the isoquant of level q=12
- Solution: q=12 → $12=L^2K$ → [it contains all the combinations (L, K) to which the production function assigns q=12] and solving → $K=12/L^2$

1. Technology and the Production Function

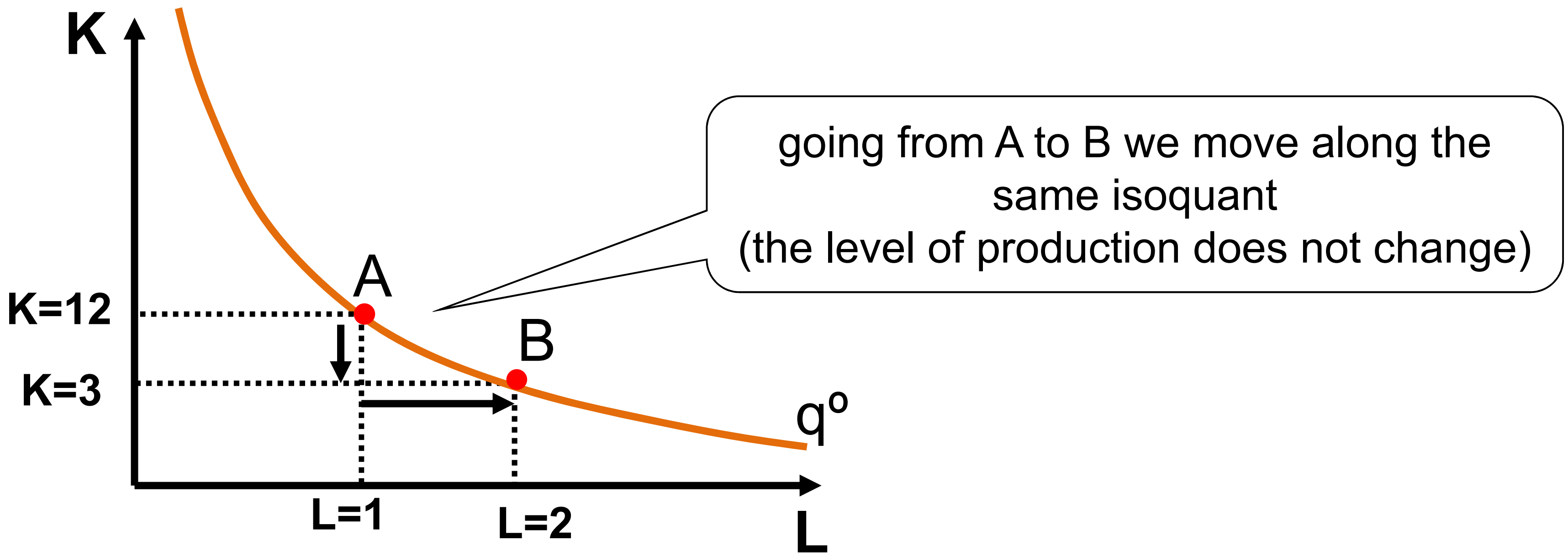
- The production function allows us to know the **isoquant map** or set of all isoquant curves as the graphical representation of the technology. This map (curves) is (are) characterised by being:
  - Infinite
  - Decreasing
  - Continuous
  - Without intersections
  - A greater distance to origin represents more output
  - Strictly convex
  - Smooth



$q^o < q^1 < q^2 < q^3 < \ldots$

2. The Marginal Rate of Technical Substitution (MRTS)

- The Marginal Rate of Technical Substitution (MRTS): Rate at which one input can be substituted by another with the amount of output being constant.
- $MRTS_L^K$ measures the quantity by which *input K* must increase (or decrease) to compensate for a decrease (or increase) of 1 unit of input L so that the level of output does not change



2. The Marginal Rate of Technical Substitution (MRTS)

Analytically:

$$MRTS_L^K = -\left.\frac{\Delta K}{\Delta L}\right|_{q=q^o}$$

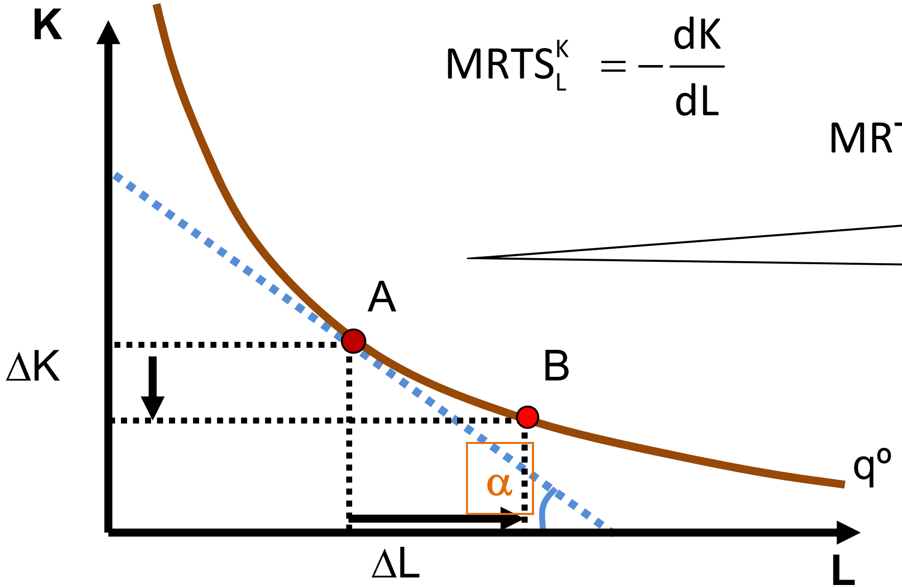
holding the level of output constant

with a negative sign to work with positive values, as ΔL and ΔK have opposite signs

Considering the expresion of the isoquant of level q°: **K=K(L)** and if we consider infinitesimal changes, the MRTS is defined as:

$$MRTS_L^K = -\frac{dK}{dL}$$

$$MRTS_L^K(A) = -\frac{dK}{dL}(A) = tg\alpha$$



the value of the MRTS in A is the slope of the isoquant at that point (with the opposite sign): **tgα**

If ΔL→0 (we consider B closer each time to A)

2. The Marginal Rate of Technical Substitution (MRTS)

Characteristics of the MRTS:

1. MRTS can be expressed as the ratio of marginal productivities

Given the production function q = f(L, K) and considering an infinitesimal change of the quantities of the we inputs, with the output level being constant, we obtain:

we require the output level to be constant: dq=0

$$dq = \frac{\partial q}{\partial L} dL + \frac{\partial q}{\partial K} dK = f_L dL + f_K dK = 0 \Rightarrow \frac{dK}{dL} = -\frac{f_L}{f_K} < 0$$

which is the expression of the slope of the isoquant curve (negative slope: decreasing isoquant).

- Thus, the MRTS can be expressed as the ratio of marginal productivities:

$$MRTS_L^K = -\frac{dK}{dL} = \frac{f_L}{f_K} > 0$$

2. The Marginal Rate of Technical Substitution (MRTS)

2. Principle of Diminishing Marginal Rate of Technical Substitution.

- As marginal productivities are functions of L and K, the MRTS is also a function of L and K:

$$MRTS_L^K = \frac{f_L(L,K)}{f_K(L,K)} = MRTS_L^K\ (L,K)$$

It gives the value of MRTS for each combination of inputs, the slope of the isoquant at each point

- The strict convexity of the isoquant makes that:

$$\frac{dMRTS_L^K}{dL} < 0 \ \rightarrow \text{along a given isoquant } MRTS_L^K \downarrow \text{if } \uparrow L$$

- It is known as the **Principle of Diminishing Marginal Rate of Technical Substitution**: moving along a given isoquant, the value of the MRTS decreases as we increase the quantity of the input L.

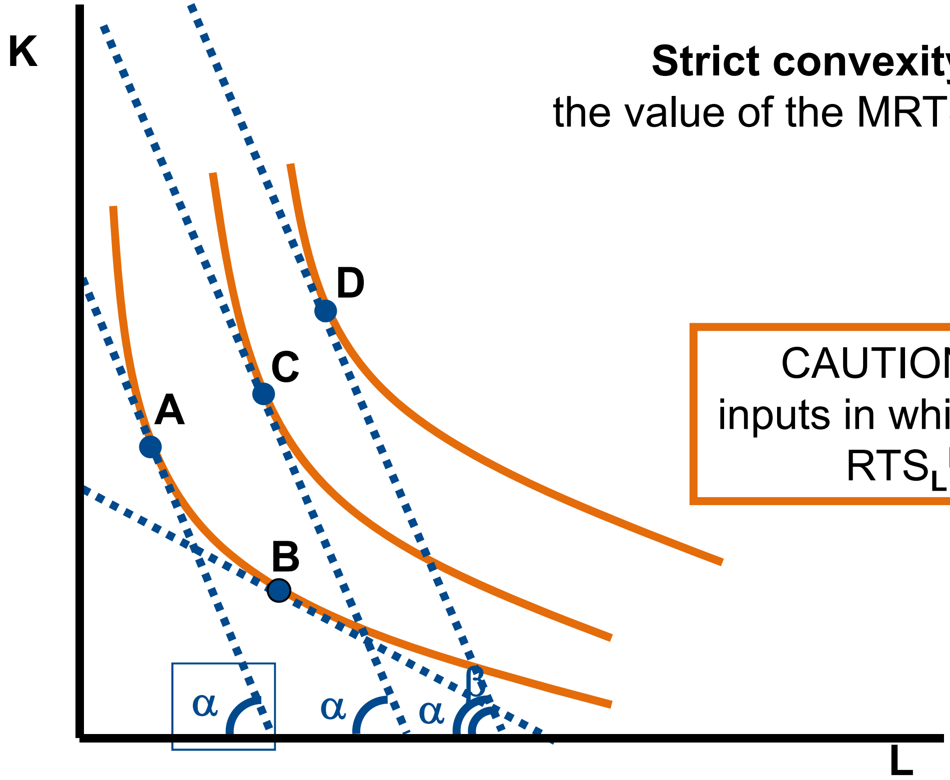
2. The Marginal Rate of Technical Substitution (MRTS)

We compare graphically how the value of the MRTS changes along a given isoquant:

$MRTS_L^K$ (A)= tg α

$MRTS_L^K$ (B)= tg β < tg α

Strict convexity → the value of the MRTS↓ as ↑L →
the value of the $MRTS_L^K$ is different for each point of an isoquant



CAUTION! There are infinite combinations of inputs in which the value of the $MRTS_L^K$ is the same
RTS_L^K (A)= RTS_L^K (C)=RTS_L^K (D)=tgα

Differences between the Production Function and Ordinal Utility Function

•Production Function

$q=f(L, K)$

•It relates each combination of inputs to the number of physical units of output that can be produced.

•Isoquant map: represents the technology (the same for all producers)

•Marginal productivity: reflects that additional physical units of output are obtained when input increases

•MRTS: reflects the rate of substitution among inputs allowed by the technology, with the production being constant

•Function of Ordinal Utility

$U=U(q_1, q_2)$

•It relates each combination of goods to a number that reflects the order of the consumer's preference.

•Indifference map: represents the preferences (different for each consumer)

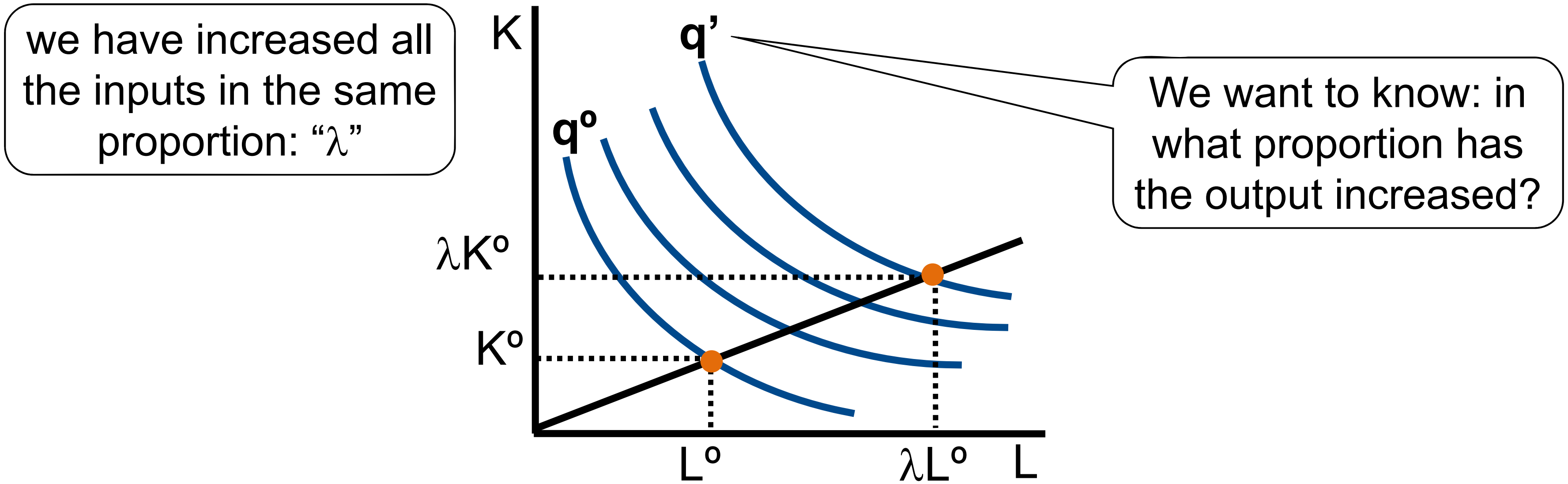
•Marginal utility: its value lacks informative value (it only tells us the sign)

•MRS: reflects the rate of substitution among goods allowed by the subjective preference of the consumer, with the utility being constant

3. Returns to scale

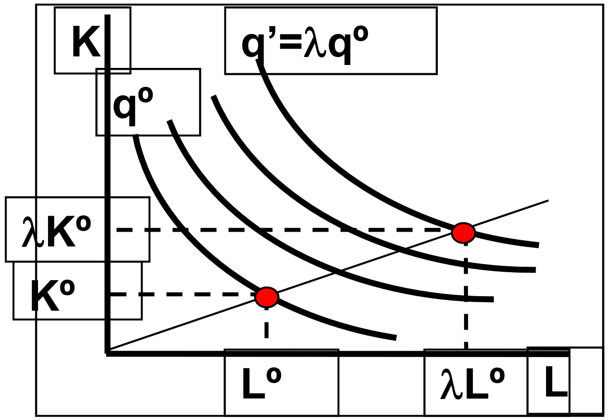
- We analyze how output evolves when we modify the quantity of all inputs (we are in the long run, as all the inputs are variable).

- With the combination (L^o, K^o) we are in the isoquant of level qº:
- If we increase the quantities of the two inputs in the same proportion: λ >1, we go to an isoquant more distant from the origin, reflecting a higher quantity of output q'>q^o



3. Returns to scale

CONSTANT RETURNS TO SCALE

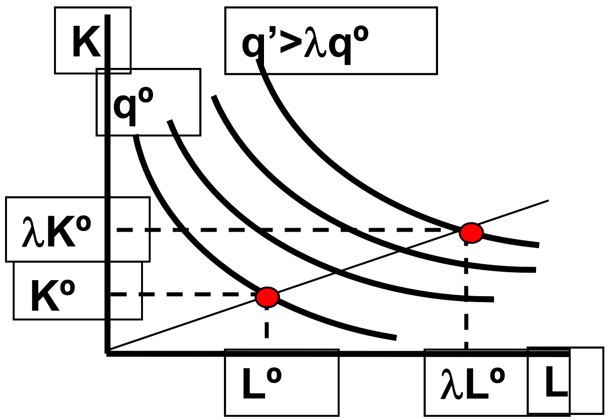


With (λL°, λK°) we obtain:

$q' = \lambda q^o$

The output has increased in the same proportion

INCREASING RETURNS TO SCALE

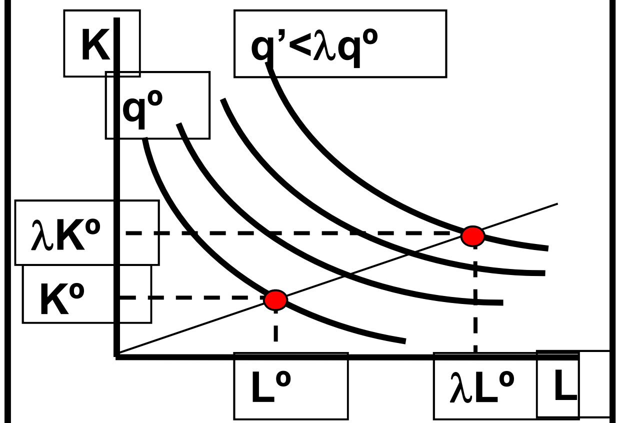


With (λL°, λK°) we obtain:

$q' > \lambda q^o$

The output has increased in a higher proportion

DECREASING RETURNS TO SCALE



With (λL°, λK°) we obtain:

$q' < \lambda q^o$

The output has increased in a lower proportion

3. Returns to scale

- **Constant returns to scale:** when the quantities of all inputs increase in the same proportion, **the output increases in THE SAME PROPORTION**

- **Increasing returns to scale:** when the quantities of all inputs increase in the same proportion, **the output increases in A HIGHER PROPORTION**

- **Decreasing returns to scale:** when the quantities of all of the inputs increase in the same proportion, **the output increases in A LOWER PROPORTION**

3. Returns to scale

- When increasing returns to scale are present, it is said that there exist "**economies of scale**": as when the scale of the firm increases (more output) the relation inputs-output improves.

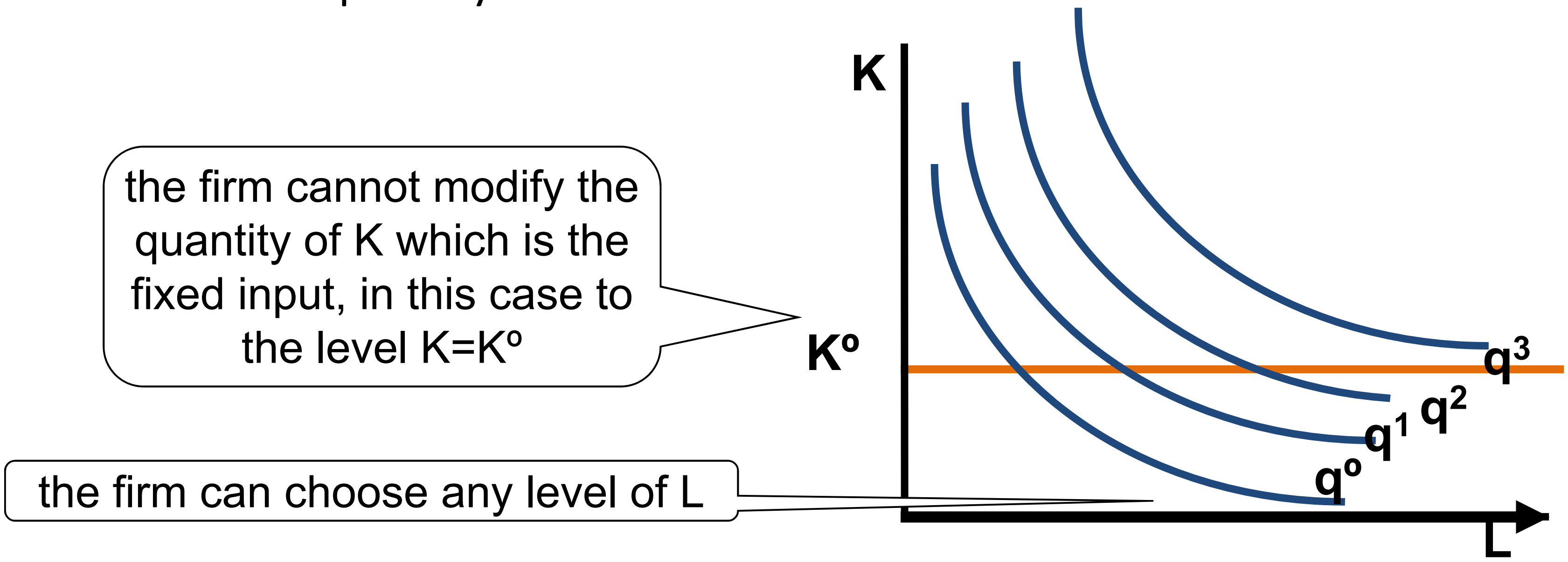
  - The reasons may be: more specialization and division of job tasks as the size of the firm increases, the use of distribution networks, etc.

- When decreasing returns to scale are present, it is said that there exist "**diseconomies of scale**": as when the scale of the firm increases (more output) the relation inputs-output declines.

  - The reasons may be: as the size of the firm increases, the management, control, and coordination of the firm is more difficult.

4. Total, average and marginal product

- We analyze how output evolves **when we modify the quantity of one of the inputs**, holding constant the rest (we are in the **short run**: there are fixed inputs).
- Considering the production function for two inputs: q=f(L,K), we assume that **in the short run it is the input K which is held constant** (to a level K=K°, that we link to the "size of the firm") while **the input L is variable**, that is to say, the firm can choose the quantity of labor.



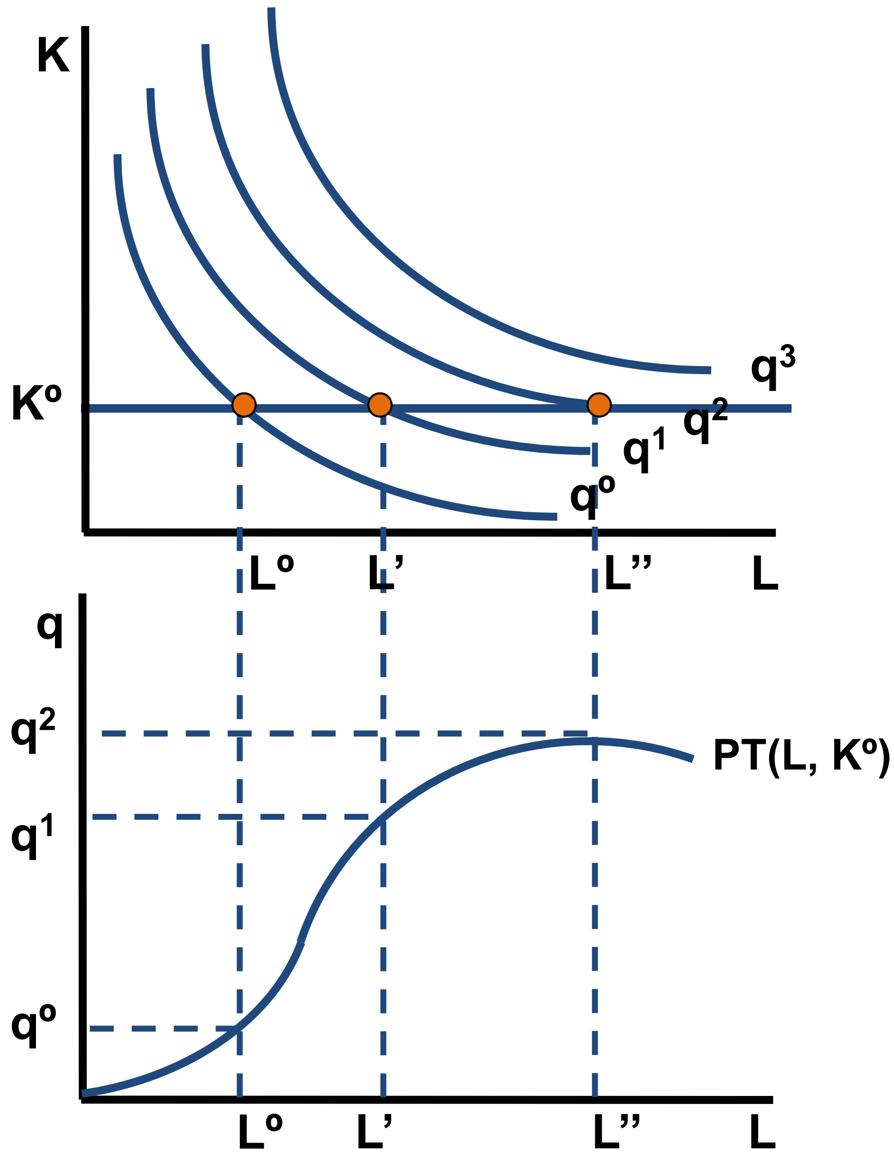
4. Total, average and marginal product

- **TOTAL PRODUCT OF FACTOR L:**

  PT(L)=f(L, K^o)

– It is the function that, for each value of L, relates the maximum output that can be obtained with the quantity of K^o (that is, with the firm size fixed at K^o).

- It starts from the origin: f(0,K^o)=0
- It is originally increasing
- It reaches a maximum, given that with a firm size fixed at K=K^o, there is a maximum level of output that can be obtained.



4. Total, average and marginal product

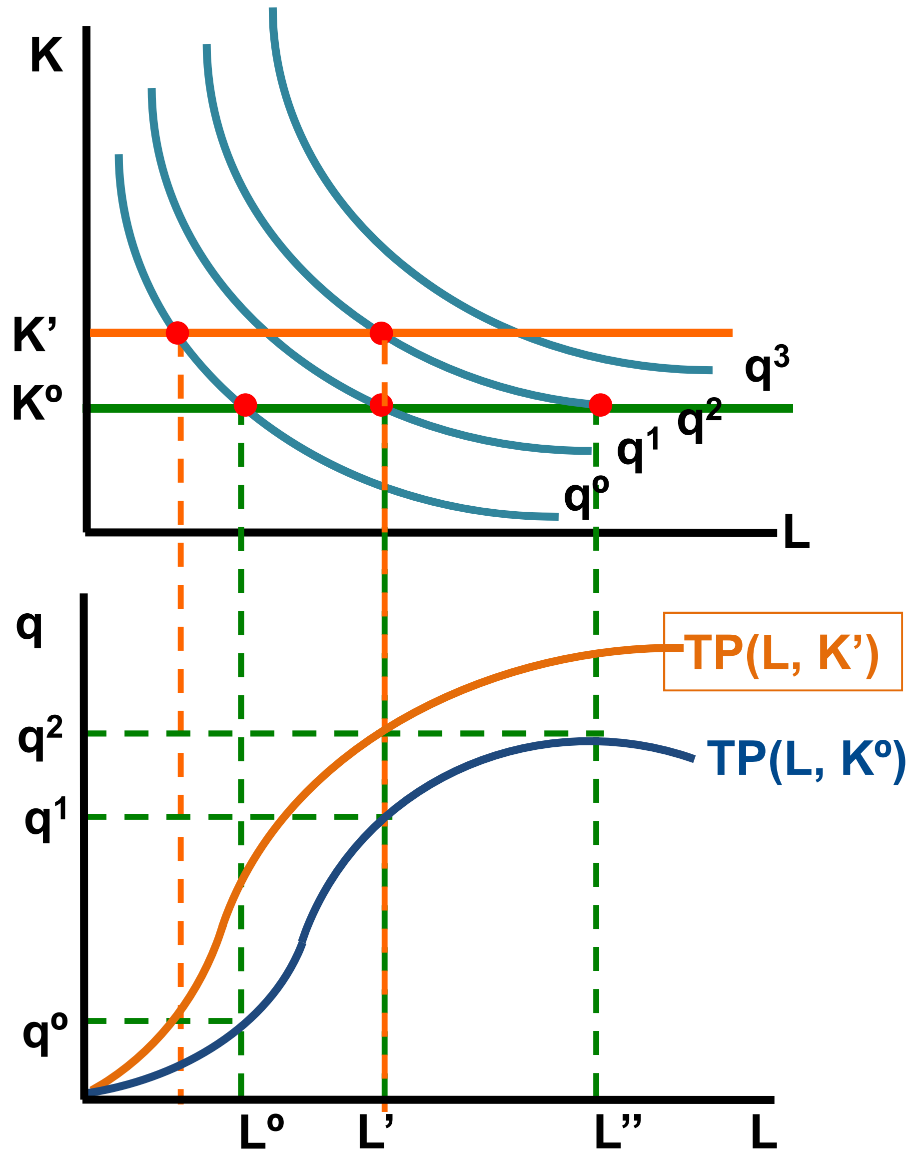
If we assume that K=K'>K^o

- the same level of output can be obtained with a lower quantity of L
- with the same quantity of input L the firm can obtain a higher level of output

We will have a new function of total product of input L:

before: TP(L)=f(L, K^o)

now: TP(L)=f(L, K')



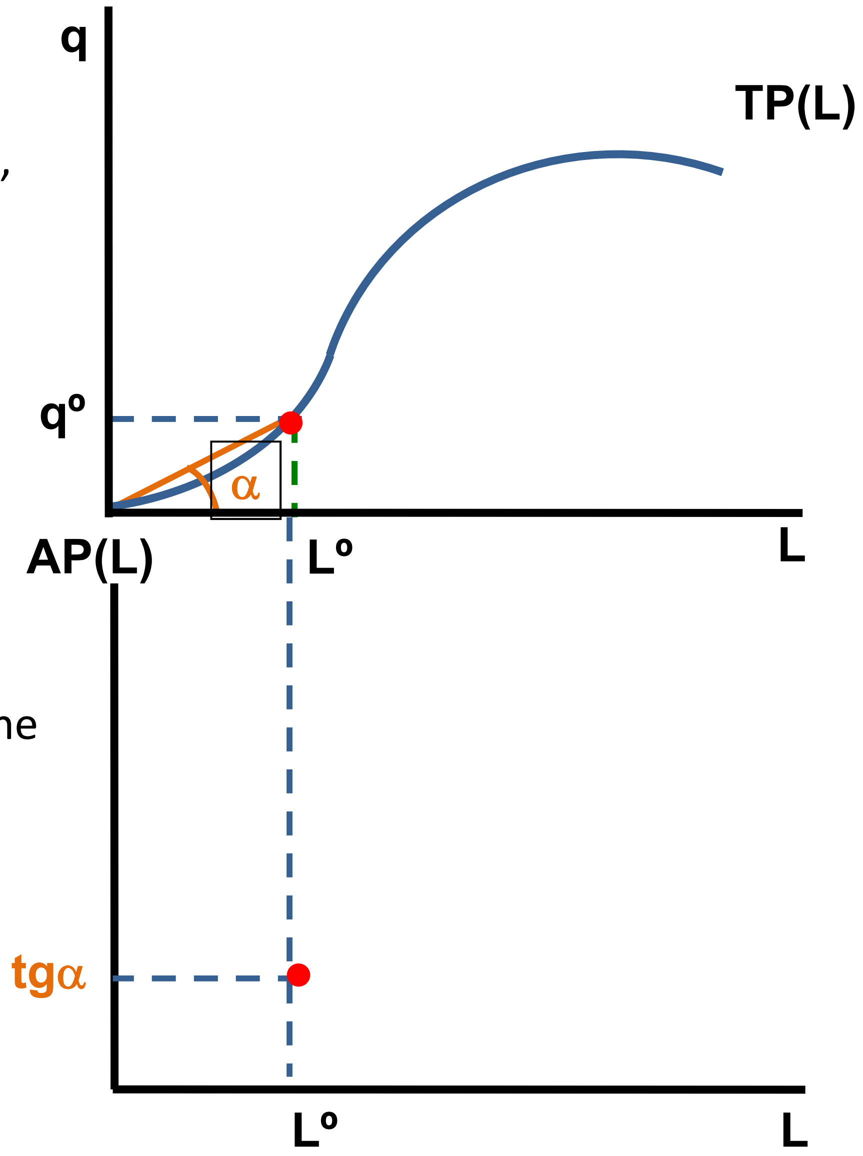
4. Total, average and marginal product

AVERAGE PRODUCT OF INPUT L:

– It is the function that links, to each value of L, the amount of output that can be produced per unit of input L:

$$AP(L) = \frac{TP(L)}{L} = \frac{f(L, K^o)}{L}$$

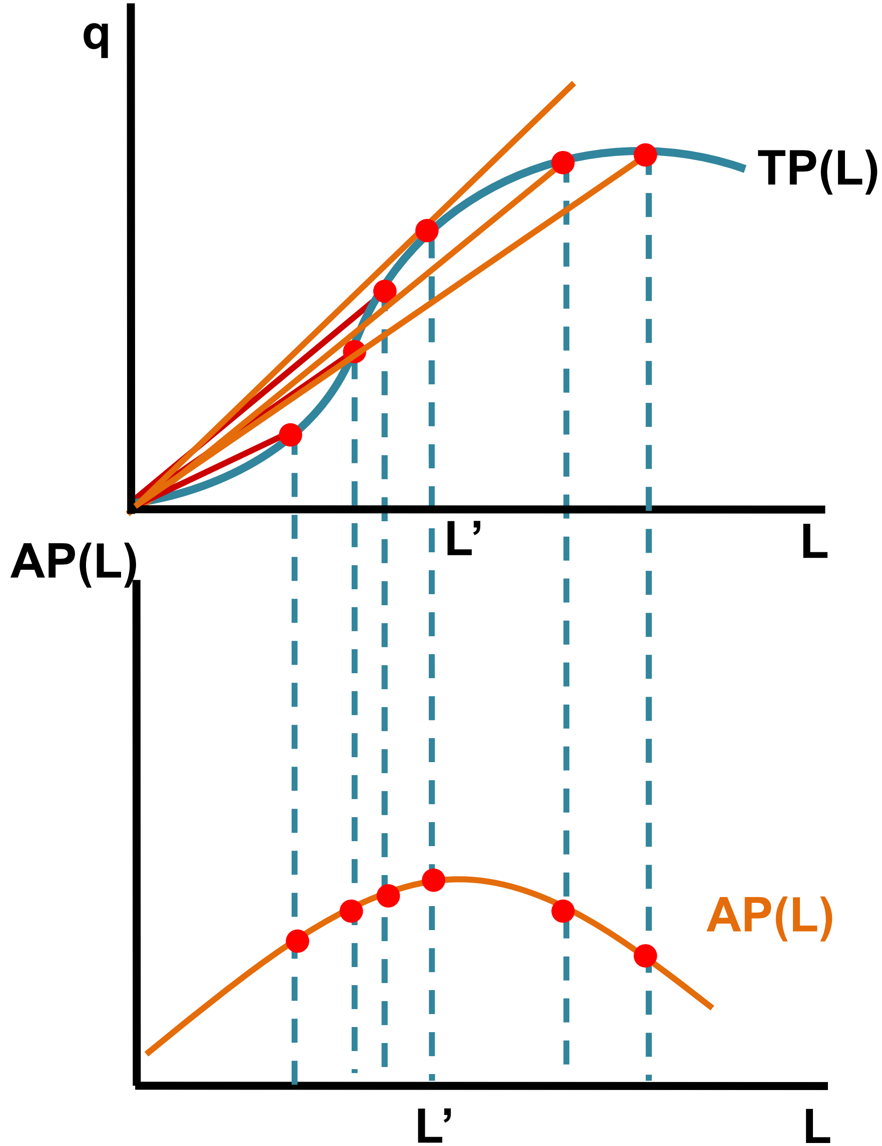
- To represent it graphically, for each level of L we must assign the output per unit of input
- for $L=L^o \rightarrow AP(L^o)=q^o/L^o$
- Graphically it is the slope of the line linking the point of Average Productivity with the origin
- for $L=L^o \rightarrow AP(L^o)=q^o/L^o = tg\alpha$



4. Total, average and marginal product

- **AVERAGE PRODUCT OF INPUT L:**

- Initially, as L increases, the slope of the line also increases: AP(L) also increases
- Up to the volume of input L where the line is tangent to the TP(L) curve, where the AP(L) reaches the maximum.
- From L=L' the line has a decreasing slope→ the AP(L) is also decreasing



4. Total, average and marginal product

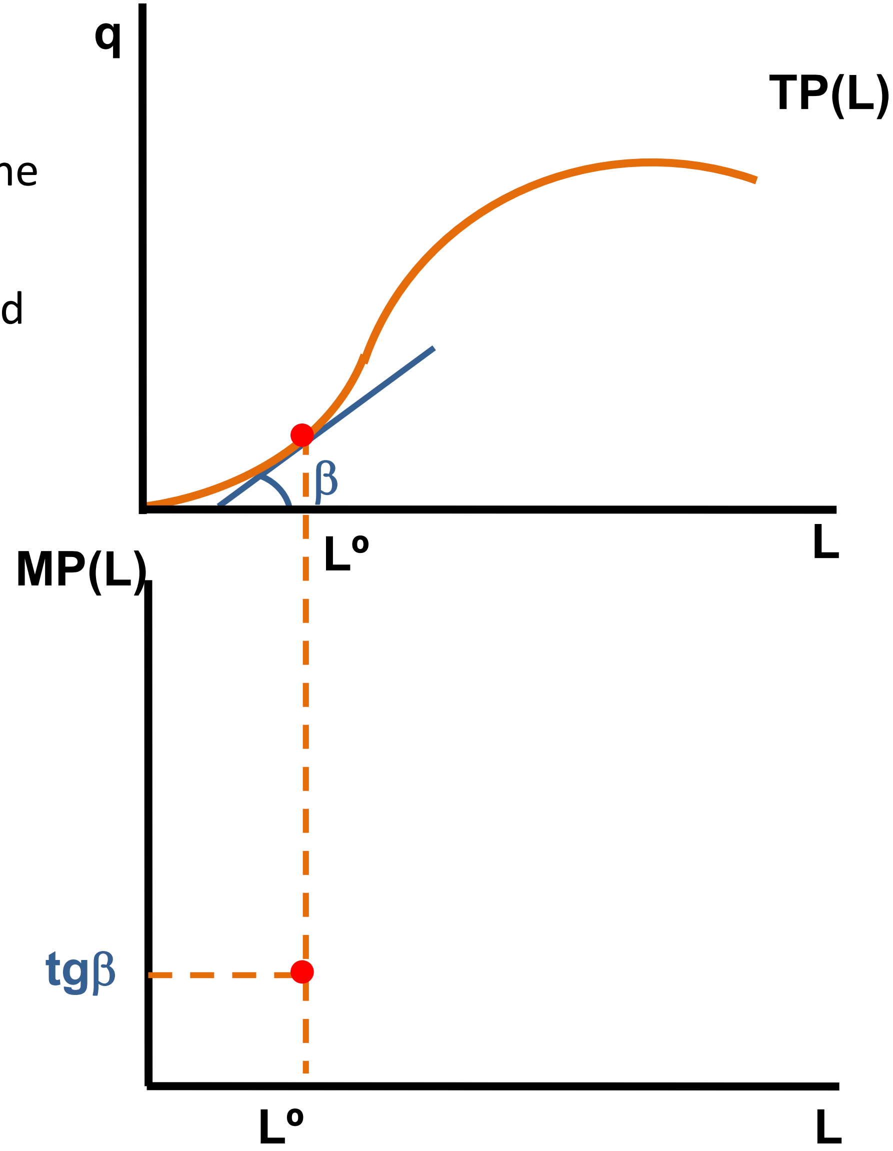
MARGINAL PRODUCT OF FACTOR L:

- It is the function that links, to each value of L, the change in the quantity of output obtained when the level of the input (L) is infinitesimally changed

$$MP(L) = \frac{dTP(L)}{dL} = \frac{\partial f}{\partial L} = f_L$$

To represent it graphically, for each level of L, we must assign the **slope of the TOTAL PRODUCT** at the point

- for $L=L^o \rightarrow MP(L^o)=$**tg**$\beta$



4. Total, average and marginal product

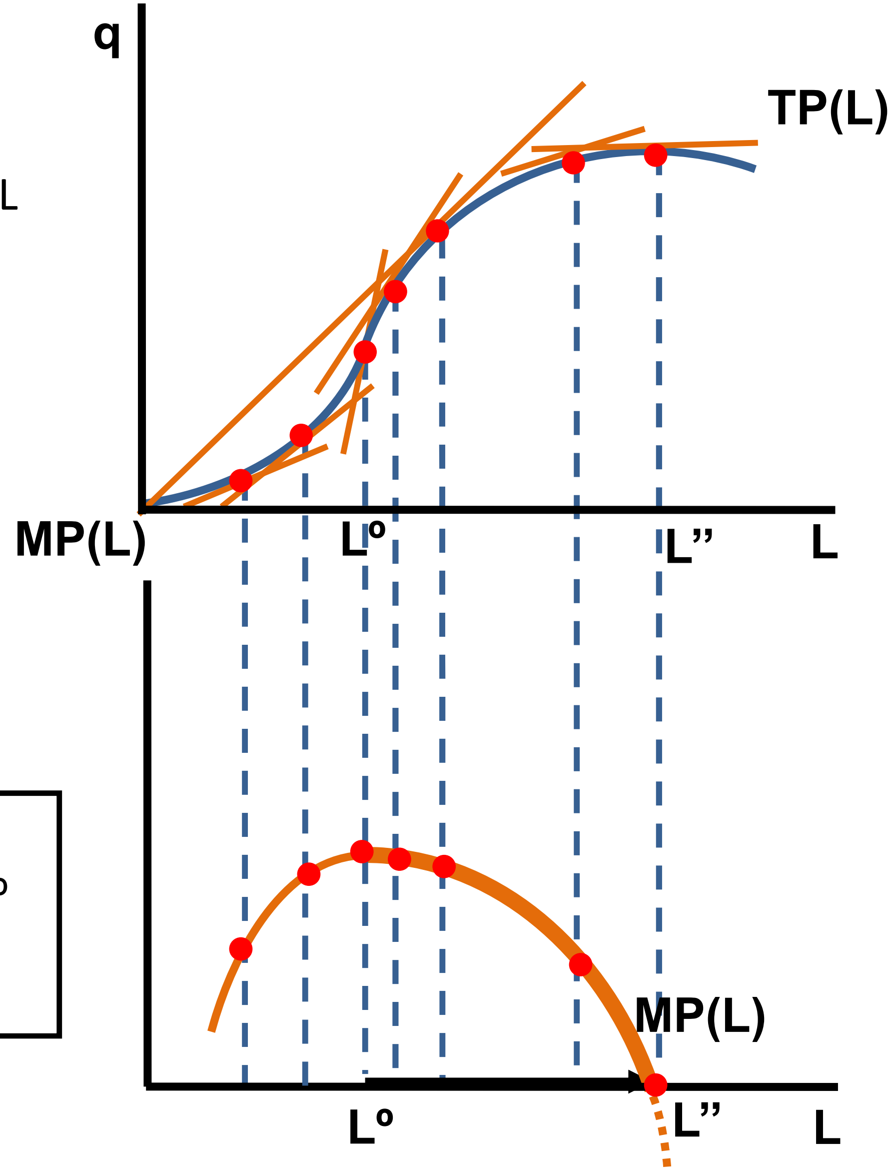
MARGINAL PRODUCT OF FACTOR L:

- Initially, as L increases, the slope of the TOTAL PRODUCT also increases: MP(L) is increasing.
- Up to the inflexion point of the TOTAL PRODUCT curve, in L=L^o, which is the point where MP(L) is at a maximum

- from L=L^o → the MP(L) is decreasing
- for L=L'' the TOTAL PRODUCT reaches its maximum→ MP(L)=0

Law of Diminishing Returns:

in the short run there is a level of input L= L^o from where the MP(L) is decreasing



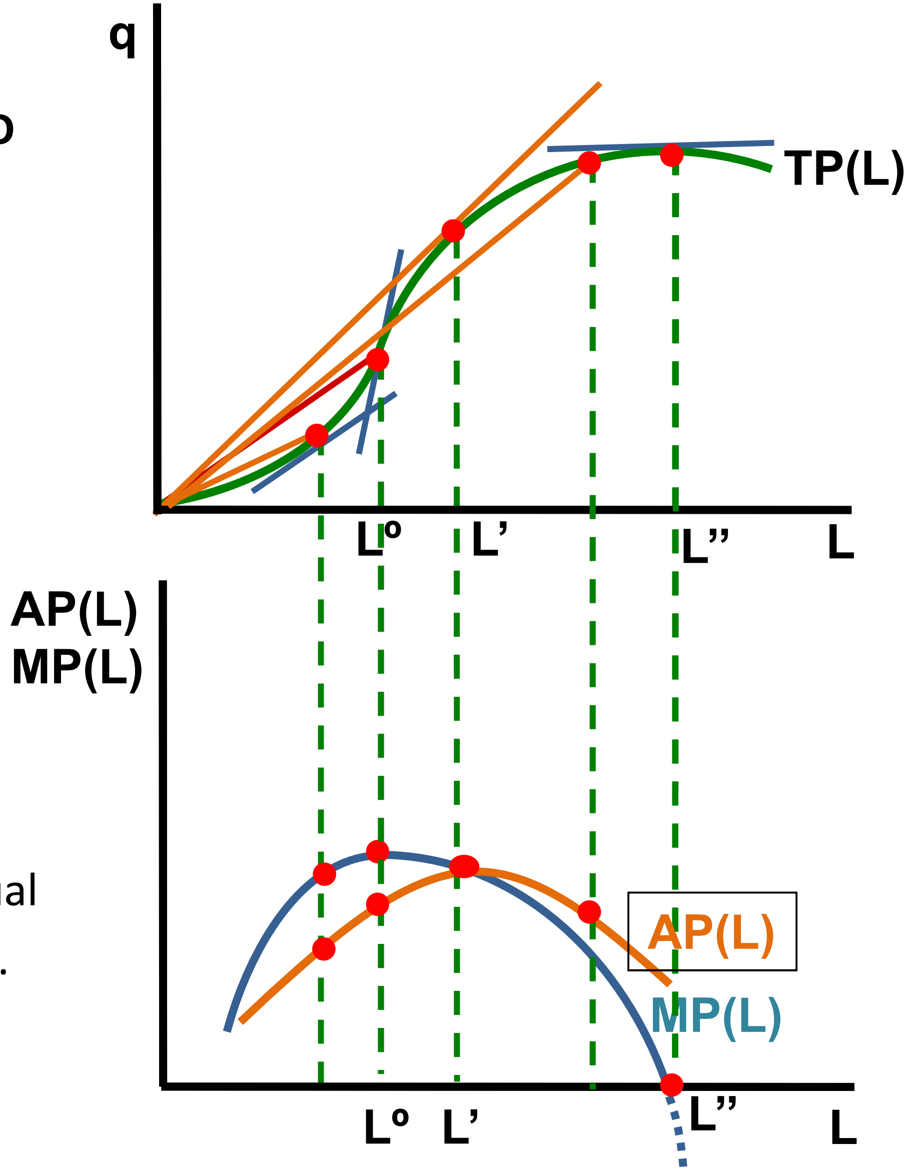
4. Total, average and marginal product

- **JOINT DEVELOPMENT OF THE TP(L), AP(L) AND MP(L) CURVES**

  - MP(L) increases to L=L^o, where it reaches its maximum, and later decreases to L=L’’where its value is 0

  - MP(L) increases to L= L’, where it reaches the maximum and then decreases

  - In L=L’ marginal and average product are equal since the vector is tangent to the TP(L) curve.



Exercises

1.- To produce a given good "q" it is necessary to use the following inputs: L (labor) and K (capital). We know the amount of inputs that are needed in the following productive processes:

Productive process		K	Q
A		1	10
B		2	10
C		2	10
D		2.5	10
E		2.5	10
F		6	10
G		2.5	10

a) Determine the production processes that are technically efficient.
b) If the prices of the inputs are w=50 for L and r=30 for K, determine the production process that is economically efficient.
c) If the prices of the inputs are w=20 for L and r=60 for K, determine the production process that is economically efficient.

2.- Consider the following production processes (L,K) where 1 unit of output can be produced as follows: A=(1,12); B=(1,8); C=(3,9); D=(2,1). Indicate and explain if the following statements are true or false:

a) All the production processes are technically efficient.
b) Only production process D is technically efficient.
c) Only production processes B and D are technically efficient.
d) Production process D is economically efficient

3.- Consider the following production processes (L,K) where 2 units of output can be produced as follows: P=(3,2); Q=(4,10); R=(2,9); S=(2,13). We know that the prices of the inputs are w=2 (L) and r=4 (K):

a) Determine what production processes are technically efficient.

b) Determine what production process is economically efficient.

c) Assign other prices to the inputs so that the answer to question b) changes

4.- Analyze the type of returns to scale present in the following production functions and explain their economic interpretation:

a) $$q = \frac{L^3K - 2L^4}{K^2}$$

b) $$q = L^{1/3}K^{1/3}$$

c) $$q = 4L^{2/3}K^{1/3}$$

5.- Given the following production function: $q = L^{\alpha}K^{\beta}$ where α>0 and β>0,

a) Determine the type of returns to scale of the production function in relation to the values of α and β.
b) Obtain the expression of the Marginal Rate of Technical Substitution (MRTS).

6.- Given the following production function: $q = L^{1/2}K^{1/2}$

a) Give, for that technology, an example of a production process that is technically inefficient.
b) Obtain the expression of the Marginal Rate of Technical Substitution (MRTS).
c) Obtain the expression of the isoquant curve of level 10. Obtain the combinations of inputs for that isoquant curve and the value of the MRTS in those combinations.
d) Show, using this MRTS, whether the isoquant curves are strictly convex. What is the economic interpretation of that property?

7.- To produce 20 units of output, a firm uses 3 units of K and 4 units of L. If it is known that the technology presents decreasing returns to scale, determine what statement is true:

a) To produce 40 units of output, the firm uses 6 units of K and 8 of L.
b) To produce 40 units of output, the firm uses 9 units of K and 12 of L.
c) To produce 40 units of output, the firm uses 21 units of K and 28 of L.
d) To produce 40 units of output, the firm uses 4,5 units of K and 6 of L.

8.- Given the following production function: $q = L^{1/2}(K+1)^{1/2}$

a) Obtain the expression of the isoquant curve of level 10.
b) Obtain the expression of the MRTS and its values in (3,26) and (9,8).
c) For K=15, obtain the total product of input L. What amount of L should be used to produce 8 units of output?

9.- Given the following production function: $q = 60 L^2 + 2KL^2 - 3L^3$ with q=output, L= labor, K=capital

a) Obtain the Total Product function of L, knowing the level of the fixed input $K^0=6$.

1) Determine the amount of output for the input levels L=2; L=3; L=4. What are the increases in the level of output?
2) Determine the amount of output for the input levels L=9; L=10; L=11. What are the increases in the level of output?
3) How can the results of questions 1) and 2) be explained?
4) Obtain the Marginal Product and Average Product functions of the variable input.
    i. Obtain the amount of output that makes the Marginal Product maximum
    ii. Obtain the amount of output that makes the Average Product maximum
5) Determine the maximum level of output that can be produced

b) Obtain the Total Product function of L, knowing the level of the fixed input $K^0=15$.

1) Obtain the Marginal Product and Average Product functions of the variable input.
    i. Obtain the amount of output that makes the Marginal Product maximum
    ii. Obtain the amount of output that makes the Average Product maximum
2) Determine the maximum level of output that can be produced

c) Represent graphically (in the same graph) the two Total Product functions

10.- Given the following production function: $q = 2L^2K + L^2 - 2L^3$, with q=output, L= labor, K=capital, obtain:

a) The expression of the MRTS and its value in (2,4). Interpret this value.
b) The Marginal Product and Average Product functions of input L.

Considering that the firm has 4 units of input K, obtain

a) The Total Product function of input L.
b) The level of input L that maximizes the total product.
c) The amount of product that can be produced from question d) and the expression of the isoquant curve related to that level.
d) The Marginal Product function of input L.
e) The amount of output that makes the Marginal Product maximum
f) The Average Product function of input L.
g) The amount of output that makes the Average Product maximum

11.- Given the following production function: $q = 2L^2K + L^2 - 2L^3$ with q=output, L= labor, K=capital, show:

a) The expression of the isoquant curve of level 30.
b) Three combinations of inputs (L,K) pertaining to that isoquant curve and the value of the MRTS at those points.
c) The economic interpretation of that property?

12.- Given the following production function: $q = 10L^2K + 100L^2 - 5L^3$ where q=amount of output, L= hours of work, and K=machinery hours. If the machinery operates for 5 hours, obtain the hours of work that are needed in order to:

a) maximize the level of production. Indicate the level of production.
b) maximize the Marginal Product of input L. What is the amount of output obtained at that point? What value of L makes the Marginal Product null? What will happen to production if the firm increases L above the level of L?
c) maximize the Average Product of input L. What is the amount of output obtained at that point?
d) Represent graphically the Marginal Product and Average Product functions of input L, and relate them to the Total Product function. Is the Law of Diminishing Returns fulfilled?

13.- To produce 20 units of output, a firm uses 4 units of the variable input. After deciding to increase the units of the variable input to 6, the level of production increased by 26 units of output. What is the Marginal Product of that input? How has the Average Product changed with the change in input?

14.- A given firm is producing at a point where the Marginal Product of input L is null. From this situation we can infer that:

a) If more units of input L are used the production will decrease.
b) If fewer units of input L are used the production will increase.
c) The Average Product of input is at its maximum.
d) If more units of input L are used the Marginal Product of input L will be negative.

15.- Give reasons for or against the following statements:

a) The production function changes when there are changes in the price of any of the inputs.
b) If there is a technical innovation, the production function of the firm changes.
c) Knowing that there are increasing returns to scale, a reduction in the level of inputs by 50% will lead to a reduction in the level of output by less than 50%.
d) If the Law of Diminishing Returns is fulfilled, the firm cannot have increasing returns to scale.
e) If the firm is producing at a point where the Marginal Product of input L is null, the firm is able to increase its production by reducing the number of workers.
f) The Law of Diminishing Returns can be interpreted as follows: as the firm adds units of a given input to the productive process, the amounts of output obtained are smaller each time.
g) With increasing returns to scale if all the inputs increase in the same proportion, the output varies in the same direction and in a higher proportion.
h) If the Total Product function of a given input is an increasing and concave function, for any level of that input the Law of Diminishing Returns is satisfied.

16.- A firm can produce in the short-run according to the production function q=f(K,L), with L being the fixed factor. It is known that the Marginal Product function of input L is increasing up to the ninth (9^{th}) worker, and then it is decreasing for additional workers. The firm is now using 7 workers, and it is considering hiring one additional worker in order to increase the Average Product of the input. Is this strategy correct? Justify your answer with the aid of a graph

17.- Give reasons for or against the following statements about the production function:

a) It uses the production processes that are technically efficient.
b) It determines the maximum output that can be obtained for any combination of inputs.
c) It shows the level of output needed to maximize the profit.
d) It should not be taken into account in the profit-maximization process.
e) It changes when there is any change in the price of any of the inputs.

18.- Give reasons for or against the following statements:

a) The constraints that the firm must face to maximize its profit are given by the technology, the price of the inputs, and the market demand.
b) If the Total Product function of an input is an increasing function that starts from the origin and is convex, for any level of that input, the Law of Diminishing Returns is satisfied.
c) If the Total Product function of an input is an increasing function that starts from the origin and is concave, for any level of that input, the Law of Diminishing Returns is satisfied.
d) The Principle of Diminishing Marginal Rate of Technical Substitution is only satisfied when the isoquant curves are strictly convex