UNIT 4. CONSUMER BEHAVIOR

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APPENDIX: Relation between expenditure and elasticities
4.1 Consumer equilibrium

**Consumer equilibrium:**

- We proceed to analyze how the consumer chooses the quantity to buy of each good or service (market basket), given his/her:
  - Preferences
  - Budget constraint
- We shall assume that the decision is made *rationally*:
  Select the quantities of goods to purchase in order to maximize the satisfaction from consumption given the available budget
- We shall conclude that this market basket maximizes the utility function:
  - The chosen market basket must be the preferred combination of goods or services from all the available baskets and, particularly,
  - It is on the budget line since we do not consider the possibility of saving money for future consumption and due to the non-satiation axiom
4.1 Consumer equilibrium

Graphical analysis

• The **equilibrium** is the point where an indifference curve intersects the budget line, with this being the upper frontier of the budget set, which gives the highest utility, that is to say, where the indifference curve is tangent to the budget line.
4.1 Consumer equilibrium

Consumer problem

• The consumer will assign limited resources (i.e., income “Y”) among different goods or services \((q_1, q_2)\) in order to maximize utility:

\[
\max_{q_1, q_2} U = U(q_1, q_2)
\]

subject to:

\[
Y \geq p_1 q_1 + p_2 q_2
\]

\[
q_1, q_2 \geq 0
\]

• Assumptions:
  – There is always some positive consumption of all goods or services
  – The consumer spends all income

• Therefore, the optimization problem can be written as follows:

\[
\max_{q_1, q_2} U = U(q_1, q_2)
\]

s.t. \( Y = p_1 q_1 + p_2 q_2 \)
4.1 Consumer equilibrium

Mathematical resolution of the consumer problem:

- **Lagrange Multipliers Method (LMM):** We use an auxiliary function called "The Lagrangian function" (λ: Lagrange multiplier):

\[
L = L(q_1, q_2, \lambda) = U(q_1, q_2) + \lambda[Y - p_1q_1 - p_2q_2]
\]

which is maximized: \[ \max_{q_1, q_2, \lambda} L = L(q_1, q_2, \lambda) \]

The First Order Conditions (F.O.C) are:

\[
\begin{align*}
L_1 &= \frac{\partial L}{\partial q_1} = U_1 - \lambda p_1 = 0 \quad \Rightarrow \quad U_1 = \lambda p_1 \quad (1) \\
L_2 &= \frac{\partial L}{\partial q_2} = U_2 - \lambda p_2 = 0 \quad \Rightarrow \quad U_2 = \lambda p_2 \quad (2) \\
L_\lambda &= \frac{\partial L}{\partial \lambda} = Y - p_1q_1 - p_2q_2 = 0 \quad \Rightarrow \quad Y = p_1q_1 + p_2q_2 \quad (3)
\end{align*}
\]

Given particular values for the exogenous variables (income and prices), we solve the system of three equations for the three endogenous variables \((q_1, q_2, \lambda)\)
4.1 Consumer equilibrium

Mathematical resolution of the problem:

• From equations (1) and (2) we derive:

\[ \lambda = \frac{U_1}{p_1} = \frac{U_2}{p_2} \]

**Equal marginal principle (EMP):** The consumer must, in equilibrium, obtain the same utility from the last monetary unit spent on either good 1 or good 2.

• In equilibrium, we obtain:

\[ \frac{U_1}{p_1} = \frac{U_2}{p_2} \iff MRS^2_1 = \frac{U_1}{U_2} = \frac{p_1}{p_2} \]

That is to say, the consumer is in equilibrium when the following two magnitudes are equated:

➢ The **subjective** exchange rate between goods \((MRS^2_1)\): the rate at which the consumer wants to exchange one good for the other with constant utility

➢ The objective exchange **rate between goods** offered in the market \((P_1/P_2)\)
4.1 Consumer equilibrium

Mathematical resolution of the problem:

• **SUMMARIZING:** the consumer problem is solved as follows:

**Problem**

\[
\begin{align*}
\text{max } & \quad U = U(q_1, q_2) \\
\text{s.t. } & \quad Y = p_1q_1 + p_2q_2
\end{align*}
\]

**From F.O.C.s:**

\[
\frac{U_1}{p_1} = \frac{U_2}{p_2}
\]

\[
Y = p_1q_1 + p_2q_2
\]

**Solution:**

Given \(p_1^0, p_2^0\) and \(Y^0\), \((q_1^*, q_2^*)\)

- Endogenous variables: \(q_1\) and \(q_2\)
- Exogenous variables: \(p_1, p_2\) and \(Y\)

In equilibrium:

\[
MRS_{12} = \frac{U_1}{U_2} = \frac{p_1}{p_2}
\]

The marginal rate of substitution must be equal to the relative price
4.1 Consumer equilibrium

Mathematical resolution of the problem:

- **SPECIAL CASE:** Corner solution \( MRS_1^2 \neq \frac{p_1}{p_2} \)

  The consumer spends the total income on just one good or service and decides not to buy any of the other good:
4.1 Consumer equilibrium

Economic interpretation of the Lagrange Multiplier

• In equilibrium, the Lagrange Multiplier is equal to the marginal utility of income:

(1) First we differentiate the utility function: 
\[ dU = U_1 dq_1 + U_2 dq_2 \]

(2) Also the budget line equation (constant prices):
\[ dY = P_1 dq_1 + P_2 dq_2 \]

(3) We know that in equilibrium: 
\[ U_1 = \lambda P_1 \quad \text{and} \quad U_2 = \lambda P_2 \]

(4) Finally, substituting (3) into (1):
\[ dU = \lambda P_1 dq_1 + \lambda P_2 dq_2 = \lambda( P_1 dq_1 + P_2 dq_2 ) = \lambda dY \]

\[ \lambda = \frac{dU}{dY} \]

The Lagrange Multiplier is equal to the marginal utility of income

And using (2)
4.2 Individual demand function

Derivation of the the Marshallian individual demand function

*Def*: The general demand function (or Marshallian demand function) is the mathematical relationship, in equilibrium, between prices and income and the corresponding quantity of the good:

\[ q_1 = q_1(p_1, p_2, Y) \quad \text{Demand function of good 1} \]

\[ q_2 = q_2(p_1, p_2, Y) \quad \text{Demand function of good 2} \]

These functions are obtained from the F.O.C.s of the consumer maximization problem, considering \( p_1, p_2 \) and \( Y \) as parameters/variables in place of specific numerical values.
4.2 Individual demand function

Properties of the Marshallian individual demand functions

• They are functions such that:
  ➢ For each three values \((p_1, p_2, Y)\) there is only one optimal quantity demanded of each good or service, that is to say, the optimal equilibrium basket is unique

• They describe optimal behavioral relations:
  ➢ They incorporate the objectives and restrictions of the consumer problem
  ➢ The consumer tastes or preferences fully determine the functional form

• They are homogeneous degree “0” functions in prices and income: If all prices and income are multiplied by the same positive constant “\(k\)”, the equilibrium remains:

\[
q_i(kp_1, kp_2, kY) = k^0q_i(p_1, p_2, Y) = q_i(p_1, p_2, Y); \quad i = 1, 2
\]
4.3 Individual demand curves and Engel curves

Once we have the individual demand functions, we can define several behavioral curves according to the particular values of specific exogenous variables ($p_1$, $p_2$, $Y$):

- **(1) Ordinary demand curve:** Shows the quantity demanded of a good as a function of its price (the income and the price of the other good are given):

  \[ q_1 = q_1(p_1, p_2^0, Y^0) = q_1(p_1) \quad \text{Ordinary demand curve of good 1} \]
  \[ q_2 = q_2(p_1^0, p_2, Y^0) = q_2(p_2) \quad \text{Ordinary demand curve of good 2} \]

- **(2) Crossed demand curve:** Shows the quantity demanded of a good as a function of the price of the other good (the income and the price of the same good are given):

  \[ q_1 = q_1(p_1^0, p_2, Y^0) = q_1(p_2) \quad \text{Crossed demand function of good 1} \]
  \[ q_2 = q_2(p_1, p_2^0, Y^0) = q_2(p_1) \quad \text{Crossed demand function of good 2} \]

- **(3) Engel curve:** Shows the quantity demanded of a good as a function of the consumer’s income (both prices are given):

  \[ q_1 = q_1(p_1^0, p_2^0, Y) = q_1(Y) \quad \text{Engel curve of good 1} \]
  \[ q_2 = q_2(p_1^0, p_2^0, Y) = q_2(Y) \quad \text{Engel curve of good 2} \]
4.3 Individual demand curves and Engel curves

**Individual demand functions**

\[ q_i = q_i(p_i, p_j, Y) \]

**Behavioral curves**

1. **Ordinary demand curve:**
   \[ q_i = q_i(p_i) \]

2. **Crossed demand curve:**
   \[ q_i = q_i(p_j) \]

3. **Engel curve:**
   \[ q_i = q_i(Y) \]
4.3 Individual demand curves and Engel curves

(1) Ordinary demand curve of good 1:

Price-consumption curve (PCC): Shows how the quantity demanded changes when prices change. First we change $p_1$:

$$CPC_1 = \left\{ (q_1, q_2) \mid MRS^2_1 = \frac{p_1}{p_2}; \; p_2, Y \text{ constant}; \; p_1 \text{ free} \right\}$$

Ordinary demand curve (ODC):

$$ODC_1 = \left\{ (q_1, p_1) \mid MRS^2_1 = \frac{p_1}{p_2}; \; p_2, Y \text{ constant} \right\}$$
4.3 Individual demand curves and Engel curves

(2) Crossed demand function of good 1:

Price-consumption curve of good 1 (PCC): Shows how the quantity demanded changes when the price of good 2 changes. Therefore, we allow $p_2$ to change:

$$CPC_2 = \left\{(q_1, q_2) \mid MRS_1 = \frac{p_1}{p_2}; \ p_1, Y \text{ constant}; \ p_2 \text{ free}\right\}$$

Crossed demand curve (CDC):

$$CDC_1 = \left\{(q_1, p_2) \mid MRS_1 = \frac{p_1}{p_2}; \ p_1, Y \text{ constant}\right\}$$

In this case $q_1$ and $q_2$ are substitutes.
4.3 Individual demand curves and Engel curves

(3) Engel curve of good 1: **NORMAL GOOD**

**Income-consumption curve (ICC):**
Shows how the quantity demanded changes when the consumer’s income changes:

\[
ICC = \left\{(q_1, q_2) \mid MRS_2 = \frac{p_1}{p_2}; \quad p_1, p_2 \text{ constant}; \quad Y \text{ free}\right\}
\]

**Engel curve (EC):**

\[
EC_1 = \left\{(q_1, Y) \mid MRS_1 = \frac{p_1}{p_2}; \quad p_1, p_2 \text{ constant}\right\}
\]

\[\uparrow Y \Rightarrow \uparrow q_1\]
Normal good (increasing EC)
4.3 Individual demand curves and Engel curves

(3) Engel curve of good 1: INFERIOR GOOD

- The Engel curve of an inferior good is decreasing
- ↑ consumer’s income ⇒ ↓ quantity demanded

\[ q_1^0 \rightarrow Y \rightarrow q_1^1 \]
4.3 Individual demand curves and Engel curves

(3) Engel curve of good 1: NEUTRAL GOOD

- The Engel curve of a neutral good is vertical.
- ↑ consumer’s income ⇒ the quantity demanded neither ↑ nor ↓

![Diagram showing Engel curve of a neutral good](image-url)
4.4 Price and income elasticities

Elasticity:

- Measures the sensitivity of one variable to changes in another variable. It is defined as the percentual variation produced in variable B due to a 1% change in variable A:

\[ E_{AB} = \frac{\Delta B}{\Delta A} = \frac{\Delta B}{A} = A \cdot \frac{\Delta B}{B \cdot \Delta A} \]

- Elasticities are quantitative measures associated with each point on a behavioral curve:

**Individual demand functions**

- Ordinary demand curve: \( q_i = q_i(p_i) \)
- Crossed demand curve: \( q_i = q_i(p_j) \)
- Engel curve: \( q_i = q_i(Y) \)

**Graphical representation**

- **Price elasticity**
- **Crossed price elasticity**
- **Income elasticity**
4.4 Price and income elasticities

(1) PRICE ELASTICITY:

Price elasticity measures the variation in the quantity demanded of a good due to a change in its own price:

\[
e_{p_1q_1} = - \frac{\text{Variation (\%) in the quantity demanded of good } 1}{\text{Variation (\%) in the price of good } 1} = - \frac{p_1}{q_1} \frac{\Delta q_1}{\Delta p_1}
\]

- As a general rule, the demand of the good decreases when its price rises (normal demand), so we typically change the sign of the elasticity to positive (it is the only elasticity defined with the opposite sign). The price elasticity allows us to **CLASSIFY DEMANDS**.

- In infinitessimal terms:

\[
e_{p_1q_1} = - \frac{p_1}{q_1} \frac{dq_1}{dp_1}
\]
4.4 Price and income elasticities

(2) CROSSED PRICE ELASTICITY:

The crossed price elasticity measures the variation in the quantity demanded of a good due to a change in the price of the other good:

\[
\epsilon_{p_2q_1} = \frac{\text{Variation (\%) in the quantity demanded of good } 1}{\text{Variation (\%) in the price of good } 2} = \frac{p_2}{q_1} \frac{\Delta q_1}{\Delta p_2}
\]

- The sign of this elasticity does not change, since it reveals information about the RELATIONSHIP BETWEEN THE GOODS.

- In infinitesimal terms:

\[
\epsilon_{p_2q_1} = \frac{p_2}{q_1} \frac{dq_1}{dp_2}
\]
4.4 Price and income elasticities

(3) INCOME ELASTICITY:

The income elasticity measures the variation in the quantity demanded of a good due to a change in the consumer’s income:

\[ e_{Yq_i} = \frac{\text{Variation (\%) in the quantity demanded of good } i}{\text{Variation (\%) in the consumer's income}} = \frac{Y}{q_i} \frac{\Delta q_i}{\Delta Y} \]

- The sign does not change, since it is informative about the NATURE OF THE GOOD.
- In infinitesimal terms: \[ e_{Yq_i} = \frac{Y}{q_i} \frac{dq_i}{dY} \]
4.5 Classification of goods and demand

- The elasticities associated with behavioral curves allow us to classify the individual demand of a good:

1. **Price elasticity** $\Rightarrow$ Classification of the demand:

   - **Normal**
   - **Abnormal**

2. **Crossed price elasticity** $\Rightarrow$ Relationship between goods:

   - Gross complements
   - Gross independent
   - Gross substitutes

3. **Consumer’s income elasticity** $\Rightarrow$ Classification of goods:

   - Normal
   - Neutral
   - Inferior
4.5 Classification of goods and demand

(1) Price-elasticity \( \Rightarrow \) **Classification of demand:**

\[
e_{p_iq_i} = -\frac{p_i}{q_i} \frac{dq_i}{dp_i}
\]

- **Normal demand:** \( \downarrow p_i \rightarrow \uparrow q_i \Rightarrow \frac{dq_i}{dp_i} < 0 \Rightarrow e > 0 \) \( \begin{cases} e > 1 \Rightarrow \text{Elastic} \\ e = 1 \Rightarrow \text{Elasticity 1} \\ e < 1 \Rightarrow \text{Inelastic} \end{cases} \)

- **Abnormal demand:** \( \downarrow p_i \rightarrow \downarrow q_i \Rightarrow \frac{dq_i}{dp_i} > 0 \Rightarrow e < 0 \)

  \( \begin{cases} \text{Totally inelastic demand:} & \frac{dq_i}{dp_i} = 0 \Rightarrow e = 0 \\ \text{Extreme cases:} & \\ \text{Totally elastic demand:} & \frac{dq_i}{dp_i} = \infty \Rightarrow e = \infty \end{cases} \)
4.5 Classification of goods and demand

(1) Price-elasticity ⇒ **Classification of demand:**

**Normal demand:** Different variations in price and quantity:

\[ \downarrow p_i \rightarrow \uparrow q_i \Rightarrow \frac{dq_i}{dp_i} < 0 \Rightarrow e_{p_iq_i} > 0 \]

**i. Elastic:** Quantity demanded changes in a higher proportion than price:

\[ \downarrow p_1 (= 1\%) \rightarrow \uparrow q_1 (> 1\%) \Rightarrow e_{p_1q_1} > 1 \]
4.5 Classification of goods and demand

(1) Price-elasticity $\Rightarrow$ Classification of demand

**Normal demand:**

**ii. Inelastic:** Quantity demanded changes in a lower proportion than price:

\[
\downarrow p_1 (= 1\%) \rightarrow \uparrow q_1 (< 1\%) \Rightarrow 0 < e_{p \rightarrow q} < 1
\]
4.5 Classification of goods and demand

(1) Price-elasticity ⇒ Classification of demand

Normal demand:

iiii. Unitary demand: Quantity demanded changes in the same proportion as price:

\[
\downarrow p_1 (= 1\%) \rightarrow \uparrow q_1 (= 1\%) \Rightarrow e_{p_1q_1} = 1
\]
4.5 Classification of goods and demand

(1) Price-elasticity ⇒ Classification of demand:

Abnormal demand: Variations in price and quantity demanded move in the same direction (also known as “Giffen goods”):

\[ \uparrow p_i \rightarrow \uparrow q_i \Rightarrow \frac{dq_i}{dp_i} > 0 \Rightarrow e_{p_i q_i} < 0 \]
4.5 Classification of goods and demand

(1) Price-elasticity ⇒ Classification of demand

Extreme cases:

- **Perfectly inelastic (rigid) demand:**
  The quantity demanded is not sensitive to changes in price:
  \[ \uparrow p_i \rightarrow q_i \text{ does not change} \Rightarrow \frac{dq_i}{dp_i} = 0 \Rightarrow e_{p_i/q_i} = 0 \]

- **Perfectly elastic demand:**
  Any quantity is demanded, but only to a single fixed price:
  \[ \frac{dq_i}{dp_i} = \infty \Rightarrow e_{p_i/q_i} = \infty \]
4.5 Classification of goods and demand

(2) Crossed price-elasticity $\Rightarrow$ Relationship between goods:

$$E_{p_iq_j} = \frac{p_i}{q_j} \frac{dq_j}{dp_i}$$

$$\begin{cases} 
\text{Complements:} & \uparrow p_i (\downarrow q_i) \downarrow q_j \Rightarrow \frac{dq_j}{dp_i} < 0 \Rightarrow E_{p_iq_j} < 0 \\
\text{Substitutes:} & \uparrow p_i (\downarrow q_i) \uparrow q_j \Rightarrow \frac{dq_j}{dp_i} > 0 \Rightarrow E_{p_iq_j} > 0 \\
\text{Independent:} & \uparrow p_i (\downarrow q_i) \text{ unchanged } q_j \Rightarrow \frac{dq_j}{dp_i} = 0 \Rightarrow E_{p_iq_j} = 0 
\end{cases}$$
4.5 Classification of goods and demand

(2) Crossed price-elasticity ⇒ Relationship between goods:

**Substitutes (gross):**
Satisfy the same need or desire. When the price of one good changes, the quantity demanded of the other good changes in the same direction:

\[ \uparrow p_2 \rightarrow \downarrow q_2 \Rightarrow \uparrow q_1 \Rightarrow E_{p_2q_1} = \frac{p_2}{q_1} \frac{dq_1}{dp_2} > 0 \]

Increasing cross demand curve
4.5 Classification of goods and demand

(2) Crossed price-elasticity ⇒ Relationship between goods:

Complements (gross):
Satisfy jointly the same need. When the price of one good changes, the quantity demanded of the other good changes in the same direction:

\[ \uparrow p_2 \rightarrow \downarrow q_2 \Rightarrow \downarrow q_1 \Rightarrow E_{p_2q_1} = \frac{p_2 \ dq_1}{q_1 \ dp_2} < 0 \]

Increasing cross demand curve
4.5 Classification of goods and demand

(2) Crossed price-elasticity ⇒ **Relationship between goods:**

**Independent (gross):**

The needs or desires satisfied by both goods are unrelated:

\[ \frac{d p_2}{d q_1} \neq 0 \]

\[ E_{p_2 q_1} = \frac{p_2}{q_1} \left( \frac{dq_1}{dp_2} \right) = 0 \]

Vertical crossed demand curve
4.5 Classification of goods and demand

(3) Income elasticity ⇒ Classification of goods:

\[
E_{Yq_i} = \frac{q_i \, dq_i}{Y \, dY}
\]

Normal good: $Y \uparrow \rightarrow q_i \uparrow \Rightarrow \frac{dq_i}{dY} > 0 \Rightarrow E_{Yq_i} > 0 \Rightarrow \begin{cases} E_{Yq_i} > 1 \Rightarrow \frac{dq_i}{q_i} > \frac{dY}{Y} \Rightarrow \text{Luxury good} \\ E_{Yq_i} = 1 \Rightarrow \frac{dq_i}{q_i} = \frac{dY}{Y} \Rightarrow \text{Unit elasticity good} \\ E_{Yq_i} < 1 \Rightarrow \frac{dq_i}{q_i} < \frac{dY}{Y} \Rightarrow \text{Basic need good} \end{cases}
\]

Neutral good: $Y \uparrow \rightarrow q_i$ does not change $\Rightarrow \frac{dq_i}{dY} = 0 \Rightarrow E_{Yq_i} = 0$

Inferior good: $Y \uparrow \rightarrow q_i \downarrow \Rightarrow \frac{dq_i}{dY} < 0 \Rightarrow E_{Yq_i} < 0$
4.5 Classification of goods and demand

(3) Income elasticity ⇒ Classification of goods:

**Normal good:**

When income rises, quantity demanded rises (increasing EC):

\[ \uparrow Y \rightarrow \uparrow q_1 \Rightarrow \frac{dq_1}{dY} > 0 \Rightarrow E_{Yq_1} > 0 \]
4.5 Classification of goods and demand

(3) Income elasticity ⇒ Classification of goods:

**Normal good:**
- **Basic need good:** quantity demanded increases in a lower proportion than income
- **Luxury good:** quantity demanded increases in a higher proportion than income

**Unit income-elasticity:** Both variables change in the same proportion

- **Basic need good**
  - $0 < E_{Yq_1} < 1$

- **Luxury good**
  - $E_{Yq_1} > 1$

- **Unit income-elasticity**
  - $E_{Yq_1} = 1$
4.5 Classification of goods and demand

(3) Income elasticity ⇒ Classification of goods:

**Inferior good:**

When income decreases the quantity demanded increases (decreasing EC):

\[ \frac{dq_1}{dY} < 0 \Rightarrow E_{Yq_1} < 0 \]

\[ \uparrow Y \Rightarrow \downarrow q_1 \]

Q1 INFERIOR GOOD (decreasing)
4.5 Classification of goods and demand

(3) Income-elasticity ⇒ **Classification of goods:**

**Neutral good:**

When income changes the quantity demanded does not change (inelastic EC):

\[ \uparrow Y \rightarrow q_1 \text{ does not change} \Rightarrow \frac{dq_1}{dY} = 0 \Rightarrow E_{Yq_1} = 0 \]

![Diagram of Q1 Neutral Good]

\[ \uparrow Y \Rightarrow \text{neither } \uparrow \text{nor } \downarrow q_1 \]

\[ Y^0 \quad Y^1 \]

\[ q_1^0 = q_1^1 \]
Examples: Inferior good

When passing from A to B both burgers and steaks behave as normal goods, but burgers become an inferior good when income increases, such that the consumer equilibrium moves from B to C: the income-consumption curve turns backwards from equilibrium point B to point C:
Examples: Inferior good
SUMMARY OF THE UNIT:

Consumer equilibrium

Demand (Marshallian) functions

1. Ordinary demand curve
   \( q_i = q_i(p_i) \)

2. Crossed demand curve
   \( q_i = q_i(p_j) \)

3. Engel curve
   \( q_i = q_i(Y) \)

Price elasticity

Classification of demand

Crossed price elasticity

Relationship between goods

Income elasticity

Classification of goods

Classification of goods

Normal

Abnormal

Elastic

Inelastic

Complements

Independent

Substitutes

Normal

Neutral

Inferior

Basic need

Luxury

Normal

Abnormal

Elastic

Inelastic
APPENDIX: Relation between expenditure and elasticities

(1) Price elasticity and its relation with expenditure

- **Expenditure on good: 1:**  
  \[ G_1 = p_1q_1 = [ODC_1 : q_1 = q_1(p_1)] = p_1q_1(p_1) \]

- **Variation in expenditure when price changes:**
  \[
  \frac{dG_1}{dp_1} = q_1 + p_1 \frac{dq_1}{dp_1} = q_1 + p_1 \frac{q_1}{q_1} \frac{dq_1}{dp_1} = q_1 \left[ 1 + p_1 \frac{dq_1}{q_1} \frac{1}{dp_1} \right] = q_1 \left[ 1 - e_{p_1q_1} \right]
  \]

- **Therefore:**
  \[
  \frac{dG_1}{dp_1} \left\{ \begin{array}{ll}
  < 0 & \text{when demand is elastic } \quad (e_{p_1q_1} > 1) \iff \uparrow p_1 \rightarrow \downarrow G_1 \\
  = 0 & \text{when elasticity is 1 } \quad (e_{p_1q_1} = 1) \iff \uparrow p_1 \rightarrow G_1 \text{ does not change} \\
  > 0 & \text{when demand is inelastic } \quad (e_{p_1q_1} < 1) \iff \uparrow p_1 \rightarrow \uparrow G_1 
  \end{array} \right.
  \]
APPENDIX: Relation between expenditure and elasticities

(1) Price elasticity and its relation with expenditure:

Particular case → Linear demand

The same demand curve can behave as elastic and inelastic when evaluated at different points along the curve:
APPENDIX: Relation between expenditure and elasticities

(1) Price elasticity and its relation with expenditure:

The variation in consumer expenditure \( G_1 = p_1 q_1 \) when the price changes depends on the price elasticity of demand:

➢ Elastic range:

\[ e_{p_1 q_1} > 1 \]

\[ \uparrow p_1 (\downarrow q_1) \Rightarrow \downarrow G_1 \]

➢ Inelastic range:

\[ 0 < e_{p_1 q_1} < 1 \]

\[ \uparrow p_1 (\uparrow q_1) \Rightarrow \uparrow G_1 \]
APPENDIX: Relation between expenditure and elasticities

(2) Income and its relation with the % share of income spent on the good:

% Share of income spent on good 1: \( g_1 = \frac{p_1 q_1}{Y} \)

Variation of the % share of income spent on good 1 when income changes:

\[
\frac{dg_1}{dY} = \frac{d(p_1 q_1)}{Y} \frac{dY}{dY} = \frac{Y p_1}{Y^2} \frac{dg_1}{dY} - \frac{p_1 q_1}{Y^2} \frac{dY}{dY} = \frac{p_1 q_1}{Y^2} \frac{dq_1}{dY} - \frac{p_1 q_1}{Y^2} = \\
\frac{p_1 q_1}{Y^2} E_{Yq_1} - \frac{p_1 q_1}{Y^2} = \frac{p_1 q_1}{Y} \left( E_{Yq_1} - 1 \right) = \frac{g_1}{Y} \left( E_{Yq_1} - 1 \right)
\]

Therefore:

\[
\frac{dg_1}{dY} = \frac{g_1}{Y} \left( E_{Yq_1} - 1 \right) \begin{cases} < 0 & \text{if } E_{Yq_1} > 1 \iff \uparrow Y \rightarrow \uparrow g_1 \\ = 0 & \text{if } E_{Yq_1} = 1 \iff \uparrow Y \rightarrow g_1 \text{ does not change} \\ > 0 & \text{if } E_{Yq_1} < 1 \iff \uparrow Y \rightarrow \downarrow g_1 \end{cases}
\]
Exercises

1.- Victor is 12 years old and has lunch at school every day. His mother gives him 1,20 euros every day for
the lunch. Victor only likes “Bollycaos” ($Q_1$) and orange juice ($Q_2$), and consumption yields him a
utility given by the following utility function: $U=U(q_1, q_2)= (q_1 \cdot q_2)^{1/2}$

Answer the following questions:

a) Let’s suppose that both “Bollycaos” and orange juice cost 60 cens per unit, how do you think
that Victor should spend the Money that his mother gives him for lunch?

b) Compute the marginal rate of substitution in equilibrium and explain its economic meaning.

2.- Marisol’s utility function is: $U=U(q_1, q_2)= 2q_1^2 \cdot q_2^2$, where $Q_1$ y $Q_2$ are the only goods available, with
market prices $p_1=2$ y $p_2=1$ respectively. The consumer’s income is $Y = 12$.

a) What will the consumed quantities of both goods be in equilibrium (Find first the individual
marshallian demand functions).

b) Compute the MRS in equilibrium and explain its economic meaning.

c) Suppose that there is a change in the price of good 2, such that now $p_2=2$. Find the new
equilibrium. Are the individual marshallian demand functions useful? Why?

3.- Helen and Rachel have identical tastes regarding goods $Q_1$ and $Q_2$. Both earn the same income ($Y$).

a. Will they demand the same quantity of both goods? Justify your answers using graphs.

b. If the prices of both goods double at the same time as Helen’s income doubles and Rachel’s
income triples, then, can we conclude with certainty that both Helen and Rachel will demand a
larger quantity of $Q_1$? Justify your answers using graphs.

c. Consider the initial situation and assume that income is constant while the price of $Q_1$ doubles
and the price of $Q_2$ is reduced by 50%. Do the utilities of both Rachel and Helen increase? Justify
your answers using graphs.
4.- Consider an economy with only two goods \((Q_1, Q_2)\) with prices \(p_1=6\) and \(p_2=3\), respectively. A consumer chooses a market basket from the choice set such that \(\text{MRS}_{1,2}=1/2\). Can we be sure that the chosen market basket is the one that maximizes utility? Justify your answers using graphs.

5.- Natalia and Pablo spent the same income consuming the two goods \(Q_1\) and \(Q_2\). Natalia’s preferences can be represented by the following utility function: \(U=U(q_1, q_2)=q_1\cdot q_2\), and Pablo’s preferences by utility function \(U=U(q_1, q_2)=\frac{1}{2}(q_1q_2)\). Can we guarantee that, under these conditions, the quantities \(Q_1\) and \(Q_2\) demanded by Natalia will be twice the quantity chosen by Pablo?

6.- Give reasons for or against the following statements regarding consumer choices in equilibrium:
   a) Prices are exogenous variables.
   b) Quantities are exogenous variables.
   c) Income is endogenous.
   d) Income should not be spent in full.
   e) The marginal rate of substitution is constant.
7.- Peter is very fond of antiquities and his preferences are given by: \( U= (q_1-2)^2(q_2+5)^3 \) where \( Q_1 \) represents the number of XIX century chairs and \( Q_2 \) the number of XVIII century wardrobes. Obtain:
   a) The individual marshallian demand functions. Check that they are homogeneous functions with degree zero with respect to prices and income. What’s the meaning of this property?
   b) The consumer equilibrium when \( Y = 95, \ p_1 = 5, \ p_2 = 3 \).
   c) The analytical expression of the Engel curve of good \( Q_1 \).
   d) The analytical expression of the income elasticity of good \( Q_1 \).
   e) The value of the income elasticity of good \( Q_1 \) for \( Y = 100 \).
   f) The analytical expression of the Engel curve of good \( Q_2 \).
   g) The analytical expression of the income elasticity of good \( Q_2 \).
   h) The value of the income elasticity of good \( Q_2 \) for \( Y = 100 \).

8.- The Engel curve of good \( Q_1 \) is given by the expression: \( q_1 = q_1(Y)=100Y-2Y^2 \). Answer the questions:
   a) Draw the graphical representation of the above curve.
   b) Obtain the analytic expression of the income elasticity of good \( Q_1 \).
   c) Classify the good using income

9.- Consider a utility function representing a given consumer’s preferences (from the Cobb-Douglas family of functions), with \( 0 < \alpha < 1 \). Income is \( Y \) and prices are \( p_1 \) and \( p_2 \). Answer the questions:
   a) Obtain the individual Marshallian demand functions of both goods.
   b) Find analytically and graphically the Engel curve of good \( Q_2 \) when prices are \( p_1=p_2=1 \).
   c) What’s the meaning of parameter \( \alpha \)? Describe its influence on the Engel curve and on the income elasticity of demand.
10. The optimal quantity of clothing ($q_1$) chosen by Clara for a given combination of prices ($p_1$ and $p_2$) can be represented by the following individual (Marshallian) demand curve:

$$q_i = \frac{3Y - p_1 + 2p_2}{2p_1}$$

where $Y = 50$, $p_1 = 10$ and $p_2 = 5$. 

Answer the following questions:

a) Find the analytical expression of the ordinary demand curve of good $Q_1$.
b) Find the analytical expression of the direct price elasticity of good $Q_1$.
c) Classify the ordinary demand curve of $Q_1$ under different intervals of the price.
d) Find the analytical expression of the crossed demand function of good $Q_1$.
e) Find the analytical expression of the crossed price elasticity of good $Q_1$.
f) How can we classify the relation between these two goods (in gross terms)? Give reasons for your answer.

11.- Peter considers coffee ($q_1$) and sugar ($q_2$) as perfect complements. For each cup of coffee he drinks he always put a single spoon of sugar. A utility function representing his preferences is the following:

$$U = \min\{q_1, q_2\}$$

Peter has a given budget $Y$ for spending on these two goods and market prices are $p_1$ and $p_2$. Answer the following questions:

a) Find the individual demand functions of both goods.
b) Find analytically the Engel curve of coffee assuming that both prices are equal to one (unitary).
c) Find the income elasticity of the good. What is the percentage change of the quantity of sugar if the government increases income taxes by 25%.
12.- Discuss whether the following statements are true or false in the light of consumer theory and give reasons: When the consumer’s income rises but the prices of the goods are constant, this:
   a) Changes the Engel curve of good Q₁.
   b) Changes the analytical expression of the income elasticity of any good.
   c) Changes the specific value of the income elasticity.
   d) Changes the price-consumption curve.

13.- Discuss whether the following statements are true or false in the light of consumer theory and give reasons: A change in the price p₂ (“caeteris paribus”):
   a) Changes the marginal rate of substitution of good 1 for good 2 (RMS₂<sub>1</sub>.
   b) Changes the Engel curve of good Q₂
   c) Changes the ordinary demand curve of good Q₂.
   d) Changes the crossed demand curve of good Q₁.
14.- Discuss whether the following statements are true or false in the light of consumer theory and give reasons: If the price-consumption curve of good Q₁ (PCC₁) is horizontal:
   a) The ordinary demand curve of Q₁ should be horizontal.
   b) The ordinary demand curve of Q₁ is vertical.
   c) The price elasticity of demand of Q₁ is positive.
Answer the previous questions for the case of the PCC₁ being vertical instead of horizontal.

15.- Discuss whether the following statements are true or false in the light of consumer theory and give reasons: A change in consumer tastes:
   a) Changes the marginal rate of substitution.
   b) Changes the consumer equilibrium.
   c) Changes the specific value of the marginal rate of substitution at equilibrium.
   d) Changes the income-consumption curve.
   e) Changes the price-consumption curve.
   f) The Engel curve does not change.
   g) The function of the price elasticity remains the same.

16.- Knowing that the demand of food is inelastic, give reasons for whether consumers will spend more or less on food when, due to a bad crop, the price of food rises.

17. The ordinary demand curve of a good is given by the following expression: q=100/p. Can you predict how will the consumer spending change when the price of the good changes?
18. Given an ordinary demand curve, which among the following situations is likely to cause a fall in the total spending on the good by the consumer?
   a) A fall in the price from a point at any inelastic section of the curve.
   b) A rise in the price from a point at any inelastic section of the curve.
   c) A fall in the price from a point at any elastic section of the curve.
   d) A fall in the price at a section of unit elasticity.
   e) None of the above is true.

19. Given the ordinary demand curve of a good: \( q = q(p) = 5 - p \), answer the following questions:
   a) Draw a graph of the curve.
   b) Calculate the point or point intervals such that \( e_{pq} = 1, e_{pq} < 1, e_{pq} > 1, e_{pq} = \), \( e_{pq} = 0 \).
   c) If you were a buyer with that curve, at which point of the curve would you prefer to be located? Why?