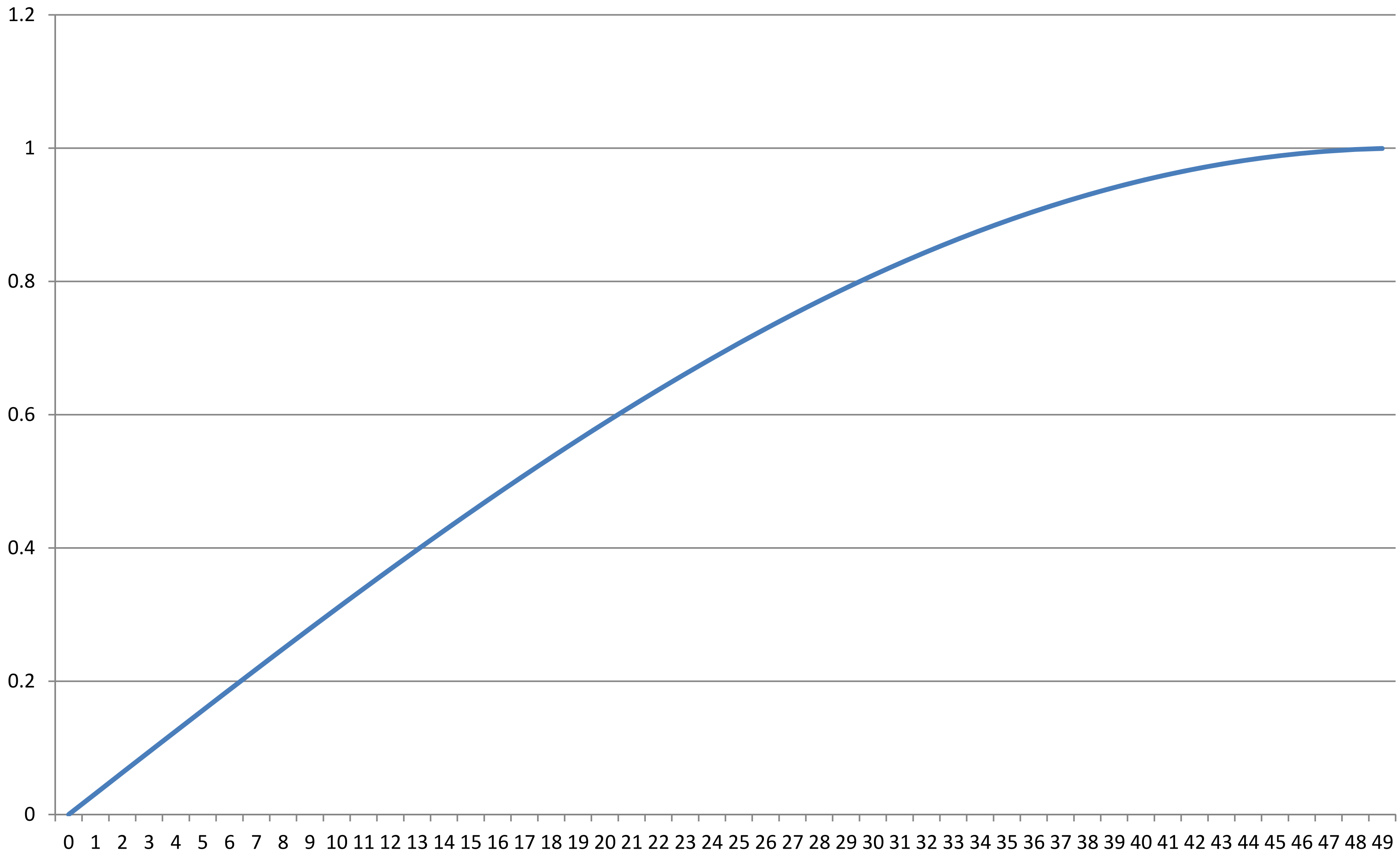
Univariate Analysis
Trends

Applied Econometrics

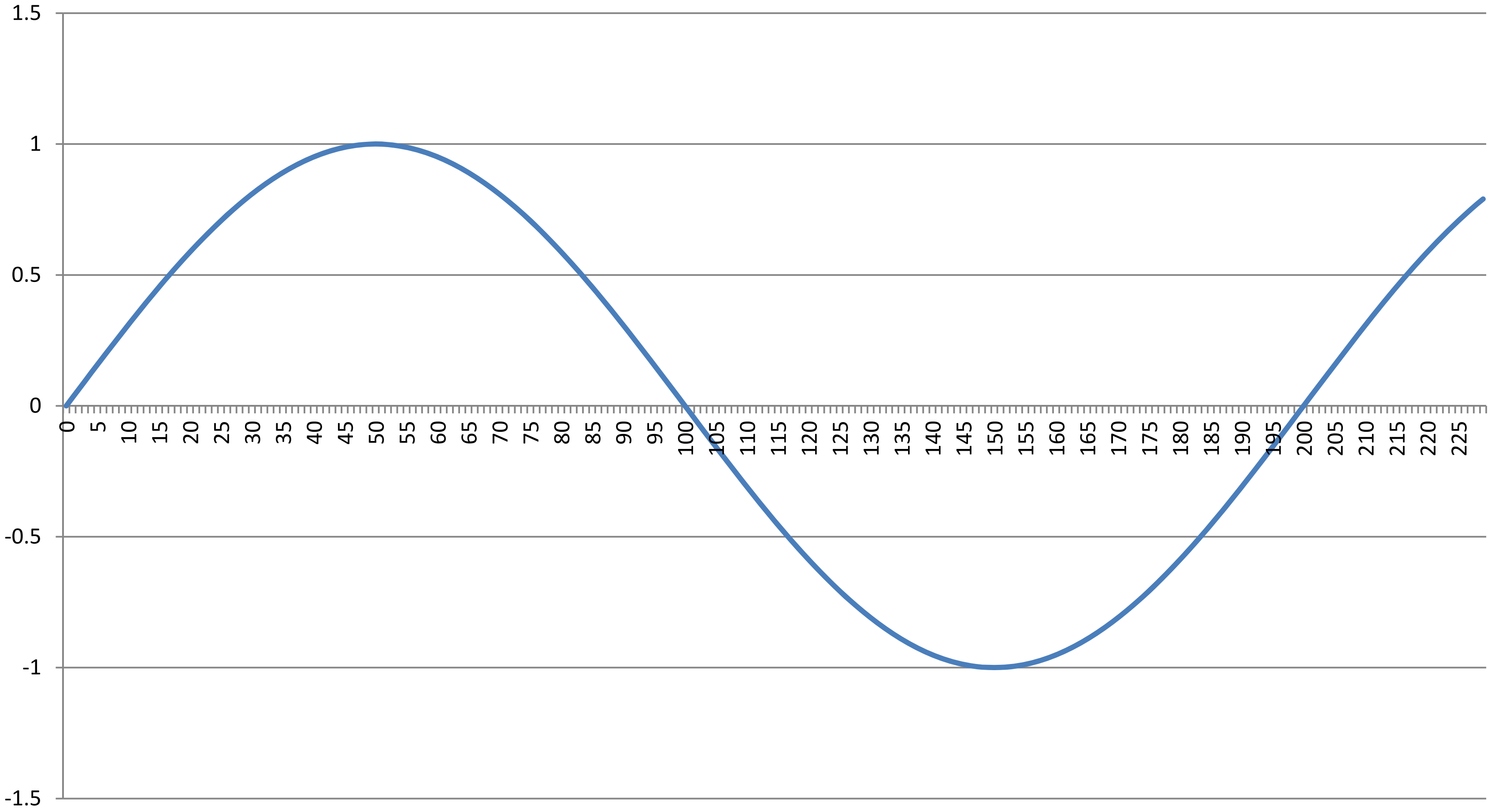
Trends

- While cycles reflect the short-run behavior of a variable, trends are related to the long-run behavior.
- There exist whenever an increase in the data of variable is observed
- However, this is a long-run concept and we should dispose a wide sample size to observer it properly

Does this variable exhibit a trend?



Are you sure?



1.5
1
0.5
0
-0.5
-1
-1.5
0 5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85 90 95 100 105 110 115 120 125 130 135 140 145 150 155 160 165 170 175 180 185 190 195 200 205 210 215 220 225

Types of trends

- Linear vs non-linear trends.

$y_t = \alpha + \beta\, t, \quad t = 1, 2, \ldots, T$

$y_t = \alpha + \beta\, t + \gamma\, t^2, \quad t = 1, 2, \ldots, T$

Why may non-linearities appear in economic variables?

Assymmetries, thresholds….

Types of trends

- Deterministic vs stochastics trends.

A) $y_t = \alpha + \beta\ t + u_t$

$u_t = \rho\ u_{t-1} + e_t$, with e_t being a WN and $|\ \rho\ |<1$

B) $y_t = \alpha + \phi\ y_{t-1} + u_t$

$u_t = \rho\ u_{t-1} + e_t$, with e_t being a WN and $|\ \rho\ |<1$ and....

$\phi=1$

Can you see the trend in model B?

Types of trends

$y_t = \alpha + y_{t-1} + u_t = \alpha + (\alpha + y_{t-2} + u_{t-1}) + u_t =$

$= 2\alpha + y_{t-2} + u_{t-1} + u_t = \ldots =$

$= \alpha + \alpha + \ldots + \alpha + u_t + u_{t-1} + u_{t-2} + \ldots + u_1 + u_0 =$

$= \alpha\, t + \sum_{i=1}^{t} u_i$

This is referred to as a stochastic trends and it is generated by the value 1 of the parameter ϕ

Trends

- Is it important to distinguish from deterministic and stochastic trends?
- Modern time series econometrics depends absolutely on this difference.
- Standard time series econometrics is still valid under the presence of deterministic trends
- If we find stochastic trends, then we should apply newest techniques: Unit root/Cointegration

Stationarity

- Let $\{y_t\}_{t=-\infty}^{t=\infty}$ be a sequence of stochastic variables. The, this process is said to be stationary whevener it is true that:

$$F(y_1, y_2, \ldots, y_m) = F(y_{k+1}, y_{k+2}, \ldots, y_{k+m})$$

for any value of t and m, where F(.) is the joint distribution function.

Weak stationary process

- A stochastic process is said to be weak stationary or 2nd-order stationary if it holds:

$E(y_t) = \mu \ \forall t$

$Var(y_t) = \sigma^2 \ \forall \ t$

$Cov(y_i \ y_j) = Cov(y_{m+i} \ y_{m+j}) = \gamma(i\text{-}j) = \gamma(j\text{-}i) \ \forall \ i,j$

Weak stationary process

REMARK

It is straightforward to see that if the process follows a Normal distribution, then weak stationarity implies strict stationarity.

Thus, we will work with this concept.

Integration

A variable y_t is said to be integrated of order d, if we should difference it d-times for it to be stationarity.

- If $y_t \sim I(d)$, then $\Delta^d yt \sim I(0)$
- Let y_t be an I(0) variable. Is it stationary?
- Which is the most habitual value of d in economics?

Testing for unit roots

Then, we have two different types of trends:

A) Trend-stationary models (TS)

B) Difference-stationary models (DS)

How can we distinguish between them?

Testing for unit roots

If we join both types of models, we obtain the following model:

$$y_t = \alpha + \beta\, t + \rho\, y_{t-1} + u_t$$

Then, if ρ =1 (and β =0), we obtain the DS model

$$y_t = a + y_{t-1} + u_t$$

Testing for unit roots

By contrast, if $|\rho|<1$ and $\beta \neq 0$, then we obtain the TS model.

$$y_t = \alpha + \beta t + \rho y_{t-1} + u_t$$

Testing for unit roots

Dickey-Fuller pseudo t-ratio

H_o: ρ =1

$$y_t = \alpha + \beta\ t + \rho\ y_{t-1} + u_t$$

$$\tau = \frac{\hat{\rho} - 1}{\sigma_{\hat{\rho}}}$$

This statistic does not follow any "standard" distribution… rather it depens on Wiener process.

Testing for unit roots

Augmented Dickey-Fuller pseudo t-ratio

H_o: $\rho = 1$

$$y_t = \alpha + \beta\, t + \rho\, y_{t-1} + \sum_{i=1}^{m} \varphi_i \Delta y_{t-i} + u_t$$

$$\tau = \frac{\hat{\rho} - 1}{\sigma_{\hat{\rho}}}$$

Where m should be selected by the researcher (MAIC)

Testing for unit roots

If the variable does not exhibit trend, it is also possible to test for unit roots by estimating the following model:

H_o: ρ =1

$$y_t = \alpha + \rho\, y_{t-1} + \sum_{i=1}^{m} \varphi_i \Delta y_{t-i} + u_t$$

$$\tau_\mu = \frac{\hat{\rho} - 1}{\sigma_{\hat{\rho}}}$$

How can we identify short-run structure?

- We should first remove the long-run behavior.
- If we cannot reject the unit root null hypothesis, the variable is integrated and we should take differences.
- If we reject the unit root null hypothesis, we should remove the deterministic trend. The best way is to use OLS and analyze the residuals for determining the short-run behavior.