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On the sustainability of bargaining solutions in family decision models

Sustainability in family decision models

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Abstract This paper analyses the sustainability of family bargaining agreements by developing a non-cooperative game between two spouses with symmetric preferences. To that end, we develop, by using a general utility function, a repeated non-cooperative game involving two players with symmetric preferences, where the characterization of a Nash sub-game perfect equilibrium allows us to demonstrate that the spouse with the greater bargaining power has a greater incentive to reach an agreement. This result is also reproduced by using a particular example of linear preferences in consumption. However, the influence of the bargaining power on the sustainability of a bargaining solution depends on the specification of the individual preferences, as well as the degree of altruism between the spouses.

Keywords Family bargaining · Sustainability of agreements · Efficiency

JEL Classifications C71 · C62 · J12

The application of bilateral bargaining models has represented an important advance in the study of family decision-making. One of the essential features of these bargaining models is that family demand does not depend solely on total family resources, but also on those controlled by each member individually. This implies that the result achieved depends on whatever is the threat point or status quo of the

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bargaining process, for example, the divorce situation. In this way, the family bargaining models reflected in the literature consider the decisions made by individuals to be the result of an explicitly defined bargaining solution (Manser and Brown 1980; McElroy and Horney 1981; Chen and Woolley 2001; Andaluz and Molina 2007).

Nevertheless, divorce does not represent the only possible threat point in a process of this nature. In this sense, a non-cooperative equilibrium could equally be the threat point in the bargaining process, in such a way that the repeated interaction between the agents over time can tacitly lead to efficient results (Lundberg and Pollak 1993; 1994; Suen et al. 2003). More specifically, and in accordance with the folk theorem, a Pareto-efficient solution can be derived as a Nash equilibrium in a repeated game, always provided that there is some strategy which penalizes all deviations from the efficient solution. Therefore, Pareto-optimum results can arise as repeated games solutions. However, it has also recently been shown that the achievement of private gains on the part of each spouse, combined with the limitations in compromising the future behavior of both spouses, can give rise to decisions that are no longer Pareto-efficient (Lundberg and Pollak 2003).

Against this background, our objective is to analyse the dynamic aspects of the bilateral bargaining process within the family and to draw conclusions with respect to the sustainability of the Pareto-efficient solutions.[1] In particular, we are interested in analyzing the effects of factors such as the bargaining power, the individual preferences and the degree of altruism of spouses on the sustainability of efficient equilibrium. To achieve this, we develop, by using a general utility function, a repeated non-cooperative game involving two players with symmetric preferences, but with bargaining power and income levels being the only sources of asymmetry between the agents.

Introducing the so-called trigger strategy (Friedman 1971) as a punishment scheme, the characterization of a Nash sub-game perfect equilibrium allows us to demonstrate that the spouse with the greater bargaining power has a greater incentive to reach an agreement. This result is also reproduced by using a particular example of linear preferences in consumption. However, the influence of the bargaining power on the sustainability of a bargaining solution depends on the specification of the individual preferences, as well as on the degree of altruism between the spouses.

The article is structured as follows. Section 1 develops the game by using a general formulation of preferences. Section 2 analyzes, according to these preferences, the sustainability of the agreements. Section 3 develops a particular example by using linear preferences and, finally, Sect. 4 closes the paper with a summary of the most relevant conclusions.

1 Developing the game

In this Section we develop a repeated game in which the two members of a family can contribute voluntarily to the supply of one household public good. After

[1] The term sustainability here means the absence of incentives to deviate from the Nash-bargaining solution; it does not refer to the ability to withstand random shocks.

assuming that we do not know the moment at which the dissolution of marriage takes place, the objective of each agent is to maximize the discounted value of a flow of utilities $\sum_{t=1}^{\infty}\delta^{t-1}W_j(U_j,U_k);(j\neq k,j,k=w,h)$, where δ denotes the discount factor, common to both agents, and $W_j(U_j,U_k)$ indicates the welfare function of agent j, which itself depends on the own utility level, U_j and on that of the spouse, U_k. Formally, each player has a welfare function of the type $W_j = U_j + s U_k$, with $s \in [0, 1]$ denoting the degree of altruism of the spouses, which it assumes, for simplification purposes, to be common to both agents.[2]

The utility of each agent takes the following general specification:

$$U_j(x_j,g_h,g_w)=x_jg_h^{\alpha}g_w^{1-\alpha};0<\alpha<1;j=h,w \tag{1}$$

in such a way that utility depends on the consumption of one private good, x_j, and on the contribution made by each spouse to the provision of a household public good, in the production of which he/she is specialised. Thus, g_j and g_k denote the available quantities of two household public goods provided by spouse j and spouse k, respectively. In this way, the preference structure considered is in line with the Lundberg and Pollak (1993) separate spheres framework, where the spouses produce different types of household public goods. These decisions, in terms of domestic production and, in consequence, in terms of labor supply, may affect the spouses' bargaining power. This introduces some difficulties in the sustainability of agreements, since it could be costly for spouses to commit to efficient solutions (Iyigun 2005).

In this context, and taking into account that the main objective of this paper is to analyse the effect of the bargaining power on the sustainability of the agreements, we assume that both individuals have identical preferences, with bargaining power and income levels being the only sources of asymmetry between the agents.

In the development of the non-cooperative equilibrium, each agent decides, given the decisions made by the other player, both the consumption of the private good and the contribution to the household public good. In this case, the solution of the one-shot game will be given by the Cournot-Nash equilibrium.[3] Formally, each individual must solve the following conditioned optimization problem:

$$\begin{gathered} Max\ W_j(U_j,U_k) \\ s.t.\ x_j+P_jg_j=Y_j \\ g_k=\bar{g}_k \end{gathered} \tag{2}$$

where P_j is the price of the household public good provided by j, Y_j is the income of individual j (we assume $Y_h > Y_w$) and, without loss of generality, the price of the consumption good is normalized to one. The solution of that problem is given by both the amounts of the private good and the public good:

[2] These preferences have been given the name "caring preferences" and have been adopted by, amongst others, Bourguignon and Chiappori (1992).

[3] Following the model proposed by Suen et al. (2003), we do not consider an internal transfer of income in the non-cooperative equilibrium. However, as we later see, these authors propose that every bargaining solution will implicitly incorporate an income transfer from one spouse to the other. In our case, we have assumed that the husband is the donor and his wife is the recipient.

$$g_h^N = \frac{\alpha[(2-\alpha-(1-\alpha)s^2)Y_h + sY_w]}{[(1+\alpha)(2-\alpha)-\alpha(1-\alpha)s^2]P_h}; \; g_w^N = \frac{(1-\alpha)[(1+\alpha-\alpha s^2)Y_w + sY_h]}{[(1+\alpha)(2-\alpha)-\alpha(1-\alpha)s^2]P_w} \quad (3)$$

$$x_h^N = \frac{(2-\alpha)Y_h - \alpha sY_w}{(1+\alpha)(2-\alpha)-\alpha(1-\alpha)s^2}; \; x_w^N = \frac{(1+\alpha)Y_w - (1-\alpha)sY_h}{(1+\alpha)(2-\alpha)-\alpha(1-\alpha)s^2} \quad (4)$$

with the level of welfare associated with that combination being:

$$W_h^N = \frac{\alpha^\alpha(1-\alpha)^{(1-\alpha)}[(2-\alpha-(1-\alpha)s^2)Y_h + sY_w]^{(1+\alpha)}[(1+\alpha-\alpha s^2)Y_w + sY_h]^{(1-\alpha)}}{[(1+\alpha)(2-\alpha)-\alpha(1-\alpha)s^2]^2 P_h^\alpha P_w^{1-\alpha}} \quad (5)$$

$$W_w^N = \frac{\alpha^\alpha(1-\alpha)^{(1-\alpha)}[(2-\alpha-(1-\alpha)s^2)Y_h + sY_w]^{\alpha}[(1+\alpha-\alpha s^2)Y_w + sY_h]^{(2-\alpha)}}{[(1+\alpha)(2-\alpha)-\alpha(1-\alpha)s^2]^2 P_h^\alpha P_w^{1-\alpha}} \quad (6)$$

We should note that the repetition of the game gives rise to multiple equilibria, some of which must represent Pareto-efficient solutions. Indeed, both agents might implicitly create some strategy that avoids all possible deviation from an optimal solution, and which guarantees the achievement of Pareto-efficiency as a Nash equilibrium in the one-shot game.

One of these possible strategies consists of penalizing the agent who deviates unilaterally from the agreement. More specifically, we adopt a relatively simple, but nevertheless commonly employed, punishment scheme, namely the so-called trigger strategy, according to which the quantities of private and public good revert forever to non-cooperative levels following a deviation from the efficient solution on the part of one of the agents.[4] The threat of punishment through the return to the non-cooperative solution is credible, and guarantees the sustainability of solutions which are more efficient than the Cournot-Nash equilibrium.

Furthermore, and again for the sake of simplicity, we consider the case of stationary trajectories,[5] arguing that a stationary trajectory is sustainable in a sub-game perfect equilibrium if, for all j, the following conditions are satisfied: $W_j^C - W_j^N \geq 0$ and $\frac{W_j^C}{(1-\delta)} \geq W_j^{Ch} + \delta\frac{W_j^N}{(1-\delta)}$, where W_j^C and W_j^{Ch} denote the levels of welfare obtained by agent j in the Pareto-efficient solution derived from the Nash-bargaining agreement, and in the deviation equilibrium, respectively. Note that the second of the restrictions can be expressed in the following form:

$$\delta \geq \frac{W_j^{Ch} - W_j^C}{W_j^{Ch} - W_j^N} \equiv \bar{\delta}_j \quad (7)$$

[4] This punishment scheme is one among others. (see Abreu 1986).

[5] It is clear that this assumption supposes a degree of simplification. As Lundberg and Pollak (2003) indicate, the environment in which the family bargaining takes place is not always stationary. Furthermore, some decisions can change the context in which the future bargaining will be carried out.

where $\bar{\delta}_j$ is the critical discount factor of individual j. Thus, the sustainability of the optimal solution requires that the discount factor, common to both individuals, is greater than or equal to the corresponding critical factor. In other words, the higher the value of that critical factor, the lower the sustainability of the Pareto-efficient equilibrium, given that the set of discount factors which guarantees the sustainability of the agreement will be smaller.

In every bargaining solution the contributions to the household public good are always Pareto-efficient. Being that t is an internal transfer from one spouse to the other, specifically, the husband transfers an amount of income to his spouse, then the set of efficient solutions is determined by the following problem:

$$\begin{aligned} \underset{x_h, x_w, g_h, g_w, t}{Max} \; W_w &= U_w + sU_h \\ s.t.\; U_h + sU_w &= W_h \\ x_h + P_h g_h &= Y_h - t \\ x_w + P_w g_w &= Y_w + t \end{aligned} \tag{8}$$

From the first-order conditions, and satisfying the second-order conditions, we can deduce the levels of private consumption and the provision of the public goods, as well as the optimum level of transfer t*:

$$g_h^C = \frac{\alpha(Y_h + Y_w)}{2P_h};\; g_w^C = \frac{(1-\alpha)(Y_h + Y_w)}{2P_w} \tag{9}$$

$$x_h^C = \frac{(2-\alpha)Y_h - \alpha Y_w - 2t^*}{2};\; x_w^C = \frac{(1+\alpha)Y_w - (1-\alpha)Y_h + 2t^*}{2} \tag{10}$$

$$t^* = \frac{[2-\alpha-s(1-\alpha)]Y_h + [(1+\alpha)s - \alpha]Y_w}{2(1-s)} - \frac{2P_h^\alpha P_w^{1-\alpha} W_h^C}{(1-s)\alpha^\alpha(1-\alpha)^{1-\alpha}(Y_h + Y_w)} \tag{11}$$

Substituting such values in the objective function, we obtain the utility possibilities frontier W_w (P_h, P_w, Y_h, Y_w, s, W_h), with $\frac{dW_w}{dW_h}$ being its slope.

Let us suppose that there is a bargaining process according to which the agents choose the generalized Nash-bargaining solution.[6,7] That is to say, they choose the stationary trajectory of amounts that maximizes the product of the utilities normalized by the levels associated with the non-cooperative equilibrium.

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[6] The dynamic strategic bargaining model of alternating offers proposed by Rubinstein (1982) stands as an alternative to the static Nash bargaining model. Given that the agents do not know when the game will end, and that they have the same discount factor, there is an exact equivalence, always provided that the time interval between successive offers tends to zero, between the Nash bargaining solution and the equilibrium obtained in a strategic model with exogenous risk of breakdown and the same time preferences (see Binmore et al. 1986).

[7] The Nash bargaining solution used here is not fully analogous to the perfect equilibrium, from a strategic bargaining model with outside options, described in Bergstrom (1997), and initially proposed by Binmore (1985). This is so because, following Bergstrom (1997), the substitution of the threat point by the outside options does not guarantee equivalence between the Nash bargaining solution and perfect equilibrium.

Formally, the equilibrium can be obtained from the solution to the following maximization problem:

$$\underset{W_h}{Max}\, J = (W_h - W_h^N)^{\beta}\left[W_w(P_h, P_w, Y_h, Y_w, s, W_h) - W_w^N\right]^{(1-\beta)} \tag{12}$$

where $\beta \in [0,1]$ denotes the bargaining power of the husband, and $(1 - \beta)$ represents that of the wife. There are various factors which determine the value of the said parameter (see Agarwal 1997). For example, in certain societies there are legal restrictions that limit women's control over property, which have a negative influence on their bargaining power. Similarly, various institutional practices, such as the refusal to grant loans to a woman without the consent of her husband, or cultural norms such as the concept of honour and submission to the will of the husband, can suppose greater bargaining power for men than for women. From an individual point of view, each spouse can rely on individual factors of bargaining power, such as education or his/her physical strength. By analogy, in marriage each spouse can employ strategies that increase his/her bargaining power. Nevertheless, such a possibility is beyond the scope of this analysis and, in its place, it is assumed that parameter β is given exogenously.

The first order condition for this problem satisfies:

$$\beta\left[W_w(P_h, P_w, Y_h, Y_w, s, W_h^C) - W_w^N\right] + (1-\beta)(W_h^C - W_h^N)\frac{dW_w}{dW_h^C} = 0 \tag{13}$$

where:

$$W_h^C = \beta(A - W_w^N) + (1-\beta)W_h^N \tag{14}$$

is the welfare level of the husband that, introduced in the utility possibilities frontier, determines the spouse's welfare level associated with the bargaining solution:

$$W_w^C = (1-\beta)(A - W_h^N) + \beta W_w^N \tag{15}$$

where:

$$A = \frac{(1+s)(Y_h + Y_w)^2 \alpha^{\alpha}(1-\alpha)^{1-\alpha}}{2P_h^{\alpha}P_w^{1-\alpha}} \tag{16}$$

Furthermore, if one of the agents decides to unilaterally deviate from the agreement, then he/she must choose the combination of the amounts of both the private good and the public good that solve the following problem:

$$\begin{gathered} Max W_j(x_j, x_k^C, g_j, g_k^C, s) \\ s.t. x_j + x_k^C + P_j g_j + P_k g_k^C = Y_j + Y_k \end{gathered} \tag{17}$$

If it is the husband who deviates from the bargaining solution, we obtain the equilibrium in deviation:

$$g_h^{Ch} = \frac{\alpha\left[(1+\alpha)(Y_h+Y_w) - 2(1-s)x_w^C\right]}{2(1+\alpha)P_h} \tag{18}$$

$$x_h^{Ch} = \frac{\left[(1+\alpha)(Y_h+Y_w) - 2(1+\alpha s)x_w^C\right]}{2(1+\alpha)} \tag{19}$$

where the associated level of satisfaction is:

$$W_h^{Ch} = \frac{\left[\alpha^{1+\alpha}(1-\alpha)^{1-\alpha}(Y_h+Y_w)^2 + 4W_h^C P_h^\alpha P_w^{1-\alpha}\right]^{1+\alpha}}{4\alpha^{\alpha^2}(1-\alpha)^{\alpha(1-\alpha)}(1+\alpha)^{1+\alpha}(Y_h+Y_w)^{2\alpha}P_h^\alpha P_w^{1-\alpha}} \tag{20}$$

Assuming that it is the wife who unilaterally deviates from the agreement, the levels of both private consumption and provision of the household public good are:

$$g_w^{Ch} = \frac{(1+\alpha)\left[(2-\alpha)(Y_h+Y_w) - 2(1-s)x_w^C\right]}{2(2-\alpha)P_h} \tag{21}$$

$$x_h^{Ch} = \frac{\left[(2-\alpha)(Y_h+Y_w) - 2[1+s(1-\alpha)]x_w^C\right]}{2(2-\alpha)} \tag{22}$$

with the welfare level being:

$$W_w^{Ch} = \frac{\left[\alpha^\alpha(1-\alpha)^{1-\alpha}(2+s-\alpha)(Y_h+Y_w)^2 - 4W_h^C P_h^\alpha P_w^{1-\alpha}\right]^{2-\alpha}}{4\alpha^{\alpha(1-\alpha)}(1-\alpha)^{(1-\alpha)^2}(2-\alpha)^{2-\alpha}(Y_h+Y_w)^{2(1-\alpha)}P_h^\alpha P_w^{1-\alpha}} \tag{23}$$

2 Analyzing the sustainability of the agreements

Having reached this point, we are now in a position to analyze the sustainability of the bargaining solution. As mentioned earlier, the sustainability of this solution requires that the discount factor of each player is not lower than that corresponding to the critical value. Introducing (5), (6), (14), (15), (20) and (23) into the definition of the critical discount factor of each of the agents (7), we can deduce a dependency of the critical discount factor of each spouse on the bargaining power (β) and to the degree of altruism $(s) : \bar{\delta}_j = \bar{\delta}_j(\beta, s);\ j = h, w$.

The evolution of that factor with respect to each of its arguments allows us to analyze their influence on the sustainability of the agreements.

Lemma 1 *The welfare gains derived from the bargaining solution are greater for the spouse with the greater bargaining power.*

Proof The difference between (14) and (15) allows us to deduce the husband's welfare surplus derived from the bargaining process. Analogously, the difference between (15) and (16) determines the wife's welfare gains derived from the agreement.

From these analyses, we can derive the following inequalities: $(W_w^C - W_w^N) > (W_h^C - W_h^N) \Leftrightarrow \beta < \frac{1}{2}$, and $(W_w^C - W_w^N) < (W_h^C - W_h^N) \Leftrightarrow \beta > \frac{1}{2}$.

Thus, the gains of the agreement, alternatively, the losses arising from the deviation, are greater for the spouse with the greater bargaining power. □

To study the evolution of the discount-critical factors in terms of the bargaining power, without loss of generality, we specify the following values: $\alpha = 0.5$, $s = 0.25$, $Y_w = 1$, $Y_h = 2$, $P_h = P_w = 1$.

Under these assumption, we can deduce the following proposition:

Proposition 1 *The sustainability of the bargaining solution is only determined by the behavior of the husband. The husband's bargaining power exerts an ambiguous effect on the sustainability of the bargaining agreement.*

Proof From (23) and (15), we can deduce that, for the wife, the difference between the welfare associated with the deviation, and that derived from the agreement, is negative for every value of both the degree of altruism and the bargaining power. Formally: $W_w^{Ch} - W_w^C < 0;\ \forall\ 0 < s < 1,\ 0 < \beta < 1$.

Consequently, under the general preferences specified, the recipient of the internal transfer never deviates from the bargaining agreement, with the sustainability of this agreement being determined by the spouse's behavior.

Figure 1 shows the evolution of the husband's critical discount factor in terms of the value of β. From this, we can deduce that such factor exhibits a decreasing trend for low values of the representative parameter of the bargaining power. Reaching a specific level, this critical factor adopts an increasing evolution. Formally: $\frac{\partial \delta_h}{\partial \beta} < 0\ \forall\ 0 < \beta < \tilde{\beta} = 0.214$, and $\frac{\partial \bar{\delta}_h}{\partial \beta} > 0\ \forall\ \tilde{\beta} = 0.214 < \beta < 1$.

Thus, if the wife's bargaining power is sufficiently higher, an increase in the relative bargaining power of the husband reinforces the sustainability of the agreement, given that the incentives to unilaterally deviate from the bargaining solution are reduced.

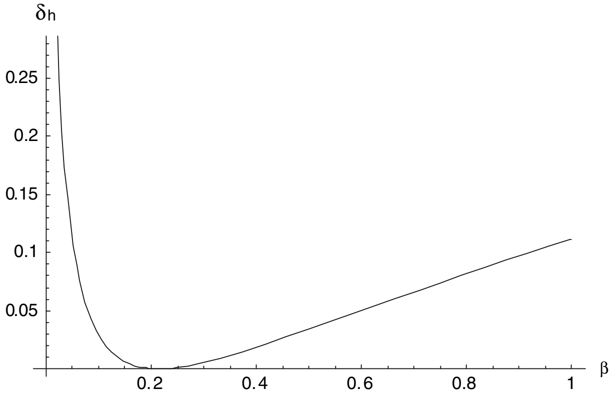

Fig. 1 Effect of β on the sustainability of the agreement

By contrast, if the husband's relative power exceeds a certain level, increasing his bargaining power reduces the possibility of maintaining the agreement given that, for such spouse, the gains derived from the deviation are much greater.

On the other hand, a more detailed analysis allows us to deduce that the higher the degree of altruism between the spouses, the less the value of $\tilde{\beta}$. In particular, we have $\tilde{\beta} \rightarrow 0$ if $s \rightarrow 1$ and $\tilde{\beta} \rightarrow 0.38$ if $s \rightarrow 0$. Thus, a higher degree of altruism reduces the interval by which the critical discount factor is decreasing. □

3 A specific linear example

With the aim of studying the robustness of the results with respect to the characterization of the individual preferences, we now develop the game by using a more restrictive specification of the utility functions. The utility of each agent is linear in the private consumption, with, as in the general case, both bargaining power and income levels being the only source of asymmetry between the agents:

$$U_j(x_j, g_h, g_w) = x_j + \alpha Lng_h + (1-\alpha)Lng_w; \;\; (j = h, w; 0<\alpha<1) \tag{24}$$

The parameter α denotes the extent to which both spouses value the contribution to each household public good. Thus, one partner happens to be good at a type of household production that both spouses value highly, whereas the other partner specializes in a household production that is less valued by each spouse.

Accordingly, the levels of private consumption and provision of the family good associated with the non-cooperative equilibrium are given by:

$$g_h^N = \frac{\alpha(1+s)}{P_h}; \;\; g_w^N \frac{(1-\alpha)(1+s)}{P_w} \tag{25}$$

$$x_h^N = Y_h - \alpha(1+s); \;\; x_w^N = Y_w - (1-\alpha)(1+s) \tag{26}$$

with the individual welfare levels being:

$$\begin{aligned} W_h^N =& Y_h + sY_w - (1+s)[\alpha + s(1-\alpha)] + (1+s) \\ &\times \left[\alpha Ln\left(\frac{\alpha}{P_h}\right) + (1-a)Ln\left(\frac{1-\alpha}{P_w}\right) + Ln(1+s)\right] \end{aligned} \tag{27}$$

$$\begin{aligned} W_w^N =& Y_w + sY_h - (1+s)(1-\alpha+\alpha s) + (1+s) \\ &\times \left[\alpha Ln\left(\frac{\alpha}{P_h}\right) + (1-a)Ln\left(\frac{1-\alpha}{P_w}\right) + Ln(1+s)\right] \end{aligned} \tag{28}$$

The resolution of problem (8) allows us to derive the Pareto-efficient values of the private consumption and the provision of the household public good. In particular, we deduce the following:

$$g_h^C = \frac{2\alpha}{P_h};\; g_w^C = \frac{2(1-\alpha)}{Pw} \tag{29}$$

$$x_h^C = Y_h - t^* - 2\alpha;\; x_w^C = Y_w + t^* - 2(1-\alpha) \tag{30}$$

with the transfer level being:

$$t^* = \frac{Y_h + sY_w - 2[\alpha + s(1-\alpha)] + (1+s)\left[\alpha Ln\left(\frac{\alpha}{P_h}\right) + (1-\alpha)Ln\left(\frac{1-\alpha}{P_w}\right) + Ln2\right] - W_h}{1-s} \tag{31}$$

According to (29), (30) and (31), the resolution of the optimization problem (12) provides the welfare levels associated with the Nash bargaining agreement:

$$W_h^C = \beta\left[(1+s)k - W_w^N\right] + (1-\beta)W_h^N \tag{32}$$

$$W_w^C = (1-\beta)\left[(1+s)k - W_h^N\right] + \beta W_w^N \tag{33}$$

where:

$$k = Y_h + Y_w + 2\left[\alpha Ln\left(\frac{\alpha}{P_h}\right) + (1-\alpha)Ln\left(\frac{1-\alpha}{P_w}\right) + Ln2 - 1\right] \tag{34}$$

whereas the welfare levels associated with the non-cooperative equilibrium are given by (27) and (28).

Finally, the resolution of problem (17) gives us the values corresponding to the equilibrium in deviation. In particular, if it is the husband who deviates from the agreement solution, then his contribution to the household public good, and the level of private consumption are:

$$g_h^{Ch} = \frac{\alpha(1+s)}{P_h};\; x_h^{Ch} = Y_h - t^* - \alpha(1+s) \tag{35}$$

with the welfare level being:

$$\begin{aligned} W_h^{Ch} &= Y_h + sY_w - (1-s)t^* - [\alpha(1-s) + 2s] \\ &+ (1+s)\left[\alpha Ln\left(\frac{\alpha(1+s)}{P_h}\right) + (1-\alpha)Ln\left(\frac{2(1-\alpha)}{P_w}\right)\right] \end{aligned} \tag{36}$$

Analogously, if it is the wife who deviates from the agreement, the levels of household public good and private consumption are:

$$g_w^{Ch} = \frac{(1-\alpha)(1+s)}{P_w};\; x_w^{Ch} = Y_w + t^* - (1-\alpha)(1+s) \tag{37}$$

where the welfare level is:

$$W_w^{Ch} = Y_w + sY_h + (1-s)t^* - [\alpha(1-s) - 1 - s] + (1+s)\left[\alpha Ln\left(\frac{2\alpha}{P_h}\right) + (1-\alpha)Ln\left(\frac{(1+s)(1-\alpha)}{P_w}\right)\right] \tag{38}$$

From (27), (28), (32) and (33), we obtain the welfare surplus for each spouse derived from the bargaining process. In particular, we derive:

$$(W_h^C - W_h^N) = \beta(1+s)\left[2Ln\left(\frac{2}{1+s}\right) - 1 + s\right] \tag{39}$$

$$(W_w^C - W_w^N) = (1-\beta)(1+s)\left[2Ln\left(\frac{2}{1+s}\right) - 1 + s\right] \tag{40}$$

and from the difference between these two expressions we deduce: $(W_w^C - W_w^N) > (W_h^C - W_h^N) \Leftrightarrow \beta < \frac{1}{2}$, and $(W_w^C - W_w^N) < (W_h^C - W_h^N) \Leftrightarrow \beta > \frac{1}{2}$.

Thus, the gains of the agreement are greater for the spouse with the greater bargaining power, conforming to Lemma 1

On the other hand, from (32), (33), (36) and (38), we deduce that the surplus associated with the deviation is independent of the bargaining power. Moreover, from the husband's point of view, the surplus exhibits a decreasing relation with respect to the degree of altruism, with this relation increasing from the wife's perspective.

Introducing the expressions (27), (28), (32), (33), (36) and (38) in the definition of the critical discount factor, given in (7), we can analyze the influence of the bargaining power on the sustainability of the bargaining agreement. Specifically, we can deduce the following:

Proposition 2 *Under the particular preferences given by (24), increases in the bargaining power increase the sustainability of the bargaining agreement.*

Proof The sustainability of every agreement requires that neither of the agents have incentives to deviate from the bargaining solution. Given that both spouses have the same discount factor, the sustainability will be maintained when such a factor is no less than the maximum of the critical discount factors, that is to say,$\delta \geq \max\{\bar{\delta}_h, \bar{\delta}_w\}$,

In order to establish a comparison between the critical discount factors, we introduce the following values $\alpha = 0.5$, $s = 0.25$, which do not reduce the generality of our conclusions. In this way, we deduce: $\bar{\delta}_h > \bar{\delta}_w \; \forall \; 0 < \beta < 0.071$, and $\bar{\delta}_h < \bar{\delta}_w \; \forall \; 0.071 < \beta < 1$.

Consequently, the husband's behavior will determine the sustainability of the bargaining agreement only when his bargaining power is reduced, given that in this situation his incentives to reach an agreement are also greatly reduced.

On the other hand, under the earlier values of α and s, Figs. 2 and 3 show the evolution of the critical discount factor in terms of the parameter β.

From Fig. 2, we can deduce a decreasing relation between the husband's critical factor and his own bargaining power: $\frac{\partial \bar{\delta}_h}{\partial \beta} < 0$. Thus, an increase in β reduces the

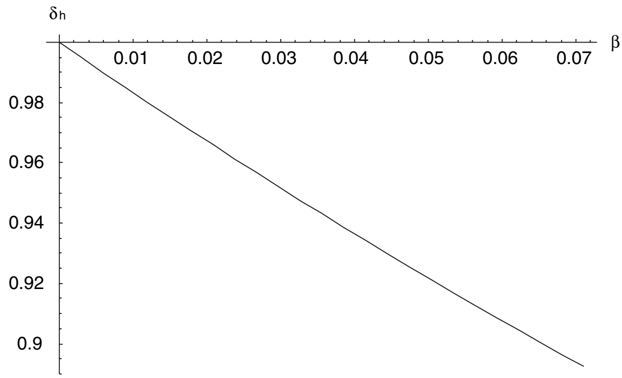

Fig. 2 Evolution of critical discount factor of the husband

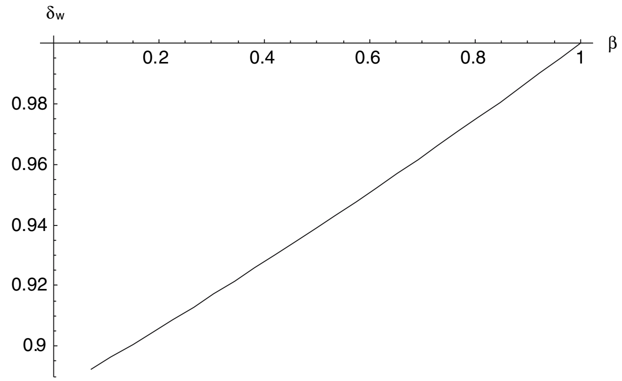

Fig. 3 Evolution of critical discount factor of the wife

value of the critical factor and, in consequence, increases the possibility that the bargaining solution will be sustained.

Analogously, Fig. 3 represents the evolution of the wife's critical factor in terms of β, deriving an increasing relation: $\frac{\partial \bar{\delta}_w}{\partial \beta} > 0$. Hence, a greater wife's bargaining power $(1 - \beta)$, reduces the value of her critical factor, as well as the incentives to deviate from the bargaining solution. □

4 Concluding remarks

Obtaining efficient allocations has been one of the aspects that has guided the economic modeling of the family decision-making process, with this being the case both in those models which view the family as a singular agent, and in those that apply the theory of bilateral bargaining.

The application of bargaining models requires that we define a priori a given threat point, which itself crucially conditions the results and has clear consequences for the intra-family distribution of the resources. Although divorce has traditionally been viewed as a representative situation of the status quo, a number of contributions have appeared in the literature which identify the threat point with a non-cooperative solution.

This latter approach requires the assumption that Pareto-efficient solutions can emerge through repeated interactions between the agents and within some implicit strategy that punishes any deviation from a Pareto-efficient solution. Nevertheless, the sustainability of agreements can be influenced by factors that similarly intervene in any bargaining process.

Against this background, in this paper we have shown how bargaining power, the degree of altruism and the preferences of the agents can determine the possibility of tacitly reaching efficient results in a context of family bargaining.

More specifically, the resolution of an infinitely repeated game, in which the two family members participate, with these being inter-related through the provision of two household public goods, a given degree of altruism and an internal transfer in every bargaining solution, has led us to the following conclusions.

First, the welfare gains derived from a bargaining agreement are greater for the spouse with a greater bargaining power.

Second, under a general specification of preferences, characterized by multiplicative levels of private consumption and household public goods, we have deduced that the sustainability of every agreement is only conditioned by the behavior of the transfer's donor, given that the recipient has no incentive to unilaterally deviate from the bargaining solution.

In this context, a husband's greater bargaining power can influence positively or negatively the sustainability of the agreement. In particular, in situations where the wife has a greater relative power in the bargaining process, increases in the husband's power will reduce his critical discount factor and, consequently, favor the sustainability of the bargaining solution. By contrast, when the husband initially has a sufficiently higher bargaining power, increases in his power will increase the incentives for him to deviate from the agreement.

On the other hand, the positive or negative influence of the bargaining power on the sustainability of the agreements, crucially depends on the degree of altruism between the spouses. A greater degree of altruism reduces the interval in which the critical discount factor exhibits a decreasing evolution and, in consequence, it is more likely that increases in the bargaining power of the spouse whose behavior is determinant, make it more difficult to maintain the bargaining solution.

Third, under a more restrictive specification of the individual preferences, by using a linear utility function in the private consumption, it is more likely that the sustainability of the bargaining solution is determined by the behavior of the wife as recipient of the internal transfer, given that her critical discount factor is dominant for a wide range of values of the parameter representing bargaining power.

In any case, we derive a decreasing relation between the critical discount factor and the bargaining power. Thus, a greater relative power on the part of the spouse whose behavior is determinant, will favor the sustainability of every efficient solution.

Finally, the conclusions presented in this paper are in line with those offered by Lundberg and Pollak (2003), who question the capacity of families to achieve efficient allocations through a bilateral bargaining process. Thus, in any bargaining model, the contribution of each individual to the family resources depends on his/her relative bargaining power and, therefore, on the control of the resources, with this being so whether or not cooperative behavior exists. As a result, a change in the value of the threat point of one of the agents can exert an influence on his/her bargaining power which, in accordance with the conclusions we have reached, could have dynamic effects leading to inefficient solutions.

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