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# MODELLING THE DEMAND BEHAVIOUR OF SPANISH CONSUMERS USING PARAMETRIC AND NON-PARAMETRIC APPROACHES

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## Abstract

This article models the demand behaviour of consumers using the parametric and non-parametric approaches. To that end, we consider Spanish time-series from 1964 to 1995 relative to the quantities and prices of five aggregated consumption goods, namely Food, Clothing, Energy, Transport and Miscellaneous Goods. With respect to the non-parametric results, we find that there is a stable demand system underlying the personal preferences structure which explains the observed quantities of goods and is equivalent to the existence of a well-behaved utility function. The parametric results show that the homogeneous and symmetric version of the Rotterdam model satisfies the econometric and theoretical requirements, and hence can properly be used for modelling the demand behaviour of Spanish consumers from 1964 to 1995.

## 1. Introduction

Economic research on the demand behaviour of consumers has usually been carried out by specifying a parametric functional form and then estimating it. However, this process is followed without any consideration being given as to whether that functional form is a good approximation to the "true" demand function, and without previously testing the consistency of the data with the neoclassical model of consumer behaviour based on the utility maximisation hypothesis. In this context, it is well known that demand theory is based on the assumption that the consumer chooses the preferred bundle of goods from among all those available for a set of

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prices. However, one prior and fundamental question that emerges in the empirical analysis of consumer behaviour is whether this choice is consistent with the utility maximisation hypothesis. An extended methodology for solving this question is the non-parametric approach derived from revealed preference theory. This does not require *ad hoc* functional specifications for demand equations because, on the basis of the available information, that is to say, quantities and prices, such an approach allows us to test whether the data are consistent with the condition of utility maximisation.

Against this background, in this paper we model the demand behaviour of consumers using two complementary approaches, that is to say, the parametric, formulated in terms of different demand equation models, and the non-parametric, derived from revealed preference theory. With respect to the first, we start from an initial general framework which allows us to derive the generic demand equations system. Thereafter, we specify different models which are estimated in order to choose the best demand system, in the sense that such a system has to satisfy a number of empirical and theoretical requirements. Specifically, we consider the most commonly employed demand systems in the relevant literature, namely the linear expenditure system (Stone 1954), the translog (Christensen, Jorgenson, and Lau, 1975), the almost ideal demand system (Deaton and Muellbauer, 1980a) and the Rotterdam model (Barten, 1964; Theil, 1965). With respect to the non-parametric approach, we carry out an analysis based on revealed preference theory, which allows us to test for the consistency of consumer behaviour with respect to the utility maximisation hypothesis. In particular, we test the weak and generalised axioms of revealed preference theory, with the purpose of determining, on the basis of the observed behaviour in the market, whether individuals choose the quantities they prefer from among the available alternatives (Afriat, 1967; Houthakker, 1950; Samuelson, 1948; Varian, 1982).

The above analyses are carried out using Spanish time-series from 1964 to 1995 on the quantities and prices of five aggregated consumption goods: Food, Clothing, Energy, Transport and Miscellaneous Goods. After a preliminary descriptive analysis, this data base first allows us to test the axioms of revealed preference theory and, secondly, to estimate the different demand equations systems in order to choose the best formulation. Given that the data takes the form of time-series, the empirical analysis of this parametric method includes an econometric procedure, which first implies testing for the presence of autocorrelation problems. Using formulations that do not exhibit this kind of problem, we then test the theoretical hypotheses, that is to say, homogeneity and symmetry, with these being imposed in the model if they are statistically accepted. In such a case, it then becomes necessary to test for the existence of autocorrelation on this restricted formulation and, if this econometric problem is rejected, we can calculate the expenditure and Marshallian and Hicksian price elasticities. With respect to the non-parametric approach, we use the NONPAR program, especially prepared by Varian (1985), with the purpose of testing the axioms of revealed preference theory.

The rest of the paper is organised as follows. In Section 2 we discuss the two theoretical approaches, the parametric and the non-parametric. The data and implementation of these approaches are considered in Section 3. Section 4 is devoted to the results of the analysis, while Section 5 closes the paper with a summary of the most relevant conclusions.

## 2. Theoretical approaches

### 2.1 The parametric approach

Let us assume a rational consumer with an available income  $y$  to spend on the purchase of  $n$  different consumption goods,  $Q = (Q_1, \dots, Q_n)$ . The consumer considers both the monetary income  $y$  and the prices of the goods  $p = (p_1, \dots, p_n)$  to be exogenous; furthermore, he has the possibility of consuming the desired quantities and does not face transaction costs. Given the prices of the goods and the monetary income, the individual chooses a particular consumption vector,  $q = (q_1, \dots, q_n)$ . This vector belongs to the consumption set defined as the non-negative space  $R_+^n$  and formed by the non-negative quantities of the  $n$  consumption goods which maximise his utility, where these quantities do not imply an expenditure higher than available income. Thus, assuming the existence of a mathematical function which represents the preferences of the consumer, that is to say, the utility function  $u(q)$ , we can formulate the consumer equilibrium as the solution of a restricted problem,  $\text{Max } u(q)$  subject to  $y = pq$ , whose first-order conditions allow us to derive the Marshallian demand functions,  $q_i = q_i(p, y)$  ( $i = 1, \dots, n$ ). Therefore, we can generically define a demand equations system as a set of  $n$  Marshallian demand functions which relate the quantities demanded with both the prices and the total expenditure:

$$\left. \begin{aligned} q_1 &= q_1(p_1, p_2, \dots, p_n, y) \\ q_2 &= q_2(p_1, p_2, \dots, p_n, y) \\ &\dots \\ q_n &= q_n(p_1, p_2, \dots, p_n, y) \end{aligned} \right\} \dots (1)$$

In order to particularise this generic demand system, let us now consider the most important specific functional forms, namely the Linear Expenditure System (LES), the transcendental-logarithmic (translog) model, the Almost Ideal Demand System (AIDS) and the Rotterdam model.

First, the LES, which is formulated from the Stone-Geary utility function,

$u(q) = \prod_{i=1}^n (q_i - \gamma_i)^{\beta_i}$ , where  $\gamma_i \geq 0$  can be interpreted as the minimum subsistence consumption and, thus,  $q_i > \gamma_i$ . Moreover  $\beta_i > 0$  in order to guarantee an increasing

utility function. The restricted maximisation of such a function allows us to derive the Marshallian demand equation of the LES:

$$q_i = \gamma_i - \beta_i \frac{y - \sum_j^n p_j \gamma_j}{p_i} \quad (i = 1, \dots, n) \quad \dots (2)$$

Secondly, the system made-up of the transcendental-logarithmic (translog) functions, formulated by a quadratic indirect utility function in the logarithms of the divisions between prices and total income or expenditure,

$$\ln v(\mathbf{p}, y) = \alpha_0 + \sum_i^n \alpha_i \ln \left( \frac{p_i}{y} \right) + \frac{1}{2} \sum_i^n \sum_j^n \beta_{ij} \ln \left( \frac{p_i}{y} \right) \ln \left( \frac{p_j}{y} \right). \text{ From Roy's Theorem,}$$

$w_i = - \frac{\partial \ln v(\mathbf{p}, y) / \partial \ln p_i}{\partial \ln v(\mathbf{p}, y) / \partial \ln y}$ , we can obtain the Marshallian demand functions of the model in terms of budget shares:

$$w_i = \frac{\alpha_i + \sum_k^n \beta_{ik} \ln \left( \frac{p_i}{y} \right)}{\sum_j^n \alpha_j + \sum_j^n \sum_k^n \beta_{jk} \ln \left( \frac{p_k}{y} \right)} \quad (i = 1, \dots, n) \quad \dots (3)$$

Given that this parametric representation of the preferences does not impose any theoretical condition, it is possible to study the integrability of the system by testing the restrictions of symmetry and negativity, having previously imposed the adding-up condition. Thus, assuming that  $\sum_j^n \alpha_j = \alpha_M = -1$  and  $\sum_j^n \beta_{jk} = \beta_{Mk}$ , the symmetry condition implies testing whether  $\beta_{ij} = \beta_{ji}$ , while the negativity restriction establishes that  $\alpha_i \leq 0$ .

Thirdly, the AIDS, derived from the following PIGLOG expenditure function,  $\log c(\mathbf{p}, u) = (1-u) \log a(\mathbf{p}) + u \log b(\mathbf{p})$ , where  $0 \leq u \leq 1$ , and where the linear homogeneous functions  $a(\mathbf{p})$  and  $b(\mathbf{p})$  can be interpreted as the subsistence expenditure ( $u = 0$ ), and the expenditure corresponding to the maximum satisfaction situation ( $u = 1$ ), respectively. This model incorporates the following functions for the prices:

$$\log a(\mathbf{p}) = \alpha_0 + \sum_k^n \alpha_k \log p_k + \frac{1}{2} \sum_k^n \sum_j^n \gamma_{kj}^* \log p_k \log p_j, \text{ and}$$

$$\log b(\mathbf{p}) = \log a(\mathbf{p}) + \beta_0 \prod_k^n p_k^{\beta_k}, \text{ with the objective of ensuring that the system is a}$$

flexible functional form. From Shephard's Lemma,  $\frac{\partial c(\mathbf{p}, u)}{\partial p_i} = h_i$ , we can obtain the

Hicksian demand function and, assuming that the consumer spends all his income, we can obtain the indirect utility function. This can then be substituted in the Hicksian function, thereby obtaining the Marshallian demand equation in terms of budget shares:

$$w_i = \alpha_i + \sum_j^n \gamma_{ij} \log p_j + \beta_i \log \left( \frac{y}{P} \right) \quad (i = 1, \dots, n) \quad \dots (4)$$

where  $\log P = \alpha_0 + \sum_k^n \alpha_k \log p_k + \frac{1}{2} \sum_k^n \sum_j^n \gamma_{kj}^* \log p_k \log p_j$ . After imposing the adding-up condition,  $\sum_i^n w_i = 1 \rightarrow \sum_i^n \alpha_i = 1, \sum_i^n \gamma_{ij} = \sum_i^n \beta_i = 0$ , demand theory implies that we test several other restrictions on the parameters of the model, namely homogeneity, which establishes that  $\sum_i^n \gamma_{ij} = 0$ , symmetry, which implies that  $\gamma_{ij} = \gamma_{ji}$  and, finally, negativity, which supposes that the matrix of substitution effects must be negative semi-definite.

Fourthly, the Rotterdam model, which characterises a demand system that is not associated to any particular utility function and which starts from a generic demand equation  $q_i = q_i(p, y)$ , that is directly approximated using its logarithmic

differentiation,  $d \log q_i = \sum_j^n e_{ij}^y d \log p_j + e_i d \log y$ , where  $e_{ij}^y$  and  $e_i$  are the

Marshallian price and the expenditure elasticities, respectively. In order to obtain the final version of this model, we consider the Slutsky equation in terms of elasticities,  $e_{ij}^y = e_{ij}^u - w_j e_i$ , where  $e_{ij}^u$  is the Hicksian price elasticity and  $w_j$  is the budget share. Substituting this equation in the logarithmic differentiation and considering that  $\theta_{ij}^* = w_i e_{ij}^u$ ,  $\mu_i = w_i e_i$  and  $\bar{y} = y/p$ , we finally derive the Marshallian demand function of the model:

$$w_i d \log q_i = \sum_j^n \theta_{ij}^* d \log p_j + \mu_i d \log \bar{y} \quad (i = 1, \dots, n) \quad \dots (5)$$

After imposing the adding-up condition,  $\sum_i^n \mu_i = 1$  and  $\sum_i^n \theta_{ij}^* = 0$ , the homogeneity condition implies that  $\sum_i^n \theta_{ji}^* = 0$  and, finally, symmetry establishes that  $\theta_{ij}^* = \theta_{ji}^*$ .

## 2.2 The non-parametric approach

The non-parametric approach to demand analysis derives algebraic conditions on the demand functions implied by utility-maximising behaviour. These conditions,

known as "revealed preference" conditions, provide a complete list of the restrictions imposed by such behaviour, in the sense that all the maximising behaviour of consumers must satisfy these conditions and, further, that all the behaviour which satisfies these conditions can be viewed as maximising. Non-parametric methods have been developed to test data for consistency with utility maximisation by means of the weak (WARP) and generalised (GARP) axioms of revealed preference. One attractive property of these tests is that they do not require a demand system to be specified, and hence do not suppose any explicit restrictions on functional form. Therefore, they offer a convenient and informative means of scanning a consumption data set for evidence of violations of demand theory.

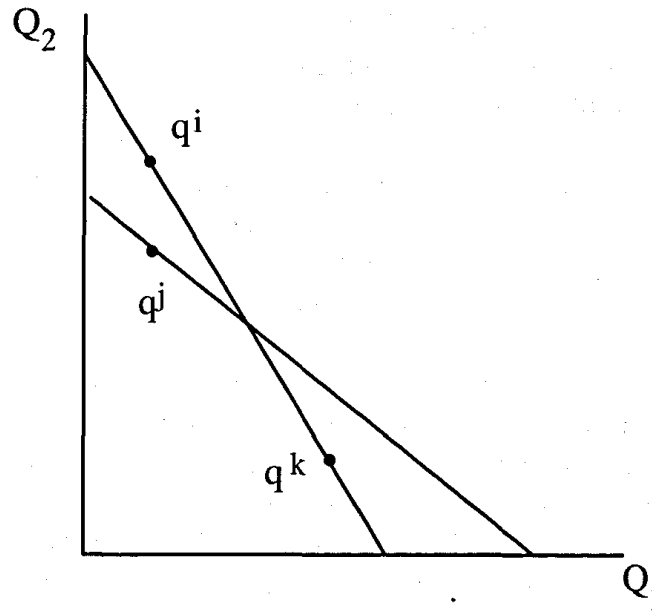
Let us now consider how we can test whether certain empirical observations regarding a representative consumer are in accordance with the utility maximisation hypothesis. Let  $\mathbf{q}^i = (q_1^i, \dots, q_n^i)$  and  $\mathbf{p}^i = (p_1^i, \dots, p_n^i)$  denote the vector of quantities and prices corresponding to  $n$  goods and let us further suppose that we have  $m$  observations  $(\mathbf{q}^i, \mathbf{p}^i)$ . If a consumer chooses a bundle of goods  $\mathbf{q}^i$  when an alternative bundle of goods  $\mathbf{q}^j$  can be obtained with the same budget outlay, he is revealing a preference for bundle  $\mathbf{q}^i$  over bundle  $\mathbf{q}^j$ , that is to say,  $\mathbf{q}^i$  is revealed preferred to  $\mathbf{q}^j$ , with this usually being denoted as  $\mathbf{q}^i R \mathbf{q}^j$ .

The weak axiom (WARP) states that if  $\mathbf{q}^i$  is revealed preferred to  $\mathbf{q}^j$ , then  $\mathbf{q}^j$  cannot be revealed preferred to  $\mathbf{q}^i$ . In other words, bundle  $\mathbf{q}^j$  will only be chosen when it is cheaper than  $\mathbf{q}^i$ , that is to say, bundle  $\mathbf{q}^i$  cannot be obtained with the same outlay. Figure 1 shows how the axiom is satisfied and how it is violated. Assuming only two goods,  $Q_1$  and  $Q_2$ , the line through  $\mathbf{q}^i$  represents the budget line when the consumer chooses bundle  $\mathbf{q}^i$ , while that through  $\mathbf{q}^j$  is the budget line when he chooses bundle  $\mathbf{q}^j$ . The weak axiom would be satisfied if commodity bundle  $\mathbf{q}^i$  is chosen when  $\mathbf{q}^j$  is available, that is to say,  $\mathbf{q}^j$  is in the budget set bounded by the line through  $\mathbf{q}^i$ , whilst  $\mathbf{q}^j$  is chosen when  $\mathbf{q}^i$  is unattainable with the given budget, that is to say, when  $\mathbf{q}^i$  is outside the budget set. By contrast, it would be violated if bundle  $\mathbf{q}^k$ , rather than  $\mathbf{q}^i$ , is chosen when  $\mathbf{q}^j$  lies within the budget set, but  $\mathbf{q}^j$  is chosen when  $\mathbf{q}^k$  is available.

This axiom can be expressed in terms of expenditures. For  $\mathbf{q}^i$  to be revealed preferred to  $\mathbf{q}^j$ , both  $\mathbf{q}^i$  and  $\mathbf{q}^j$  must be available for a given income,  $\mathbf{p}^i \mathbf{q}^i \geq \mathbf{p}^i \mathbf{q}^j$ , where  $\mathbf{p}^i$  denotes the set of prices when  $\mathbf{q}^i$  is chosen. Thus, the expenditure on  $\mathbf{q}^i$  is at least as great as the expenditure on  $\mathbf{q}^j$ . Thus, if the weak axiom is satisfied, then,  $\mathbf{p}^i \mathbf{q}^i \geq \mathbf{p}^i \mathbf{q}^j$  and  $\mathbf{p}^j \mathbf{q}^i > \mathbf{p}^j \mathbf{q}^j$ .

The second consistency condition relates to the transitivity of consumer choices. Thus, the generalised axiom (GARP) states that if for some sequence of bundles

$(q^i, q^j, q^k, \dots, q^m)$ ,  $q^i$  is revealed preferred to  $q^j$  ( $p^i q^i \geq p^i q^j$ ),  $q^j$  is, in turn, revealed preferred to a third bundle  $q^k$  ( $p^j q^j \geq p^j q^k$ ), and so on until  $q^l$  is revealed preferred to  $q^m$  ( $p^l q^l \geq p^l q^m$ ), then bundle  $q^m$  cannot be strictly revealed preferred to  $q^i$  ( $p^m q^m < p^m q^i$ ).



**Figure 1: The Weak Axiom of Revealed Preference Theory**

If some data satisfies GARP, there is a satisfactory utility function  $u(q)$  that will rationalise the observed behaviour; that is to say, the utility derived from the revealed preferred bundle is greater than, or equal to, the utility corresponding to the bundle which is not chosen ( $p^i q^i \geq p^i q^j \leftrightarrow u(q^i) \geq u(q^j)$ ). By contrast, if the data contains a violation of this Axiom, then a non-satiated utility function that will rationalise the data does not exist. However, when can the observations be rationalised by a sufficiently well behaved non-degenerate utility function? The best answer to this question is provided by Afriat's Theorem, which holds that: "The following conditions are equivalent: (1) there is a non-satiated utility function that rationalises the data; (2) the data satisfies GARP; (3) there are numbers  $U^i, \lambda^i > 0$ , with these being interpreted as measures of the utility level and marginal utility of income, that satisfy the Afriat inequalities:  $U^i \leq U^j + \lambda^j p^j (q^i - q^j)$  for  $i, j = 1, 2, \dots, n$ ; and (4) there is a concave, monotonic, continuous, non-satiated utility function that rationalises the data". Thus, condition (3) provides directly testable conditions that the data must satisfy in order for it to be consistent with the maximisation model.



### 3. Data and implementations

#### 3.1 Data

The data used in this paper are Spanish annual time-series covering the period 1964 to 1995, including total consumption and prices obtained from several issues of the National Accounts, vol. II (OECD). Total consumption is disaggregated into the five categories with the highest budget shares, that is to say, 1. Food, beverages and tobacco (Food), 2. Clothing and footwear (Clothing), 3. Gross rent, fuel and power (Energy), 4. Transport and communication (Transport) and, finally, 5. Miscellaneous goods and services (Miscellaneous Goods), with this latter group including furniture, furnishings and household equipment, medical care and health expenses, recreational, entertainment, education and cultural services, personal care and expenditures in restaurants, cafes and hotels.

Table 1 provides a brief descriptive analysis using three indicators. First, we carry out an analysis of the rates of growth, obtaining the annual average rates for all categories for both the sample period and for various subperiods. We have chosen the limit years in accordance with those that are the most representative, from an economic point of view, of the last three decades, namely the oil crisis years of 1973 and 1979. Secondly, we calculate the annual average rates of inflation for all magnitudes, as well as for the same periods taken into account in the above analysis. Thirdly, we carry out a budget shares analysis, calculating the average shares and different values along the whole sample period.

Table 1 presents the average annual rates of growth. Here, we can observe that the highest average rate along the whole sample period appears in Transport, 6,96%, whereas Food shows the lowest value, 2%. As regards time evolution, we find that the highest values are basically concentrated in the years running up to 1973; by contrast, the lowest rates are distributed in the subperiods after 1979. Secondly, with respect to the rates of inflation, the highest mean rates along the whole sample period correspond to Miscellaneous Goods, 11,13%, and Energy, 11,03%, whereas Food displays the lowest, 8,95%. In this regard, the time evolution shows that the years immediately following both oil crises exhibit the highest rates; by contrast, the lowest values are concentrated in the last years of the whole sample period. Finally, the highest mean budget shares correspond to the categories Miscellaneous Goods and Food, 34,18% and 30,19%, respectively, whereas the lowest appears in the Clothing group, 9,46%. We can also observe the decreasing time evolution of budget shares for Food and, by contrast, the increasing trends followed by the values of Transport and Miscellaneous Goods.

An important question which arises when aggregate data are used is the aggregation conditions. In this situation, and following the authors of the four models considered in this paper who also use aggregate data in their respective studies, we assume the necessary conditions under which it is possible to consider aggregate consumer behaviour as if it were the outcome of the decisions of a single utility

maximising consumer, that is to say, the exact aggregation conditions. Thus, we assume both that individual available income is exogenous and that the prices are common to all consumers (Deaton and Muellbauer, 1980b).

**Table 1: Descriptive analysis**

	1965-69	1970-73	1974-79	1980-84	1985-89	1990-95	1965-95	
Rates of growth (%)								
Food	4,39	4,64	2,88	-0,61	0,54	0,79	2,00	
Clothing	4,38	5,63	0,48	0,31	4,59	0,29	2,37	
Energy	5,54	4,94	3,33	0,60	1,71	-0,65	2,42	
Transport	18,47	13,58	3,92	0,01	8,82	0,22	6,96	
Miscellaneous	8,18	7,07	3,05	1,40	5,79	2,54	4,47	
Total	6,57	6,33	2,85	0,36	4,25	1,17	3,40	
Rates of inflation (%)								
Food	6,19	8,79	16,21	9,99	7,25	4,66	8,95	
Clothing	7,64	10,37	17,77	11,30	8,56	4,21	10,03	
Energy	5,96	6,99	19,87	16,40	5,32	9,39	11,03	
Transport	1,81	4,56	21,21	15,51	5,16	6,96	9,67	
Miscellaneous	7,22	9,95	20,45	14,56	7,52	6,03	11,13	
Total	6,22	8,59	18,68	13,12	6,90	6,13	10,14	
	1964	1970	1975	1980	1985	1990	1995	Mean
Budget shares (%)								
Food	42,26	36,94	35,17	28,05	24,91	21,76	19,71	30,19
Clothing	10,90	10,44	10,22	8,07	8,64	8,88	7,62	9,46
Energy	15,57	14,44	13,32	16,49	14,50	12,55	13,21	13,94
Transport	6,87	9,53	10,47	13,54	13,68	15,20	15,38	12,21
Miscellaneous	24,38	28,72	30,79	33,84	38,39	41,59	44,05	34,18

### 3.2 Implementations

With respect to the empirical implementation of the parametric analysis, we begin from the stochastic formulation of the general demand model, obtained by adding an error term to every equation and expressed, as usual, in terms of budget shares. These terms capture tastes shifts, measurement errors and the effects of left-out variables. The adding-up restriction,  $\sum_i w_i = 1$ , implies that  $\sum_i u_i = 0$  and, therefore, the covariance matrix is singular and the likelihood function undefined. In this situation, the usual procedure is to drop one of the equations, estimate the remaining system and calculate the parameters in the omitted equation via the adding-up condition. Thus, the general demand system to be estimated is:

$$\left. \begin{aligned} w_1 &= w_1(p_1, p_2, \dots, p_n, y) + u_1 \\ w_2 &= w_2(p_1, p_2, \dots, p_n, y) + u_2 \\ &\dots \\ w_{n-1} &= w_{n-1}(p_1, p_2, \dots, p_n, y) + u_{n-1} \end{aligned} \right\} \dots(6)$$

Moreover, given the habitual assumptions of error terms, which are contemporaneously correlated but serially uncorrelated, the model is jointly estimated by using the SURE method of Zellner (1962). This provides estimations that are both efficient and asymptotically equivalent to the maximum likelihood estimations. After formulating the model in its stochastic terms, we must test for the presence of joint autocorrelation by means of a diagnostic test which recognises the adding-up condition and, hence, allows us to consider the system globally. In particular, we use the test proposed by Harvey (1982), which is asymptotically distributed as a chi-square variable with degrees of freedom equal to the number of equations being estimated.

Once we have shown that the stochastic model does not exhibit autocorrelation problems, we must then test the theoretical hypotheses and, if they are accepted, we can impose them on the specification of the model. To that end, we use the corrected Wald test, obtained as a product of the initial Wald test and a correction factor. The use of the corrected test is justified given the bias of the initial test towards the rejection of the null hypothesis. The correction test we use is that proposed by Mauleon (1984),  $CF = (1 - n/T)(1 - k/T)$ , where  $n$  is the number of estimated equations,  $k$  is the average number of parameters per equation and, finally,  $T$  is the sample size. The corrected Wald test is asymptotically distributed as a chi-square variable with degrees of freedom equal to the number of restrictions being tested.

With respect to the non-parametric analysis, we first set-out to show that microeconomic consumer behaviour is consistent with utility maximisation. Thus, we test the two axioms of revealed preference theory: WARP and GARP. First, the weak axiom, where the corresponding non-parametric test proceeds as follows. We consider  $N$  goods and  $T$  periods; then let  $P$  ( $T \times N$ ) and  $Q$  ( $T \times N$ ) denote the matrix of prices and quantities, respectively, and let the matrix  $C = PQ'$  whose elements,  $C_{ij}$  represent the cost, at prices of time  $i$ , of buying the bundle of goods of period  $j$ . Therefore, the elements in column  $j$  give the cost, at various price vectors, of obtaining the consumption bundle  $q^j$ , while the elements in any row  $i$  allow for a comparison of the costs of various bundles at the fixed set of prices  $p^i$ . The leading diagonal represents the actual expenditure in each period  $i$ . We then use a new matrix  $\Phi$ , which is defined by dividing every element of  $C$ ,  $C_{ij}$ , by the corresponding diagonal element,  $C_{ii}$ , that is to say,  $\Phi_{ij} = C_{ij} / C_{ii}$ . If any element  $\Phi_{ij} \leq 1$ , then  $q^j$  has been revealed preferred to  $q^i$ , i.e., commodity bundle  $q^i$  was affordable at period  $j$  prices, but bundle  $q^j$  was selected. If  $\Phi_{ij} \leq 1$  and  $\Phi_{ji} \leq 1$ , the

weak axiom is violated. Thus, the elements of matrix  $\Phi$  provide the basis for testing this axiom. Moreover, the NONPAR program allows us to test the GARP directly.

#### 4. Empirical results

We first present matrix  $\Phi$  in Table 2. As we can see,  $\Phi_{ij} \leq 1$  and  $\Phi_{ji} > 1$ , that is to say, the particular data satisfies the weak axiom of revealed preference theory. Although we do not find WARP violations, it is nevertheless necessary to check for consistency with the generalised axiom. Following the earlier-mentioned Afriat's Theorem, the table shows the Afriat numbers  $U^i$ ,  $\lambda^i > 0$ , which satisfy the Afriat inequalities:  $U^i \leq U^j + \lambda^j p^j (q^i - q^j)$ . As the theorem establishes, this is equivalent to accepting GARP. In other words, we show that the demand behaviour is consistent with this axiom; that is to say, there is a stable demand system underlying the personal preferences structure which explains the observed quantities of goods, and which is equivalent to the existence of a well-behaved utility function.

The final results of this non-parametric analysis could be interpreted in terms of the stability of consumer preferences. In other words, the fact that data are consistent with axioms of revealed preference theory indicates that shifts in the patterns of consumption are attributable to variations in conventional economic factors, prices and total income, and not to changes in consumer tastes. In this paper, we have shown that the absence of violations of the WARP and GARP axioms indicates that we can rationalise the data and, further, can consider that the observations have been generated according to the utility maximisation of a representative consumer. That is to say, as we do not detect violations, we cannot reject the hypothesis of stability of preferences, which implies that the evolution of quantities demanded can only be explained by changes in economic variables.

The parametric results are obtained from the initial estimation of all the demand models. We test for the presence of autocorrelation problems by using the Harvey test, obtaining the following results. First, the initial LES (2) exhibits a value of 58.61, which is clearly higher than the critical value of the test at the 5% level of significance,  $\chi^2(4)_{0.05} = 9.49$ . Therefore, we propose the estimation of several dynamic specifications of the LES. In particular, we follow Pollak and Wales (1969), and incorporate habit formation in the static version, specifying the parameter  $\gamma_i$  to be a linear function of dynamic variables, e.g., a time trend, the lagged endogenous variable or another variable which also indicates the lagged consumption. Thus, we specify the following three dynamic versions of the LES: i)  $\gamma_{it} = \gamma_i + \gamma_{it}$  ii)  $\gamma_{it} = \gamma_i + \gamma_{it} q_{it-1}$  iii)  $\gamma_{it} = \gamma_i + \gamma_{it} z_{it-1}$ , where  $z_{it-1}$  is, for example, the average consumption during the three years immediately prior to the current year. From the estimation of these dynamic versions, we find that the initial autocorrelation problems do not disappear. Specifically, we obtain the Harvey values of 52.25, 28.28 and 20.58, respectively, and can therefore conclude that the static and dynamic versions of the LES model exhibit important autocorrelation problems.



Secondly, we estimate the translog model (3), obtaining a value of the Harvey test, 60,84, which also indicates the presence of clear autocorrelation problems. Given this result, we dynamise the static version, introducing a time trend as a new exogenous variable. After estimating this dynamic version of the model, we note that the Harvey test, 25,39, is also higher than the critical value,  $\chi^2(4)_{0,05} = 9,49$ , which implies that this transformation does not correct the autocorrelation problems detected in the static version.

Thirdly, we estimate the AIDS (4), once again finding autocorrelation problems, given the value of the Harvey test, 32,89. Therefore, with the objective of including the effects of consumption habits, we follow Deaton and Muellbauer (1980b) and dynamise the static version of the model, specifying the intercept to be linear functions of a lagged endogenous variable and a time trend,  $\alpha_{it} = \alpha_i + \alpha w_{it-1} + \alpha_{i1} t$ . We find that this dynamic specification does solve the initial problems of autocorrelation and, subsequently, we test the theoretical hypotheses of homogeneity and symmetry by using the corrected Wald test. The particular values, 18,14 for homogeneity and 59,45 for joint homogeneity and symmetry, are clearly higher than the critical values at the 5% level of significance,  $\chi^2(4)_{0,05} = 9,49$  and  $\chi^2(10)_{0,05} = 18,30$ , and, therefore, this dynamic version does not satisfy the theoretical conditions derived from demand theory. Given this result, we specify two further dynamic specifications of the intercept,  $\alpha_{it} = \alpha_i + \alpha_{i1} t$  and  $\alpha_{it} = \alpha_i + \alpha w_{it-1}$ , noting that whilst the former gives a value of the Harvey test higher than the critical value, the latter solves the autocorrelation problems, given its particular value, 5,30. Therefore, we again test the theoretical conditions, obtaining the corrected Wald values of 24,29 and 90,97 for homogeneity and joint homogeneity and symmetry, respectively. However, this new dynamic version, although without autocorrelation problems, does not satisfy the theoretical properties. Thus, we find that only two dynamic versions of the initial AIDS are without autocorrelation problems, but neither of them satisfy the theoretical conditions derived from demand theory. Therefore neither the static nor the dynamic versions of the AIDS satisfy all the econometric and theoretical requirements.

Fourthly, we estimate the Rotterdam model (5), obtaining a value of the Harvey test, 4,59, lower than the critical value at the 5% level of significance. It is clear, therefore, that this model does not exhibit autocorrelation problems and thus we can check the theoretical conditions of homogeneity and symmetry by means of the corrected Wald test. The results, 7,21 for homogeneity and 16,72 for joint homogeneity and symmetry, are lower than the critical values at the 5% level of significance,  $\chi^2(4)_{0,05} = 9,49$  and  $\chi^2(10)_{0,05} = 18,30$ , and, hence, this formulation also meets the requirement of satisfying the theoretical properties derived from demand theory. As a result, we impose these properties, thereby obtaining the restricted version of the Rotterdam model, which is again estimated and checked for the presence of autocorrelation problems. We can observe that this new restricted version does not present this kind of econometric problems, given that the value of

the Harvey test, 497, is also lower than the critical value at the 5% level of significance. In conclusion, we find that the homogeneous and symmetric version of the Rotterdam model satisfies both the econometric and theoretical requirements, and hence can be used for the purpose of representing the economic behaviour of Spanish consumers from 1964 to 1995.

Having chosen our best model, its estimated parameters allow us to derive the values of the expenditure, and Marshallian and Hicksian price elasticities. All these effects are presented in Table 3, being calculated at the mean point of the explanatory variables. Here, we can first observe that the five expenditure elasticities are individually significant at the 5% level of significance. The particular values indicate that Food, Clothing and Energy are necessities, whereas Transport and Miscellaneous Goods are luxuries. All the Marshallian own-price elasticities are negative, as theory predicts for decreasing demands, with four effects being statistically significant at the 5% level. The values, in absolute terms, lie between the highest, corresponding to Energy, -0,71, and the lowest, that of Clothing, -0,26; therefore, all the values indicate inelastic demands. The Hicksian own-price elasticities are also negative at the usual 5% level, indicating decreasing Hicksian demands. With respect to the cross-price effects, we obtain the same sign in all the symmetric pairs, with a significant and positive sign, characteristic of substitute goods, in the pairs Energy-Transport and Energy-Miscellaneous Goods.

**Table 3: Mean elasticities**

	Food	Clothing	Energy	Transport	Miscellaneous
Expenditure	0,7206* (8,02)	0.8913* (6,76)	0,7424* (4,76)	1,6714* (9,99)	1,1420* (10,96)
Marshallian Price					
Food	-0,3400* (-4,42)	-0,0171 (-0,42)	-0,0875* (-2,18)	-0,0946 (-1,93)	-0,1812 (-1,71)
Clothing	-0,1063 (-0,93)	-0,2062 (-1,51)	-0,1511* (-2,37)	-0,0871 (-0,82)	-0,3404 (-1,60)
Energy	-0,1962* (-2,09)	-0,0885 (-1,73)	-0,7197* (-8,18)	0,1036 (1,55)	0,1583 (1,02)
Transport	-0,5210* (-4,39)	-0,1413 (-1,56)	-0,0111 (-0,15)	-0,5997* (-3,83)	-0,3980 (-1,43)
Miscellaneous	-0,2872* (-3,91)	-0,1179* (-2,23)	0,0088 (0,18)	-0,0775 (-0,97)	-0,6680* (-3,73)
Hicksian Price					
Food	-0,1224 (-1,55)	0,0510 (1,32)	0,0129 (0,30)	-0,0066 (-0,13)	0,0651 (0,72)
Clothing	0,1628 (1,35)	-0,1218 (-0,91)	-0,0268 (-0,39)	0,0216 (0,9)	-0,0357 (-0,19)
Energy	0,0279 (0,30)	-0,0182 (-0,39)	-0,6162* (-6,65)	0,1943* (2,94)	0,4121* (3,24)
Transport	-0,0164 (-0,13)	0,0168 (0,19)	0,2218* (2,94)	-0,3956* (-2,37)	0,1733 (0,72)
Miscellaneous	0,0575 (0,72)	-0,0099 (-0,19)	0,1680* (3,24)	0,0619 (0,72)	-0,2776 (-1,78)

The asterisk (\*) indicates significant at the 5% level. t-values between parentheses

Finally, we should mention the most important previous demand studies devoted to the Spanish case, and compare our results with those of the most similar analysis. In this sense, an important difference between the great majority of these studies, indeed, all of them save for one, and this work is that we initially consider the four most important demand models and use empirical tests to choose the best formulation in accordance with the statistical data, rather than imposing one particular formulation on the data base. Thus, Lluch (1971a), Abadía (1984) and Labeaga and López (1994) estimate several versions of the linear expenditure model using different cross-sections; Contreras, Miravete and Sancho (1991) employ time-series for testing the conditions implied by the integrability in a translog system; Molina (1994 and 1997), Ramajo (1994 and 1997) and Labeaga and López (1996) estimate several specifications of the almost ideal demand system with time-series and micro-data surveys; and, finally, Lluch (1971b) uses two surveys to estimate some versions of the Rotterdam model. The single exception to this rule of estimating only one demand model is that of Lorenzo (1988), who estimates some versions of the linear expenditure system, the almost ideal demand system and the Rotterdam model.

Considering the statistical information used in these articles, that is to say, both the type and length of the time-series, and the number and characteristics of the different groups of goods, the most similar study to ours is that of Ramajo (1997), and thus we compare our results with those presented in that paper. This author starts by considering time-series from 1964 to 1999 for five groups of goods comparable to ours, that is to say, Food, Clothing, Energy, Housing and Miscellaneous Goods. This data base is then employed in order to estimate some versions of the almost ideal demand system using a joint estimation method. Again in a way that is comparable to the approach followed in our paper, some diagnostic tests are then carried out, the theoretical hypotheses are tested and, finally, the expenditure and Marshallian and Hicksian price elasticities are calculated. A comparison of the results presented in the the two papers reveals, first, that after using the diagnostic tests in both cases to choose the best formulation, our results do not allow us to reject the homogeneity and symmetry conditions, whereas those of Ramajo (1997) fail to comply with this theoretical requirement. Secondly, our expenditure effects are similar to those presented in that paper, with Food and Energy being necessities, whereas Clothing, Housing and Miscellaneous Goods are luxuries, which implies the same classification as ours for Food, Energy and Miscellaneous Goods. Moreover, both sets of price elasticities show that all categories have decreasing demand functions. Thus, despite the fact that the elasticity results are similar, we think that our methodology constitutes an improvement on that employed in Ramajo (1997), given that the initial consideration of different demand models allows us to finally choose the best specification which satisfies all the econometric and theoretical conditions.



## 5. Summary and conclusions

In this paper, we have proposed a way to model the economic behaviour of consumers using two complementary approaches, namely the parametric, formulated in terms of different demand equation models, and the non-parametric, derived from revealed preference theory. This analysis has been carried out using Spanish time-series data from 1964 to 1995 of the quantities and prices of five aggregated consumption goods: Food, Clothing, Energy, Transport and Miscellaneous Goods. After a preliminary descriptive analysis, this data base has allowed us to test the axioms of revealed preference theory and to estimate the different demand equations systems in order to choose the best formulation.

With respect to the non-parametric results, we have shown that the demand behaviour is consistent with the weak and generalised axioms of revealed preference theory, which indicates that shifts in the patterns of consumption are attributable to variations in prices and total income and, therefore, that we cannot reject the hypothesis of stability of preferences.

For their part, the parametric results show that the static and dynamic versions of the LES, translog and AIDS models do not satisfy all the econometric and theoretical requirements for them to constitute an adequate demand model to represent the demand behaviour of Spanish consumers during the period 1964-1995. However, the homogeneous and symmetric version of the Rotterdam model does satisfy these requirements, and thus can be used for the purpose of representing such behaviour.

The values of the expenditure and Marshallian and Hicksian price elasticities indicate that Food, Clothing and Energy are necessities, whereas Transport and Miscellaneous Goods are luxuries, with an increasing trend in the effect for Food and a decreasing one for both Transport and Miscellaneous Goods. All the groups have decreasing demands, with all the values therefore indicating inelastic demands. The Hicksian own-price elasticities also indicate decreasing Hicksian demands. With respect to the cross-price effects, we obtain the same sign in all the symmetric pairs, with a significant and positive sign, characteristic of substitute goods, in the pairs Energy-Transport and Energy-Miscellaneous Goods.

Finally, we present some policy implications and draw attention to the limitations of our study. With respect to the former, the estimation of a complete model of consumer demand can be used as a simulation tool to illustrate the welfare effects and the impact on government revenue of changes in relative commodity prices, for example, following a reform of indirect taxation. This is potentially of considerable importance from a policy point of view, given that it allows the policy-maker to formulate a set of recommendations on the basis of the marginal tax reform results derived from a demand system. However, this policy implication is itself closely related with the most important limitation of our study, namely the type of data used. Thus, if data is relatively scarce, as may be the case for time-series, then there

will only be limited information upon which to base recommendations for tax reform. In particular, time-series can only provide a single average measure of welfare effects, with data at household level providing the distribution results of these effects. Moreover, household databases also imply the possibility of incorporating socio-economic factors into the analysis, which allows the researcher, first, to make welfare comparisons across households; secondly, to combine heterogeneous households within the same specification for the equation of interest; and, thirdly, to throw some light on the behaviour of families with different demographic structures. Furthermore, the inclusion of these household characteristics into the demand systems can also be used to calculate summary statistics of inequality in the distribution of household income. This will indicate how inequality is affected when individuals at the lower end of the distribution receive a relatively higher or lower weight. Given that these limitation-related issues, derived from the type of data employed, are worthy of further investigation, such aspects are being considered in our current research, in which individual microdata are used.

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