

Is Spanish consumer behaviour consistent with the utility maximization? A non-parametric response

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Received 16 August 1995

The objective of this paper is to check Spanish data on consumption expenditures and leisure for consistency with maximization of a non-satiated, continuous, concave, monotonic utility function. To this end, we apply the non-parametric approach using annual time-series covering the period 1964 to 1992. The results indicate that Spanish data are consistent with the weak and generalized axioms of the revealed preference theory, that is to say, the sample national data are consistent with the neoclassical theory of consumer behaviour or, in other words, the hypothesis of stability of individual preferences is accepted.

I. INTRODUCTION

In demand theory the most important assumption is utility maximization. Specifically, it is supposed that the consumer chooses the bundle that is preferred from among all available bundles for a given set of prices and expenditure (Barten, 1964, 1967; Christensen *et al.*, 1975; Deaton and Muellbauer, 1980; Stone, 1954; Theil, 1965). However, despite this extensive use, is the observed behaviour consistent with the utility maximization hypothesis?

To answer this question, we can adopt two approaches. The first consists in specifying the parametric functional forms for demand equations and fitting them to the observed data. However, this procedure will be satisfactory only if the functional forms are good approximations to the 'true' demand equations, an aspect which is not directly tested. This problem has motivated the appearance of an alternative methodology, namely the non-parametric approach, derived from the revealed preference theory (Afriat, 1967; Houthakker, 1950; Koo, 1963 and 1971; Samuelson, 1938 and 1948; Varian, 1982 and 1983). This second procedure does not need *ad hoc* functional specifications for demand equations because, on the basis of available information, quantities and prices, this non-parametric approach allows us to test the utility maximization hypothesis.

Using the non-parametric approach, the objective of this paper is to check Spanish data on consumption expenditures and leisure for consistency with maximization of a non-satiated, continuous, concave, monotonic utility function. We apply

this technique using Spanish annual time series data covering the period from 1964 to 1992. Total consumption is divided into six goods: (1) food, beverages and tobacco; (2) clothing and footwear; (3) gross rent, fuel and power; (4) furniture, furnishings and household equipment, (5) miscellaneous goods and services, and (6) leisure. The results of this non-parametric analysis will be interpreted in terms of the stability of consumer preferences. That is to say, the fact that data are consistent with axioms of the revealed preference theory indicates that shifts in the patterns of consumption are attributable to variations in conventional economic factors (relative prices and total expenditure) and are not due to changes in consumer tastes.

The paper proceeds as follows. In the next section, the non-parametric approach is briefly explained. Section III is dedicated to describing the sample data and to presenting the results. Finally, the conclusions are summarized in Section IV.

II. THE NON-PARAMETRIC APPROACH

The consumer's allocation problem is:

$$\left. \begin{array}{l} \text{Max. } U = U(\mathbf{q}) \\ \text{s.t. } \mathbf{p}\mathbf{q} = y \end{array} \right\}$$

where $U = U(\mathbf{q})$ is the utility function, $\mathbf{q} = (q_1, \dots, q_n, q_l)$ and $\mathbf{p} = (p_1, \dots, p_n, p_l)$ denote the vector of quantities and prices (n consumption goods and leisure), and y is available income.

Given the classical properties of the utility function, the solution of the optimization problem allows us to obtain the Marshallian demand functions:

$$q = q(p_1, \dots, p_n, p_y, y)$$

The non-parametric approach to demand analysis derives algebraic conditions on the demand functions implied by maximizing behaviour. These conditions, known as ‘revealed preference’ conditions, provide a complete list of the restrictions imposed by maximizing behaviour, in the sense that all the maximizing behaviour of consumers must satisfy these conditions and, further, that all behaviour that satisfies these conditions can be viewed as maximizing behaviour. Revealed preference theory is based solely on observed and measurable phenomena, namely the commodity bundles actually purchased by consumers, the prevailing prices and consumer income or total expenditure.

Non-parametric methods have been developed to test data for consistency with utility maximization by means of the weak (WARP) and generalized (GARP) axioms of revealed preference. One attractive property of these tests is that they do not require a demand system to be specified and, hence, they suppose no explicit restrictions on functional form. Thus, these tests offer a convenient and informative means of scanning a consumption data set for evidence of violations of demand theory. Although it would seem reasonable to expect researchers to apply the tests for consistency, at the very least at the minimum level, WARP, in fact only a very small fraction of journal articles that report parametric demand systems also report whether the data were tested for consistency with GARP.

Let us now analyse how we can test if certain observations belonging to a representative consumer are in accordance with the utility maximization hypothesis. Let $q^i = (q_1^i, \dots, q_n^i, q_y^i)$ and $p^i = (p_1^i, \dots, p_n^i, p_y^i)$ denote the vector of quantities and prices corresponding to $n + 1$ goods and let us suppose that we have m observations (q^i, p^i) ($i = 1, 2, \dots, m$). If a consumer chooses a bundle of goods q^i when an alternative bundle of goods q^j is obtained with the same budget outlay, the agent is revealing a preference for bundle q^i over bundle q^j , that is, q^i is revealed preferred to q^j , usually denoted as $q^i R q^j$.

The weak axiom (WARP) states that if q^i is revealed preferred to q^j , then q^j cannot be revealed preferred to q^i . In other words, bundle q^j will only be chosen when it is cheaper than q^i , that is, bundle q^j is not obtainable with the same outlay. Figure 1 shows those situations in which the axiom is satisfied and those in which it is violated. Assuming only two goods, Q_1 and Q_2 , the line through q^i represents the budget line when the consumer chooses bundle q^i , and the line through q^j is the budget line when bundle q^j is chosen. The weak axiom would be satisfied if commodity bundle q^i is chosen when q^j is available, that is, q^j is in the budget set bounded by the line through q^i , and q^j is chosen when q^i is unattainable with the given budget, that is when q^i is outside the budget set. On the

other hand, the weak axiom would be violated if bundle q^k , rather than q^i , is chosen when q^j lies within the budget set but q^i is chosen when q^k is available.

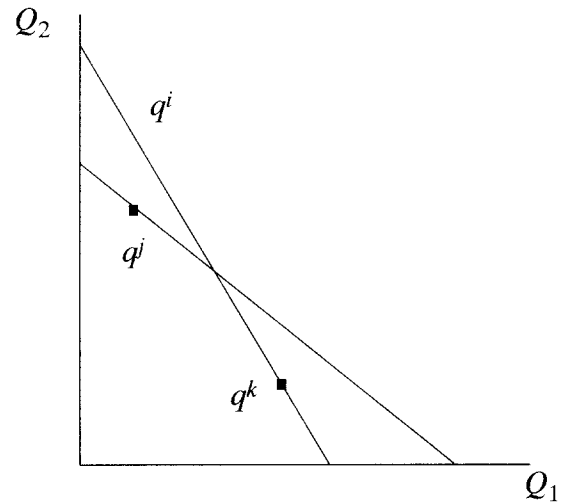


Fig. 1. WARP

This axiom can be expressed in terms of expenditures. For q^i to be revealed preferred to q^j , then both q^i and q^j must be available for a given income: $p^i q^i \geq p^i q^j$, where p^i denotes the set of prices when q^i is chosen. Thus, the expenditure on q^i is at least as great as the expenditure on q^j . If the weak axiom is satisfied, then: $p^i q^i \geq p^i q^j$ and $p^j q^j > p^j q^i$.

The second consistency condition concerns transitivity of consumer choices. On the other hand, the generalized axiom states that if q^i is revealed preferred to q^j ($p^i q^i \geq p^i q^j$), q^j is revealed preferred to a third bundle, q^k ($p^j q^j \geq p^j q^k$), and so on until q^l is revealed preferred to q^m ($p^l q^l \geq p^l q^m$), for some sequence of bundles $(q^i, q^j, q^k, \dots, q^m)$, then bundle q^m cannot be strictly revealed preferred to q^i ($p^m q^m < p^m q^i$).

If some data satisfies GARP, then there is a satisfactory utility function $U(q)$ that will rationalize the observed behaviour, that is, the utility derived from the revealed preferred bundle is greater or equal to the utility corresponding to the bundle not chosen ($p^i q^i \geq p^i q^j \leftrightarrow U(q^i) \geq U(q^j)$). By contrast, if the data contain a violation of this axiom, then a non-satiated utility function that will rationalize the data does not exist. However, when can the observations be rationalized by a sufficiently well behaved non-degenerate utility function? The best answer to this question is provided by Afriat’s theorem (Afriat, 1967):

‘The following conditions are equivalent:

- (1) there exists a nonsatiated utility function that rationalizes the data;
- (2) the data satisfies GARP;
- (3) there exist numbers $U^i, \lambda^i > 0, i = 1, 2, \dots, n$, that satisfy the Afriat inequalities: $U^i \leq U^j + \lambda^j p^j (q^i - q^j)$ for $i, j = 1, 2, \dots, n$;
- (4) there exists a concave, monotonic, continuous, non-satiated utility function that rationalizes the data.’

Attention should be drawn to some features of Afriat's theorem. First, the equivalence of Equations 1 and 4 shows that if a particular data set can be rationalized by any non-trivial utility function at all, then it can in fact be rationalized by a very satisfactory utility function. Secondly, the numbers U^i and λ^i which appear in Equation 3 can actually be used to construct a utility function that rationalizes the data. U^i and λ^i can be interpreted as measures of the utility level and marginal utility of income. Thirdly, parts 2 and 3 give directly testable conditions that the data must satisfy if it is to be consistent with the maximization model. Condition 3, for example, simply asks whether there exists a non-negative solution to a set of linear inequalities. The existence of such a solution can be checked by solving a linear program with $2n$ variables and n^2 constraints. Unfortunately, the fact that the number of constraints rises with the square of the number of observations makes this condition difficult to verify in practice for computational reasons.

III. DATA AND RESULTS

Spanish annual time series covering 1964–92 are used in the paper. The personal consumption expenditures and prices are obtained from several issues of the *National Accounts, Vol. 2* (OECD), and the nominal wage and labour supplied are obtained from the *Year Book of Labour Statistics* (OIT).

We have specified six goods, with five corresponding to consumption goods and the sixth corresponding to leisure. As regards the first group, we have disaggregated the total expenditure in consumption goods as follows: (1) food, beverages and tobacco; (2) clothing and footwear; (3) gross rent, fuel and power; (4) furniture, furnishings and household equipment and operations and (5) miscellaneous goods and services. With respect to leisure, the nominal wage is the earnings of a production worker in non-agricultural activities. Thereafter, consumption good expenditures are divided by employment in non-agricultural sectors, obtaining per capita values.

We now set out to prove that microeconomic consumer behaviour in Spain is consistent with utility maximization. To this end, we will test both axioms of the revealed preference theory: WARP and GARP.

The main results of the non-parametric approach are obtained using the software routine described by Varian (1985), which is especially designed to carry out non-parametric analysis based on revealed preference.

First, let us test the weak axiom. The corresponding non-parametric test proceeds as follows. We consider six goods and 29 periods; then, let P (26*6) and Q (26*6) denote the matrix of prices and quantities, respectively, and let the matrix $C = PQ'$, whose elements, C_{ij} represent the cost, at prices of time i , of buying the bundle of goods of period j . Thus, the elements in column j give the cost, at various price vectors, of obtaining the consumption bundle q^j , while the elements in any row i allow a comparison of the costs of various bundles

at the fixed set of prices p^i . The leading diagonal represents the actual expenditure in each period i .

We then use a new matrix Φ , which is defined by dividing every element of C , C_{ij} , by the corresponding diagonal element, C_{ii} , that is $\Phi_{ij} = C_{ij} / C_{ii}$. If any element $\Phi_{ij} \leq 1$, then q^j has been revealed preferred to q^i , that is, commodity bundle q^j was affordable at period j prices, but bundle q^i was selected. If $\Phi_{ij} \leq 1$ and $\Phi_{ji} > 1$, then the weak axiom is violated. Therefore, the elements of matrix Φ provides the basis for the test of the weak axiom.

In Table 1 we present the matrix Φ . As can be seen, $\Phi_{ij} \leq 1$ and $\Phi_{ji} > 1$, that is to say, the Spanish data satisfy the weak axiom of the theory of revealed preference.

Although we do not find WARP violations, it is also necessary to check for consistency with the generalized axioms. The NONPAR software allows us to test the GARP directly. According to Afriat's theorem explained above, Table 1 shows the Afriat numbers U^i , $\lambda^i > 0$, which satisfy the Afriat inequalities: $U^i \leq U^j + \lambda^j p^j (q^i - q^j)$. As the theorem establishes, this is equivalent to the acceptance of GARP in Spain. In other words, we have proved that Spanish consumer behaviour is consistent with this axiom; that is to say, this data set could have been generated by a single neoclassical representative consumer.

The non-parametric results can be interpreted in terms of a hypothetical structural change in consumer behaviour. The existence of changes in the patterns of good purchases are due to modifications of the economic variables (relative prices and total expenditure) or are attributable to changes in the preferences of a representative consumer.

Research on the existence of structural change in the behaviour of consumers can take two forms. The first consists in specifying a functional form for demand equations, to estimate it and to test the hypothesis of stability of the parameters. However, as we have noted in the introduction to the paper, this parametric approach has the problem that all results are conditional on the functional forms being correct. To avoid this limitation, a second method to test for structural change has been proposed, namely the non-parametric approach, where the null hypothesis assumes the stability of preferences and thus the variations of quantities consumed can be explained by changes in relative prices and total expenditure. If consumers satisfy GARP, then there exists a stable demand system underlying the personal preferences structure, which explains the observed quantities of goods. This is because GARP is equivalent to the existence of a well-behaved utility function.

In this paper, we have proved that the existence of no violations of GARP indicates the acceptance of the hypothesis of stability of preferences in Spain. In other words, the evolution of quantities demanded can be explained by changes in economic variables. That is to say, as we do not detect violations, we can rationalize the data and, further, we can consider that observations have been generated according to the utility maximization of a representative consumer.

Table 1 *Noni-parametric analysis*

	1964	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	1992		
WARP: Matrix Φ																															
1964	1	0.90	0.84	0.80	0.76	0.73	0.69	0.63	0.58	0.52	0.44	0.38	0.33	0.26	0.22	0.19	0.17	0.14	0.12	0.11	0.10	0.09	0.08	0.08	0.07	0.07	0.06	0.06	0.05		
1965	1.10	1	0.93	0.88	0.83	0.81	0.75	0.70	0.64	0.57	0.48	0.41	0.36	0.28	0.24	0.21	0.18	0.16	0.14	0.12	0.11	0.10	0.09	0.08	0.08	0.07	0.07	0.06	0.06		
1966	1.18	1.07	1	0.94	0.90	0.87	0.81	0.75	0.69	0.61	0.52	0.45	0.38	0.30	0.26	0.22	0.20	0.17	0.15	0.13	0.11	0.11	0.10	0.09	0.08	0.08	0.07	0.07	0.06		
1967	1.24	1.13	1.05	1	0.95	0.91	0.86	0.79	0.72	0.65	0.55	0.47	0.41	0.32	0.27	0.24	0.21	0.18	0.15	0.14	0.12	0.11	0.10	0.10	0.09	0.08	0.08	0.07	0.07		
1968	1.31	1.19	1.11	1.05	1	0.96	0.90	0.83	0.76	0.68	0.58	0.50	0.43	0.34	0.29	0.25	0.22	0.19	0.16	0.14	0.13	0.12	0.11	0.10	0.09	0.09	0.08	0.08	0.07		
1969	1.35	1.23	1.14	1.08	1.03	1	0.93	0.86	0.79	0.70	0.60	0.51	0.44	0.35	0.30	0.26	0.23	0.20	0.17	0.15	0.13	0.12	0.11	0.10	0.09	0.09	0.08	0.08	0.07		
1970	1.44	1.31	1.22	1.15	1.10	1.06	1	0.92	0.84	0.75	0.64	0.55	0.47	0.38	0.32	0.27	0.25	0.21	0.18	0.16	0.14	0.13	0.12	0.11	0.11	0.10	0.09	0.09	0.08		
1971	1.56	1.41	1.32	1.25	1.19	1.15	1.08	1	0.91	0.81	0.70	0.60	0.51	0.41	0.34	0.30	0.27	0.23	0.20	0.17	0.16	0.14	0.13	0.12	0.11	0.10	0.09	0.09	0.09		
1972	1.70	1.54	1.43	1.36	1.29	1.25	1.17	1.08	1	0.89	0.76	0.65	0.56	0.45	0.38	0.32	0.29	0.25	0.21	0.19	0.17	0.16	0.14	0.13	0.12	0.11	0.10	0.09	0.09		
1973	1.91	1.73	1.61	1.53	1.45	1.40	1.31	1.21	1.12	1	0.85	0.73	0.63	0.50	0.42	0.36	0.32	0.28	0.24	0.21	0.19	0.18	0.16	0.15	0.14	0.13	0.12	0.11	0.11		
1974	2.23	2.02	1.88	1.79	1.70	1.64	1.54	1.42	1.31	1.17	1	0.85	0.74	0.59	0.49	0.43	0.38	0.33	0.28	0.25	0.23	0.21	0.19	0.18	0.17	0.16	0.15	0.14	0.13		
1975	2.59	2.35	2.19	2.08	1.97	1.91	1.79	1.66	1.52	1.36	1.16	1	0.86	0.68	0.58	0.50	0.45	0.38	0.33	0.29	0.26	0.24	0.22	0.21	0.20	0.18	0.17	0.16	0.15		
1976	2.99	2.71	2.53	2.40	2.28	2.21	2.07	1.91	1.76	1.57	1.34	1.15	1	0.79	0.67	0.58	0.52	0.44	0.38	0.34	0.30	0.28	0.26	0.24	0.23	0.21	0.20	0.18	0.17		
1977	3.71	3.36	3.14	2.98	2.84	2.75	2.59	2.39	2.20	1.96	1.68	1.44	1.25	1	0.84	0.73	0.65	0.56	0.48	0.43	0.39	0.36	0.33	0.31	0.29	0.27	0.25	0.24	0.22		
1978	4.42	4.00	3.74	3.55	3.37	3.27	3.07	2.84	2.61	2.33	1.99	1.71	1.48	1.18	1	0.86	0.77	0.66	0.57	0.51	0.46	0.42	0.39	0.36	0.34	0.32	0.30	0.28	0.26		
1979	5.07	4.60	4.29	4.08	3.88	3.76	3.53	3.27	3.00	2.68	2.29	1.97	1.70	1.36	1.15	1	0.89	0.77	0.66	0.59	0.53	0.49	0.45	0.42	0.40	0.37	0.35	0.32	0.30		
1980	5.50	4.98	4.66	4.44	4.22	4.10	3.86	3.57	3.28	2.92	2.50	2.15	1.86	1.48	1.25	1.09	1	0.86	0.74	0.66	0.59	0.55	0.50	0.47	0.44	0.41	0.39	0.36	0.34		
1981	6.32	5.72	5.35	5.10	4.86	4.72	4.44	4.11	3.78	3.37	2.88	2.48	2.15	1.71	1.45	1.26	1.15	1	0.86	0.77	0.69	0.64	0.58	0.55	0.52	0.48	0.45	0.42	0.39		
1982	7.28	6.59	6.16	5.88	5.60	5.44	5.12	4.74	4.36	3.88	3.32	2.86	2.48	1.98	1.67	1.46	1.33	1.15	1	0.88	0.79	0.74	0.67	0.63	0.60	0.56	0.52	0.49	0.45		
1983	8.14	7.36	6.90	6.58	6.27	6.10	5.74	5.31	4.89	4.36	3.73	3.21	2.79	2.22	1.88	1.64	1.49	1.29	1.12	1	0.89	0.83	0.76	0.71	0.67	0.63	0.59	0.55	0.51		
1984	9.07	8.21	7.69	7.33	6.99	6.80	6.39	5.92	5.45	4.85	4.16	3.58	3.11	2.48	2.09	1.83	1.66	1.44	1.25	1.11	1	0.92	0.84	0.79	0.75	0.70	0.66	0.61	0.57		
1985	9.55	8.63	8.11	7.74	7.38	7.19	6.78	6.29	5.80	5.17	4.43	3.82	3.31	2.65	2.24	1.95	1.77	1.54	1.33	1.18	1.06	1	0.91	0.86	0.81	0.76	0.71	0.66	0.62		
1986	10.47	9.47	8.89	8.49	8.09	7.88	7.43	6.89	6.36	5.66	4.85	4.18	3.63	2.90	2.45	2.14	1.94	1.68	1.45	1.29	1.16	1.09	1	0.94	0.89	0.83	0.78	0.73	0.68		
1987	11.09	10.03	9.42	8.99	8.57	8.35	7.87	7.30	6.74	6.00	5.14	4.43	3.85	3.08	2.60	2.26	2.05	1.78	1.54	1.37	1.23	1.15	1.06	1	0.94	0.88	0.83	0.77	0.72		
1988	11.64	10.53	9.89	9.45	9.01	8.78	8.27	7.68	7.09	6.31	5.41	4.66	4.05	3.24	2.74	2.38	2.16	1.87	1.62	1.44	1.30	1.22	1.11	1.05	1	0.93	0.87	0.81	0.76		
1989	12.47	11.28	10.60	10.13	9.65	9.41	8.86	8.23	7.59	6.76	5.80	5.00	4.33	3.47	2.93	2.55	2.32	2.01	1.74	1.55	1.39	1.30	1.19	1.13	1.07	1	0.93	0.87	0.82		
1990	13.28	12.01	11.28	10.78	10.27	10.01	9.44	8.76	8.08	7.20	6.17	5.32	4.62	3.70	3.12	2.72	2.47	2.14	1.86	1.65	1.48	1.39	1.27	1.20	1.14	1.06	1	0.93	0.87		
1991	14.05	12.71	11.95	11.42	10.89	10.62	10.01	9.30	8.58	7.64	6.55	5.66	4.92	3.93	3.32	2.89	2.63	2.28	1.98	1.76	1.58	1.49	1.36	1.28	1.22	1.13	1.06	1	0.93		
1992	14.88	13.45	12.65	12.10	11.54	11.26	10.62	9.88	9.11	8.11	6.96	6.01	5.22	4.18	3.53	3.08	2.80	2.43	2.11	1.88	1.69	1.59	1.45	1.37	1.30	1.21	1.14	1.06	1		
GARP: Afriat numbers																															
U^i	97.5	97.6	97.7	97.8	97.9	98.0	98.1	98.2	98.3	98.5	98.7	98.9	99.0	99.2	99.4	99.6	99.7	99.9	100.1	100.2	100.3	100.4	100.5	100.6	100.6	100.7	100.8	100.9	101.1		
λ^i	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00		

IV. CONCLUSIONS

In this paper, we have used a non-parametric method derived from the revealed preference theory to test whether or not the Spanish data are consistent with a conventional static demand system, that is, with stable consumer preferences. To do this, we have employed a time series data set (1964–92) of per capita consumption and prices for Spain to test the weak and generalized axioms of revealed preference theory.

We have proved that there are no violations of WARP nor GARP in Spain. That is to say, the sample national data are consistent with the neoclassical theory of consumer behaviour and, therefore, a well-behaved preference map can be constructed which accords with the observed consumption pattern.

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