UNIT 5: INDIVIDUAL AND MARKET DEMAND FUNCTIONS

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UNIT 5: INDIVIDUAL AND MARKET DEMAND FUNCTIONS

5.1 Income effect and substitution effect (Pindyck → 4.2 & Appendix 4)

5.2 The consumer surplus (Pindyck → 4.4)

**5.3 From the individual demand function to the market demand function (Pindyck → 4.3)**

UNIT 5: INDIVIDUAL AND MARKET DEMAND FUNCTIONS

- **Aims of the lesson:**

  ➢ To analyze the effect of variations in the price of a good on the quantity demanded of the same or different good (decomposing this total variation in both substitution and income effects).

  ➢ To be able to classify the complement and substitute goods in both gross and net terms.

  ➢ To measure the consumer welfare effect of a change in a price by the "consumer surplus".

 ➢ To obtain, by aggegation, the market demand curve from the individual demand curves.

5.1 Income and substitution effects

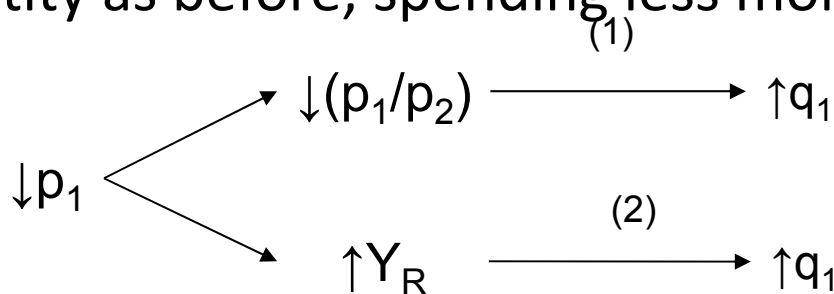- The change in the quantity demanded when the price changes (*caeteris paribus*) is the **TOTAL EFFECT:**

Initial situation: $E_0 = (q^0_1, q^0_2)$

$\downarrow p_1 \Rightarrow E_1 = (q^1_1, q^1_2)$

$\downarrow p_1$
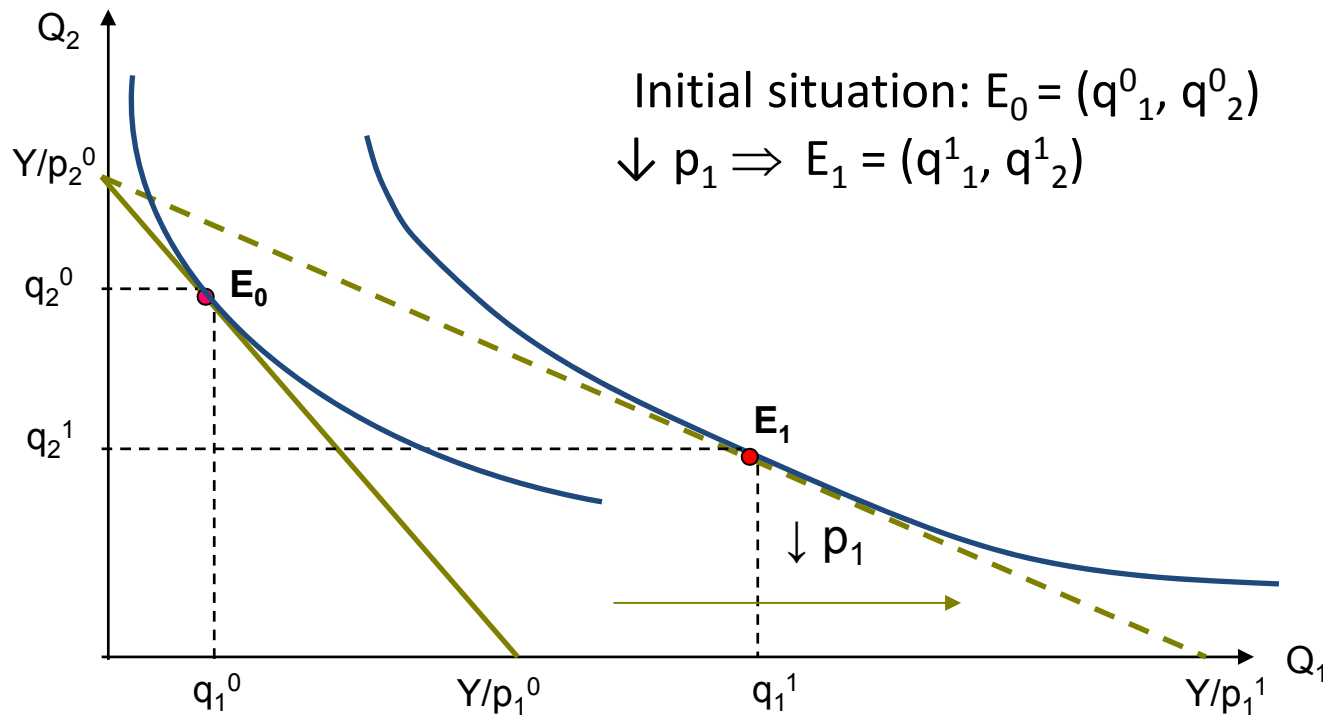
5.1 Income and substitution effects

- **We can consider:**

  - ➢ **Total Direct effect** **(T_{11})**: change in the quantity of q_1 when p_1 changes

  - ➢ **Total Crossed effect** **(T_{21})**: change in the quantity of q_2 when p_1 changes

$$|T_{11}| = |q_1^1 - q_1^0| = \text{Total direct effect}$$

$$T_{11} < 0$$

$$|T_{21}| = |q_2^1 - q_2^0| = \text{Total crossed effect}$$

$$T_{21} > 0$$
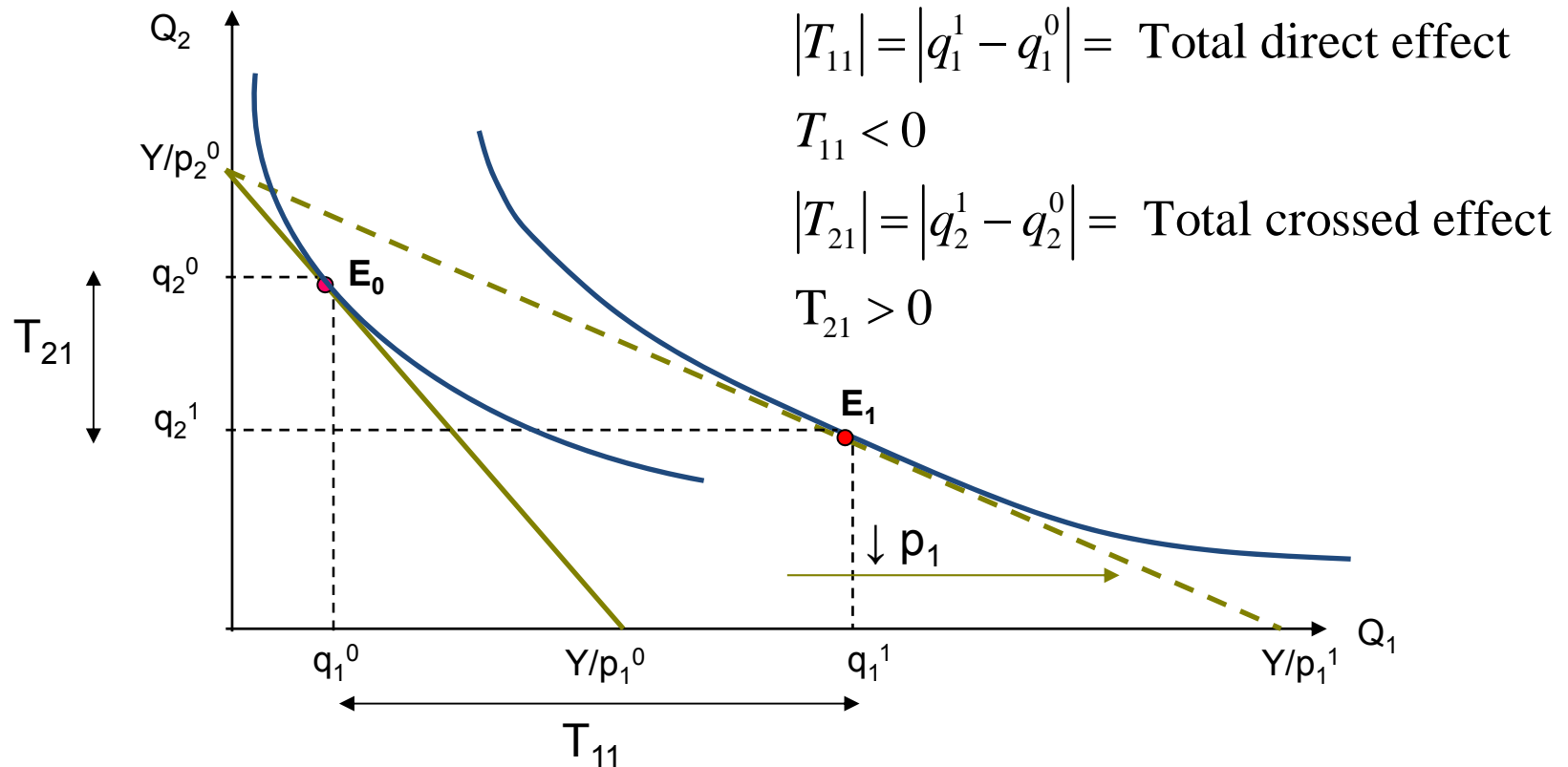
5.1 Income and substitution effects

- The variation in the price of a good (Total Effect), with income and prices of the other goods being constant, has two simultaneous effects on the quantity:

1) Consumers tend to buy a larger quantity of the good because it is now relatively cheaper than before, in terms of the other good, since the relative price of the good has decreased: **substitution effect**

2) Consumers purchasing power increases in real terms, because, while nominal income is the same, the good is now cheaper and, therefore, they can buy the same quantity as before, spending less money: **income effect**

$$\downarrow p_1 \begin{cases} \nearrow \downarrow(p_1/p_2) \xrightarrow{\text{(1)}} \uparrow q_1 \\ \searrow \uparrow Y_R \xrightarrow{\text{(2)}} \uparrow q_1 \end{cases}$$

5.1 Income and substitution effects

The total effect can be, theoretically, assumed as the sum of two different effects:

- **Substitution Effect** : Change in consumption of the good due to the change in a price, with the income and other prices being constant:

  S_{ij}= Substitution effect of good Q_i when p_j changes

- **Income Effect** : Change in consumption of the good when the real income and the relative prices are constant:

  R_{ij}= Income effect of good Q_i when p_j changes

$$T_{ij} = S_{ij} + R_{ij}$$

$T_{11} = S_{11} + R_{11}$ **Total direct effect**

$T_{21} = S_{21} + R_{21}$ **Total crossed effect**

5.1 Income and substitution effects

- The decomposition depends on how we define **"real income"**:

  The **Slutsky criterion** identifies real income with purchasing power: after the change in the price, the consumer can purchase, if he/she wishes, the market basket initially chosen before the price change:

  - The substitution effect is the change in the quantity demanded due to the change in one price, with purchasing power being constant ($\downarrow P_1/P_2$)

  - The income effect is the change in the quantity demanded as a consequence of the change in the purchasing power after the change in the price of the good. ($\uparrow Y/P_1$)
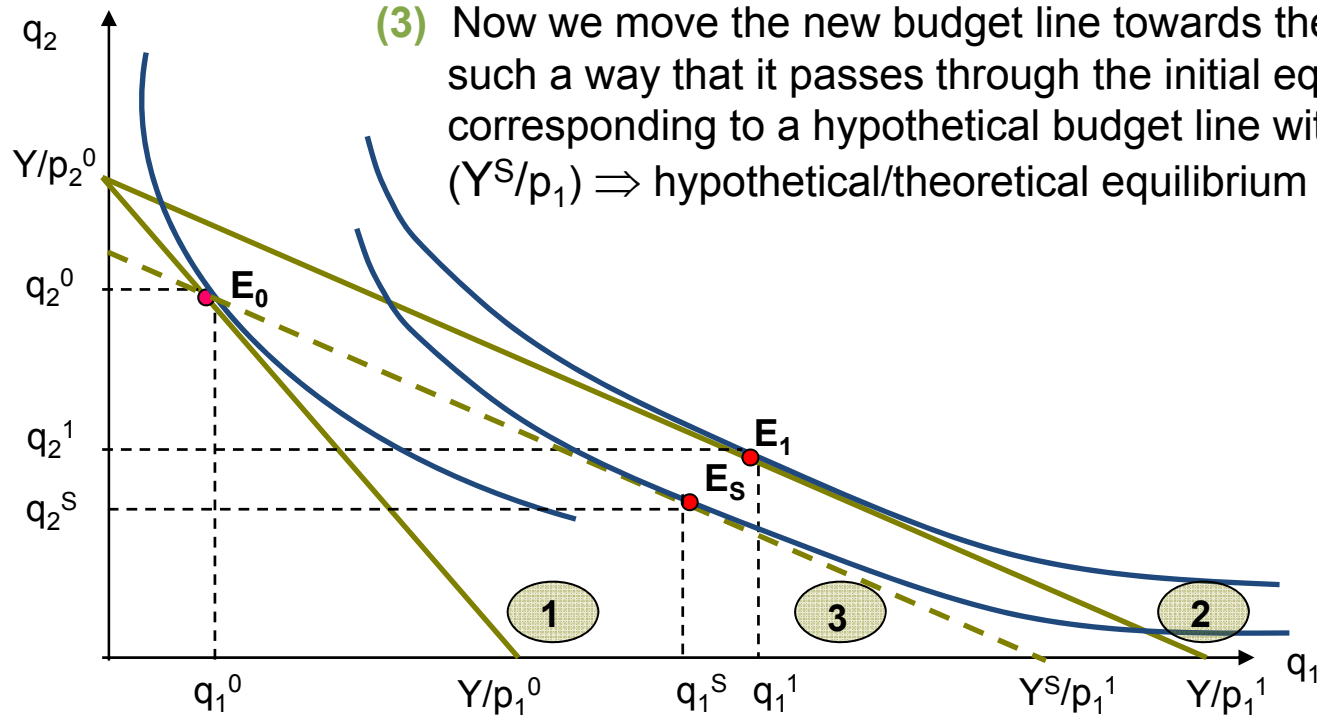
5.1 Income and substitution effects

- **Graphical analysis:**

(1) Initial budget line $\Rightarrow$ Initial equilibrium $\mathbf{E_0}$

(2) $\downarrow p_1 \Rightarrow \uparrow (Y/p_1) \Rightarrow$ New budget line $\Rightarrow$ New equilibrium $\mathbf{E_1}$

(3) Now we move the new budget line towards the origin, in parallel, in such a way that it passes through the initial equilibrium, corresponding to a hypothetical budget line with purchasing power $(Y^S/p_1) \Rightarrow$ hypothetical/theoretical equilibrium $\mathbf{E_S}$
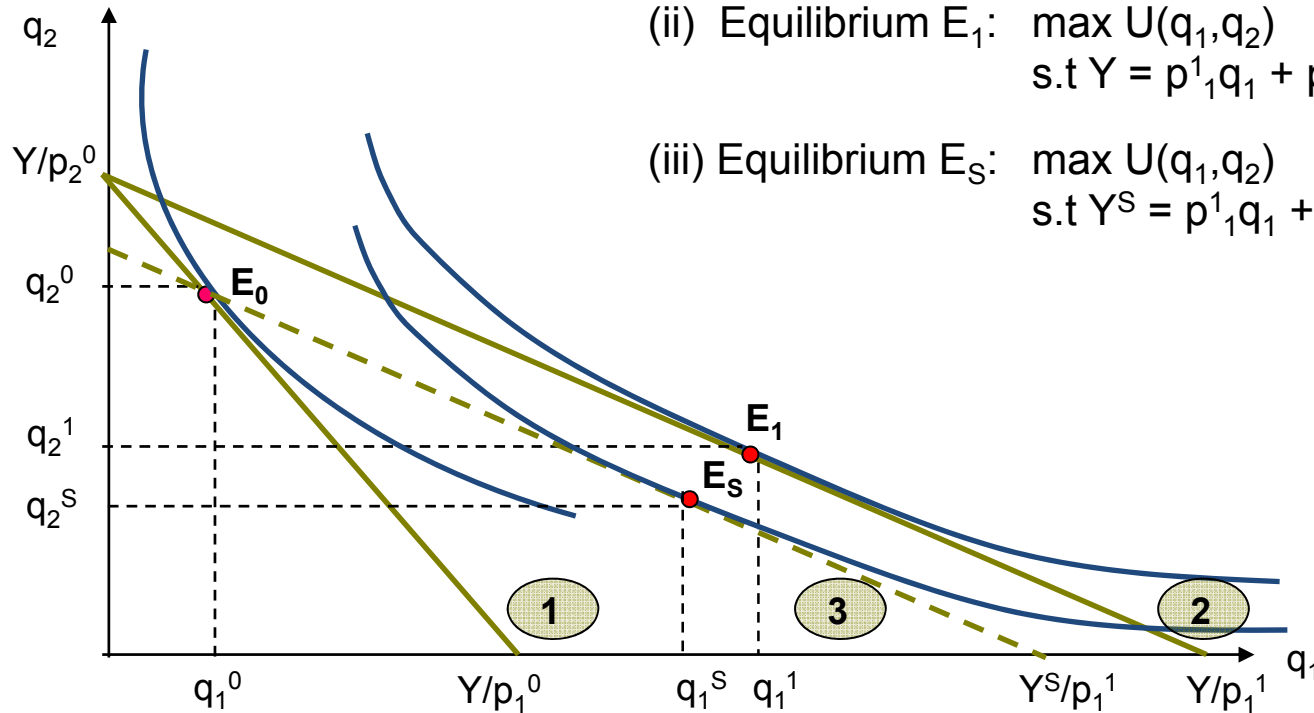
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5.1 Income and substitution effects

- **Graphical analysis:**

(i) Equilibrium E_0: max $U(q_1,q_2)$
  s.t $Y = p^0_1 q_1 + p^0_2 q_2$

(ii) Equilibrium E_1: max $U(q_1,q_2)$
  s.t $Y = p^1_1 q_1 + p^0_2 q_2$

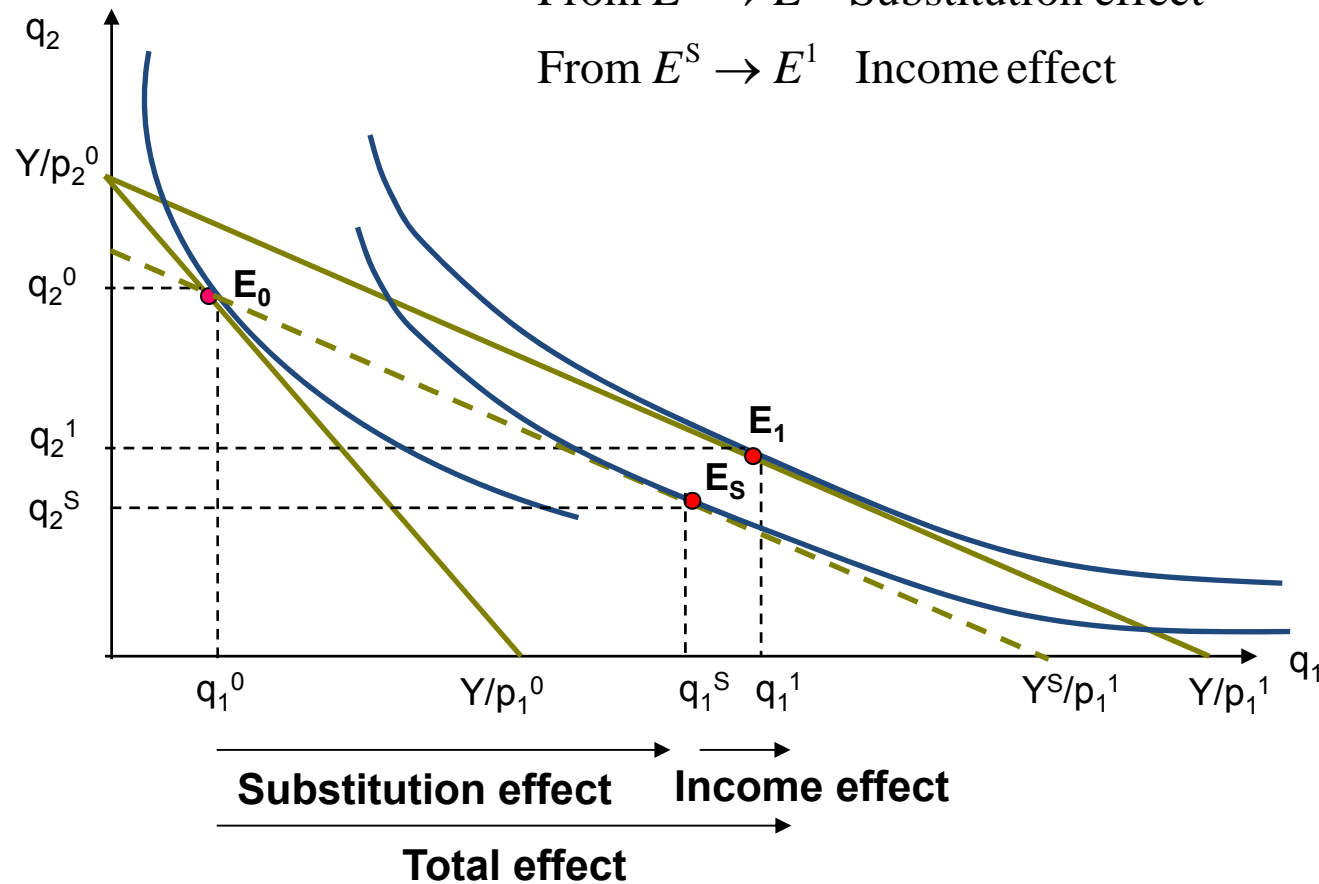(iii) Equilibrium E_S: max $U(q_1,q_2)$
  s.t $Y^S = p^1_1 q_1 + p^0_2 q_2$

5.1 Income and substitution effects

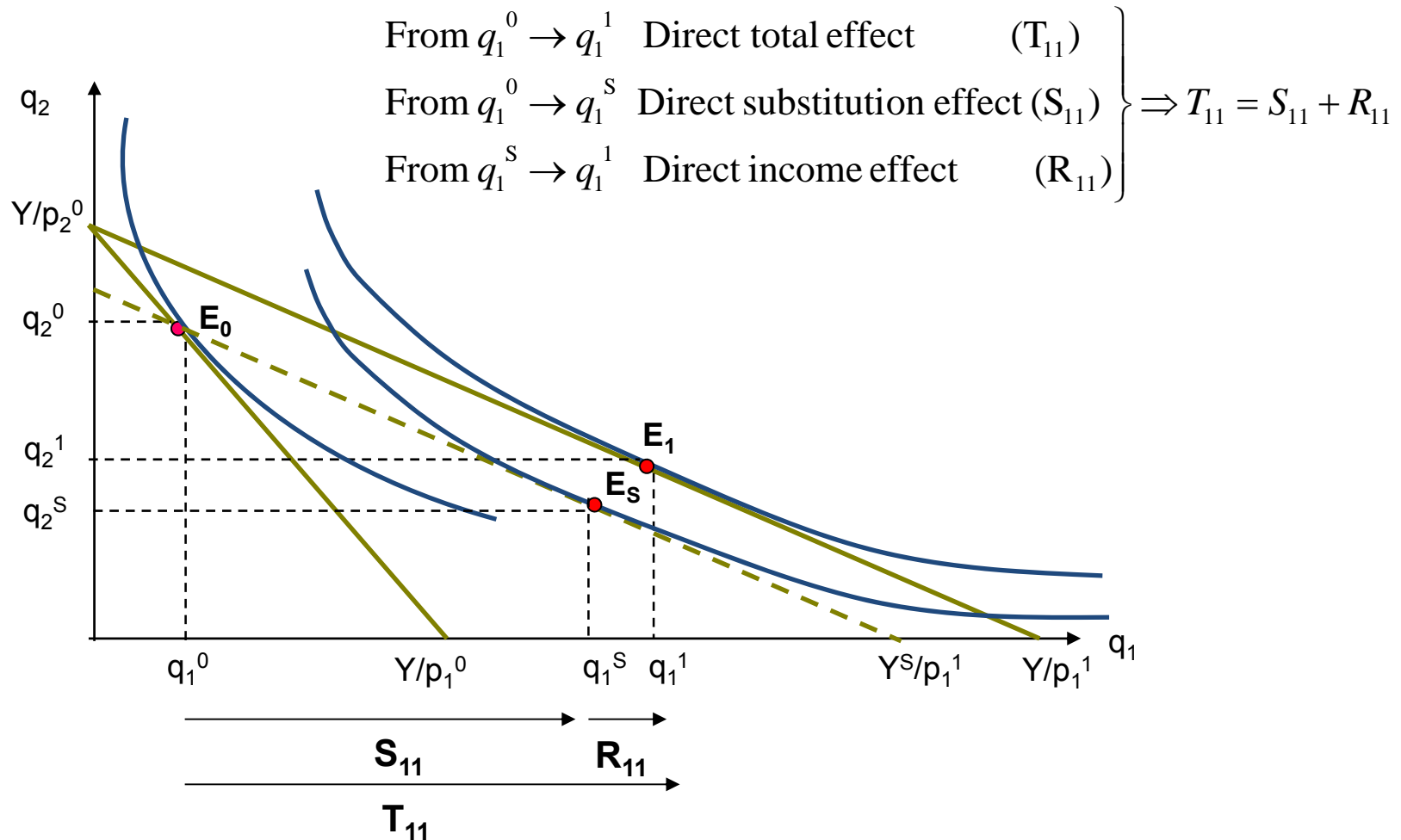- **Graphical analysis:**

From $E^0 \to E^1$ Total effect

From $E^0 \to E^S$ Substitution effect

From $E^S \to E^1$ Income effect

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5.1 Income and substitution effects

- **Graphical analysis: Direct effects**

From $q_1^0 \rightarrow q_1^1$ Direct total effect (T_{11})

From $q_1^0 \rightarrow q_1^S$ Direct substitution effect (S_{11}) $\Rightarrow T_{11} = S_{11} + R_{11}$
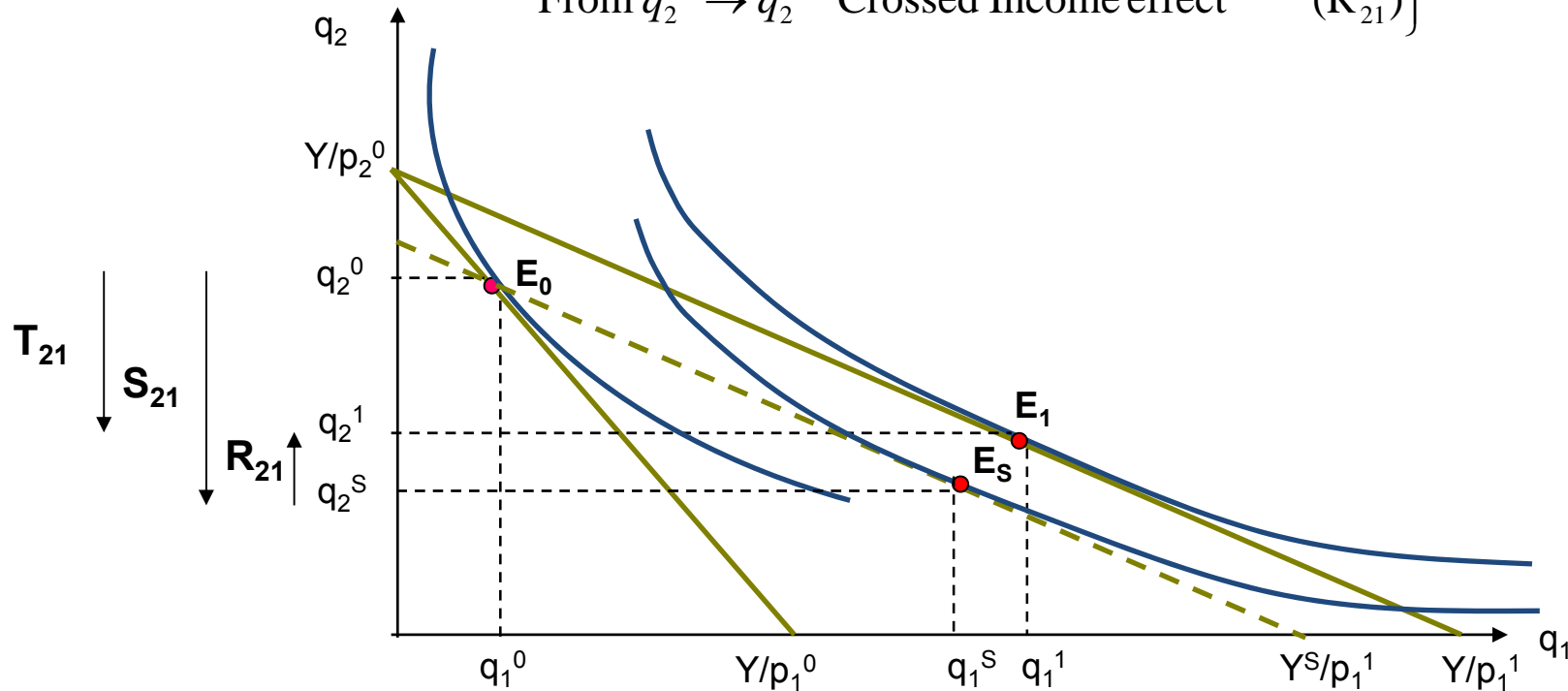
From $q_1^S \rightarrow q_1^1$ Direct income effect (R_{11})

5.1 Income and substitution effects

- **Graphical analysis: Crossed effects**

From $q_2^0 \rightarrow q_2^1$ Crossed Total effect $\quad\quad\quad (T_{21})$

From $q_2^0 \rightarrow q_2^S$ Crossed Substitution effect (S_{21}) $\left.\begin{array}{c}\\\\\end{array}\right\} \Rightarrow T_{21} = S_{21} + R_{21}$

From $q_2^S \rightarrow q_2^1$ Crossed Income effect $\quad\quad (R_{21})$

5.1 Income and substitution effects

Sign and size of the effects:

- The sign of the effects shows whether the change in prices and the change in quantities goes in the same or the opposite direction:

  – Same direction → Positive sign

  – Opposite direction → Negative sign

- The sign and size of the total, direct and crossed, effects determine the relation between the goods in gross terms

- The sign and size of the substitution effects determine the relation between the goods in net terms

- The sign of the direct total effect indicates the type of the individual demand: normal (-) or abnormal (+)

- The sign of the crossed total effect indicates the type of the crossed demand

- The sign of the total income effect indicates the type and shape of the Engel curve: normal, inferior or neutral good

5.1 Income and substitution effects

Sign and size of the effects:

- The **direct substitution effect S_{11}** is always negative (as a consequence of the strict convexity of preferences).

- The **crossed susbtitution effect S_{21}** is always positive when there are only two goods (Q_1 and Q_2). When there are more than two goods, it can exhibit any sign:

$$(\downarrow p_1 \rightarrow \downarrow p_1 / p_2 \rightarrow \downarrow q_2)$$

- The **income effect** may have any sign:

  - $R_{11} < 0$ if the good Q_1 is NORMAL $\quad (\downarrow p_1 \rightarrow \uparrow Y/p_1 \rightarrow \uparrow q_1)$

  - $R_{11} = 0$ if the good Q_1 is NEUTRAL $\quad (\downarrow p_1 \rightarrow \uparrow Y/p_1 \rightarrow ni \uparrow ni \downarrow q_1)$

  - $R_{11} > 0$ if the good Q_1 is INFERIOR $\quad (\downarrow p_1 \rightarrow \uparrow Y/p_1 \rightarrow \downarrow q_1)$

5.1 Income and substitution effects

Classification of the goods in gross-net terms:

(1) GROSS TERMS (using the crossed price elasticity): $\downarrow P_1$

➤ If T_{11} and T_{21} have the same sign $\Rightarrow$ Gross complements

➤ If T_{11} and T_{21} have opposite sign $\Rightarrow$ Gross substitutes

➤ If $T_{21} = 0$ $\Rightarrow$ Gross independent

(2) NET TERMS (given $S_{11} < 0$):

➤ If $S_{21} > 0 \Rightarrow \downarrow p_1 \rightarrow \uparrow q_1 \rightarrow \downarrow q_2$ $\Rightarrow$ Net substitutes

➤ If $S_{21} < 0 \Rightarrow \downarrow p_1 \rightarrow \uparrow q_1 \rightarrow \uparrow q_2$ $\Rightarrow$ Net complements

➤ If $S_{21} = 0 \Rightarrow \downarrow p_1 \rightarrow \uparrow q_1 \rightarrow$ neither $\uparrow$ nor $\downarrow q_1$ $\Rightarrow$ Net independent

In a two-goods economy (Q_1 and Q_2) the crossed substitution effects are always POSITIVE $\Rightarrow$ the goods are always NET SUBSTITUTES

5.1 Income and substitution effects

Classification of demand by means of the total effect:

Decreases the price of good 1:

Type of good (sign of R_{11})	S_{11}	R_{11}	T_{11}	Demand
(1) Normal good ($R_{11}<0$)	<0	<0	NEGATIVE	Normal
(2) Neutral good ($R_{11}=0$)	<0	0	NEGATIVE	Normal
(3) Inferior good ($R_{11}>0$)	<0	>0	i) $\|S_{11}\|>\|R_{11}\| \Rightarrow$ NEGATIVE ii) $\|S_{11}\|=\|R_{11}\| \Rightarrow$ ZERO iii) $\|S_{11}\|<\|R_{11}\| \Rightarrow$ POSITIVE	Normal Rigid Abnormal

5.1)¡ Income and substitution effects

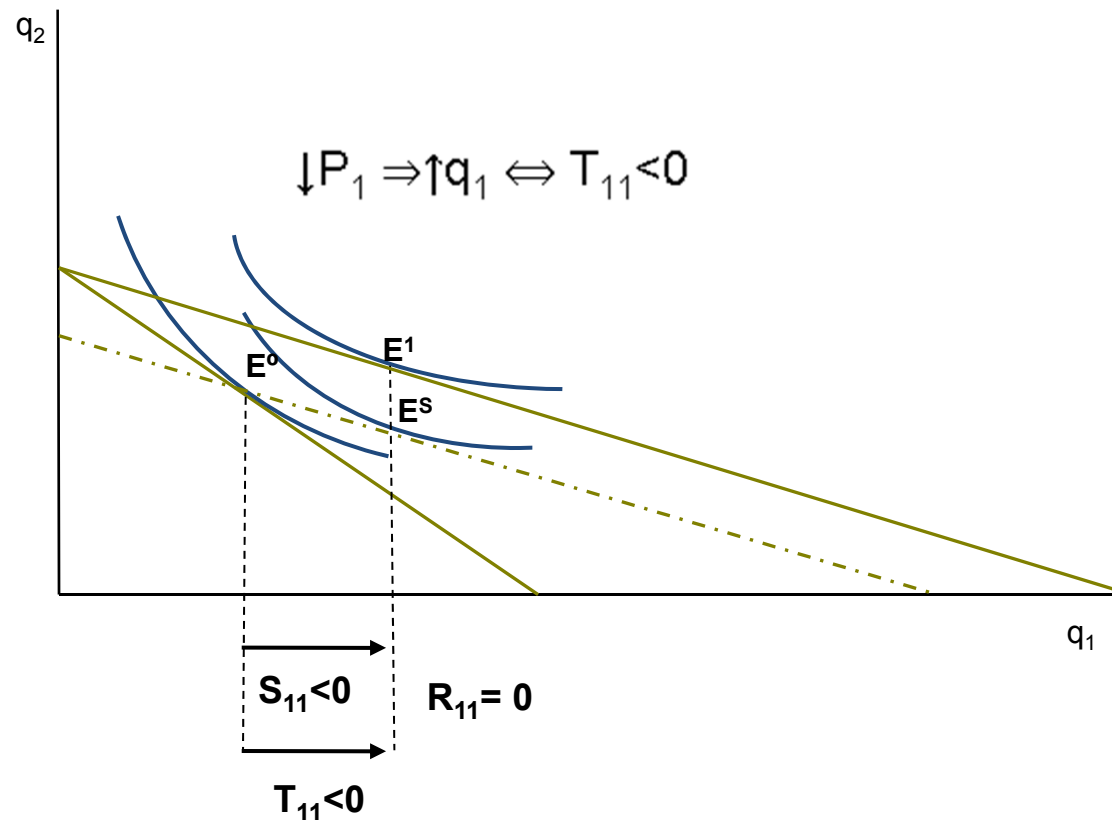Classification of demand by means of the total effect:

(1) Normal good ($R_{11} < 0$) & Normal Demand $\Rightarrow T_{11} = S_{11} + R_{11} < 0$

$$\downarrow P_1 \Rightarrow \uparrow q_1 \Leftrightarrow T_{11} < 0$$

$S_{11} < 0$ $\quad R_{11} < 0$

$T_{11} < 0$
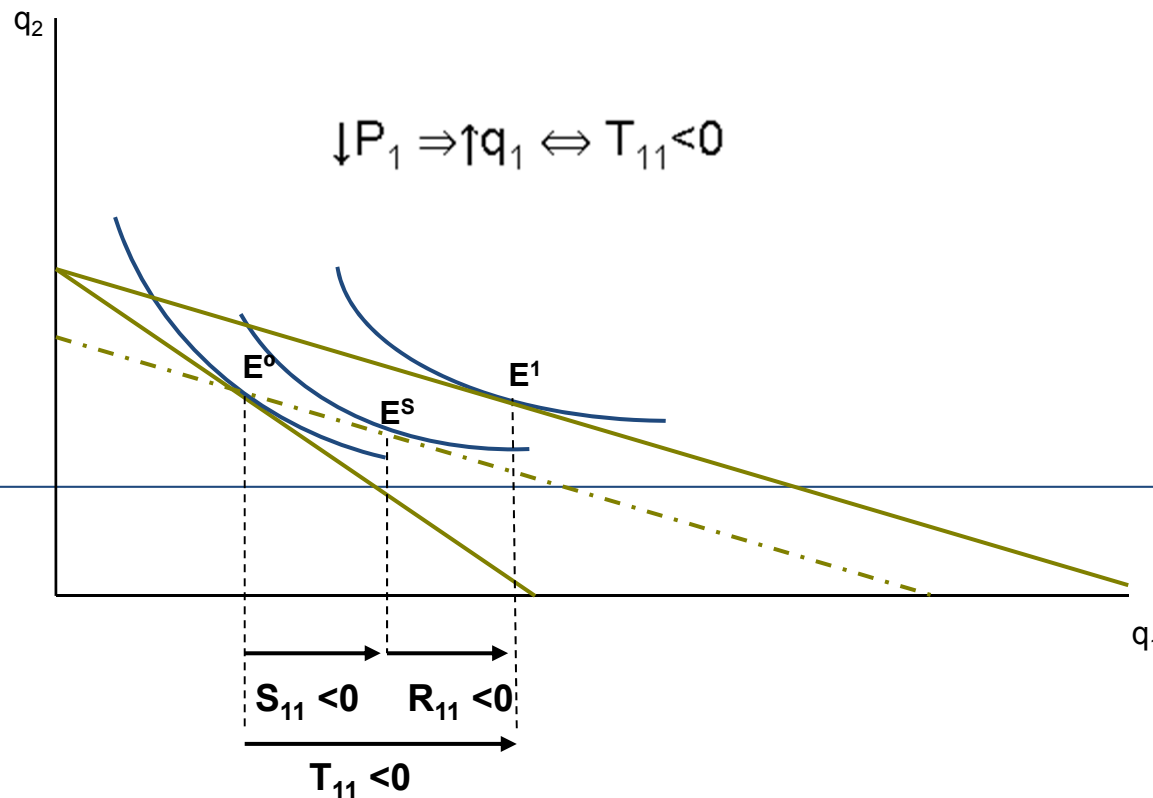
5.1 Income and substitution effects

- Classification of demand by means of the total effect:

(2) Neutral good ($R_{11}= 0$) & Normal Demand $\Rightarrow T_{11} = S_{11}+ R_{11} <0$

$\downarrow P_1 \Rightarrow \uparrow q_1 \Leftrightarrow T_{11}<0$

$S_{11}<0 \qquad R_{11}= 0$

$T_{11}<0$

5.1) Income and substitution effects

- **Classification of demand by means of the total effect:**

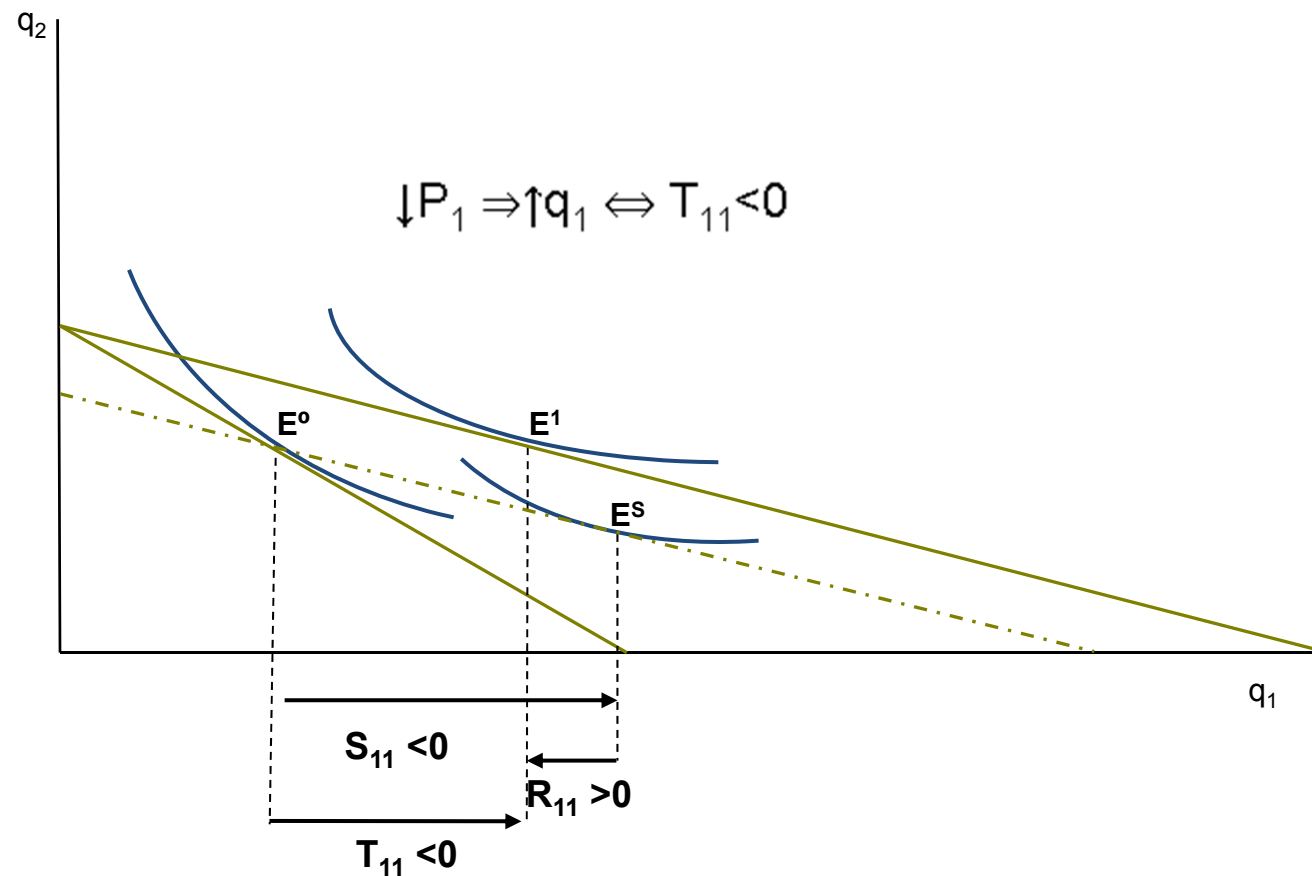**(3) Inferior good ($R_{11} > 0$) & Normal Demand: (i) if $S_{11} > R_{11} \Rightarrow T_{11} < 0$**

$$\downarrow P_1 \Rightarrow \uparrow q_1 \Leftrightarrow T_{11} < 0$$

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5.1 Income and substitution effects

- **Classification of demand by means of the total effect:**

(3) Inferior good ($R_{11} > 0$)& Rigid Demand:(ii) $S_{11} = R_{11} \Rightarrow T_{11} = 0$
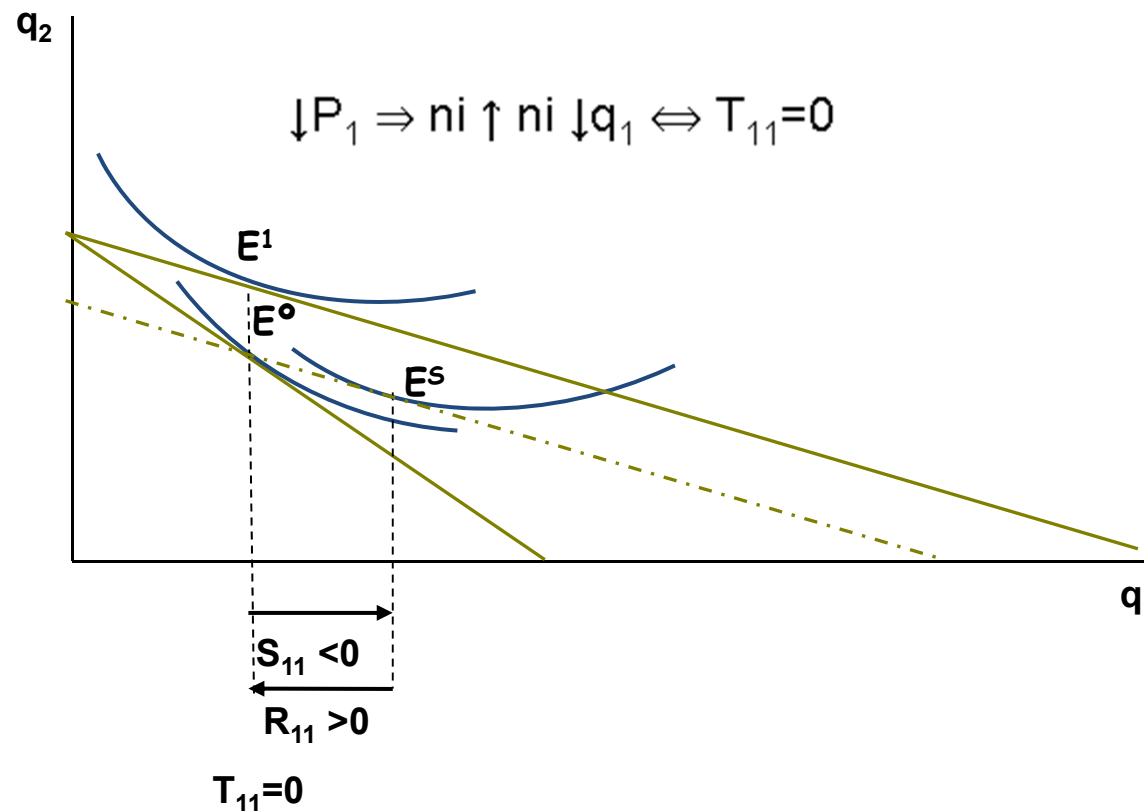
q_2

$\downarrow P_1 \Rightarrow ni \uparrow ni \downarrow q_1 \Leftrightarrow T_{11} = 0$

E^1

E^o

E^s

q_1

$S_{11} < 0$

$R_{11} > 0$

$T_{11} = 0$

5.1 Income and substitution effects

- Classification of demand by means of the total effect:

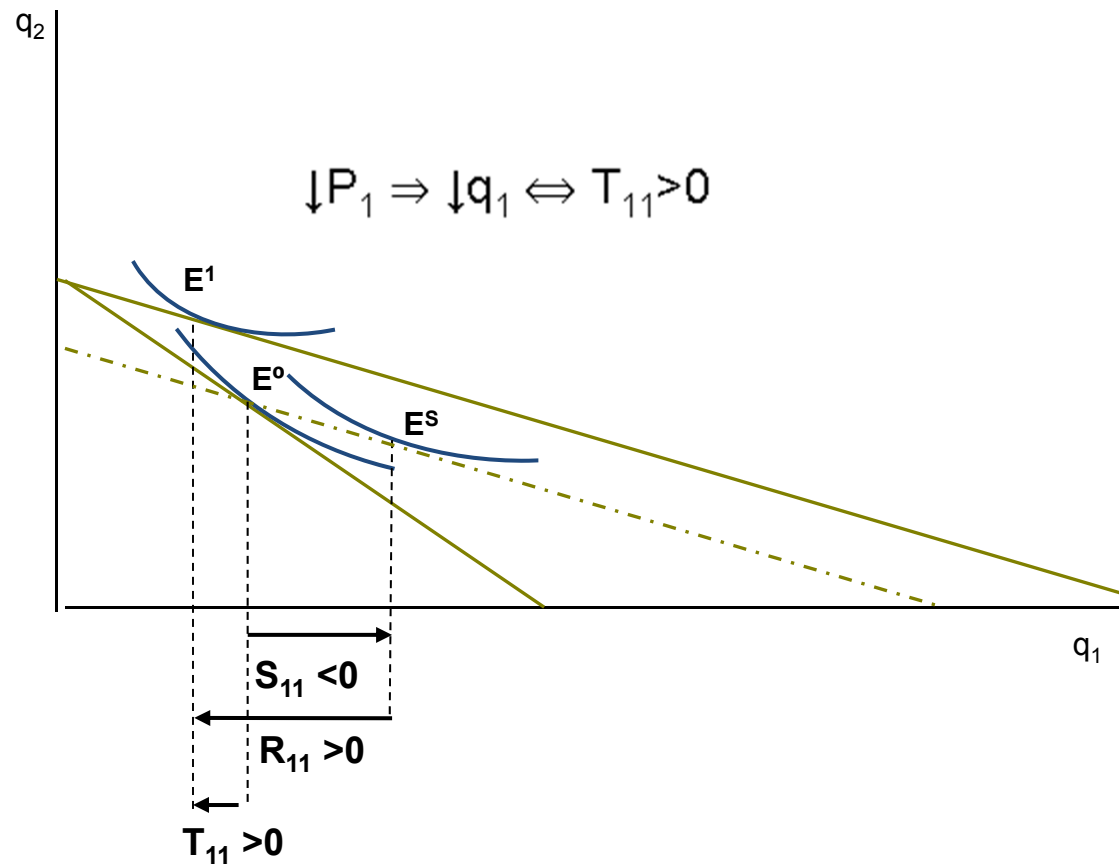**(3) Inferior good ($R_{11} > 0$) & Abnormal Demand: (iii) if $S_{11} < R_{11} \Rightarrow T_{11} > 0$**

q_2

$\downarrow P_1 \Rightarrow \downarrow q_1 \Leftrightarrow T_{11} > 0$

E^1

E^0

E^S

q_1

$S_{11} < 0$

$R_{11} > 0$

$T_{11} > 0$

5.2 The consumer surplus

- **Definition:**

    The **consumer surplus** is a quantitative indicator of the welfare improvement for a consumer according to his/her purchases of goods.

    Since every consumer obtains a different satisfaction from the consumption of a good, the maximum amount of money he/she will be willing to pay for the good will also be different.
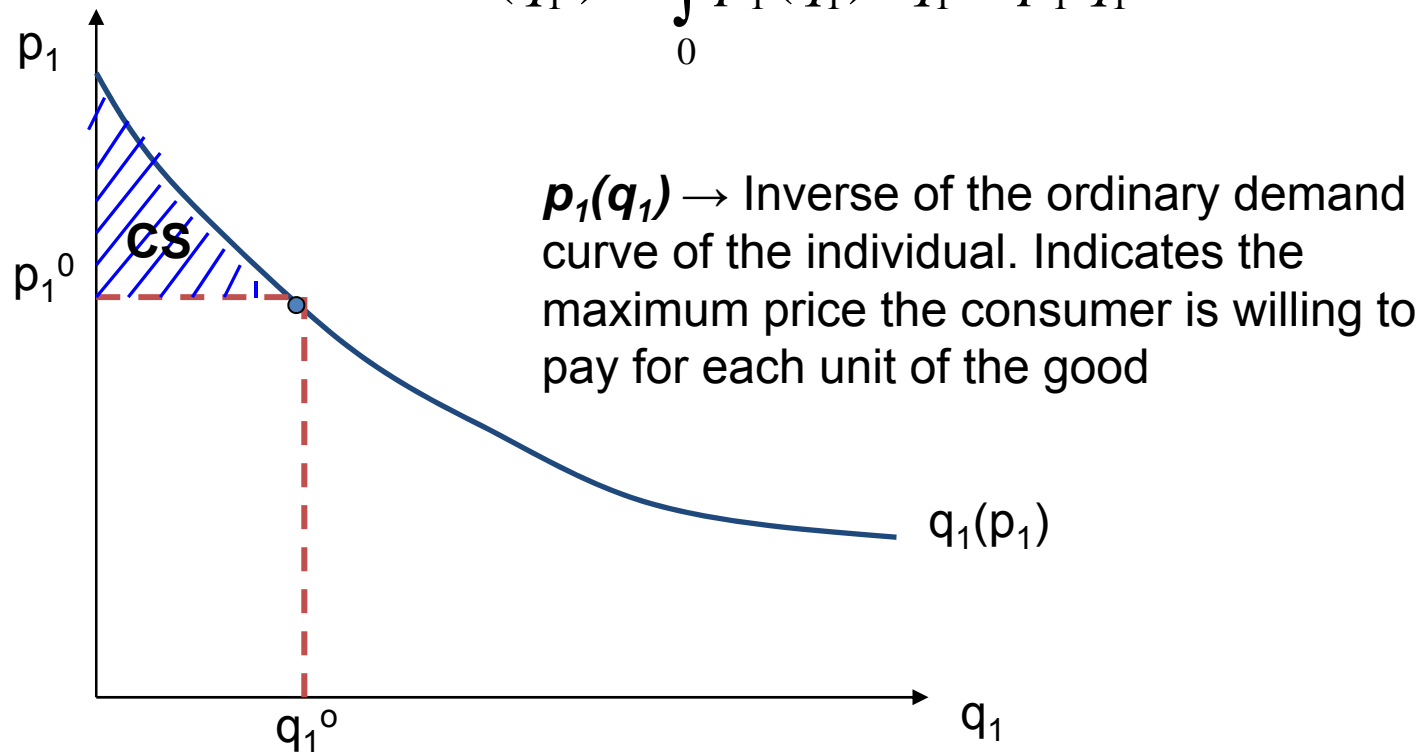
- The consumer surplus is the difference between the maximum willingness to pay for a given amount of the good, and the amount of money the consumer actually pays for it in the market (the market price).

- The consumer surplus is calculated using the individual (inverse) demand curve of the good. Consider good Q1:

$$ODC_1 : q_1 = d_1(p_1) \implies ODC_1^{-1} : p_1 = d_1^{-1}(q_1) = p_1(q_1)$$

5.2 The consumer surplus

- The consumer surplus is the consumer net gain from consuming all desired units of the good (maximum willingness to pay minus the real cost of the consumption):

$$CS(q_1^0) = \int_0^{q_1^0} p_1(q_1)dq_1 - p_1^0 q_1^0$$

$p_1(q_1) \rightarrow$ Inverse of the ordinary demand curve of the individual. Indicates the maximum price the consumer is willing to pay for each unit of the good
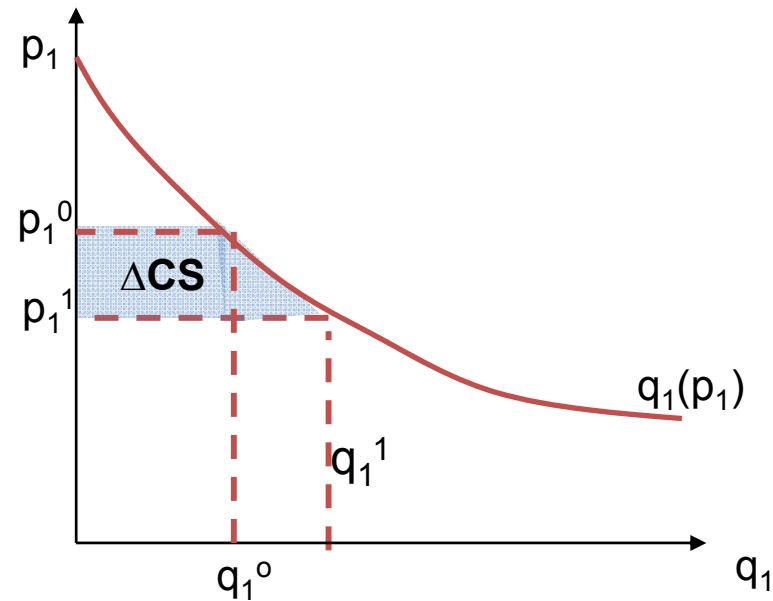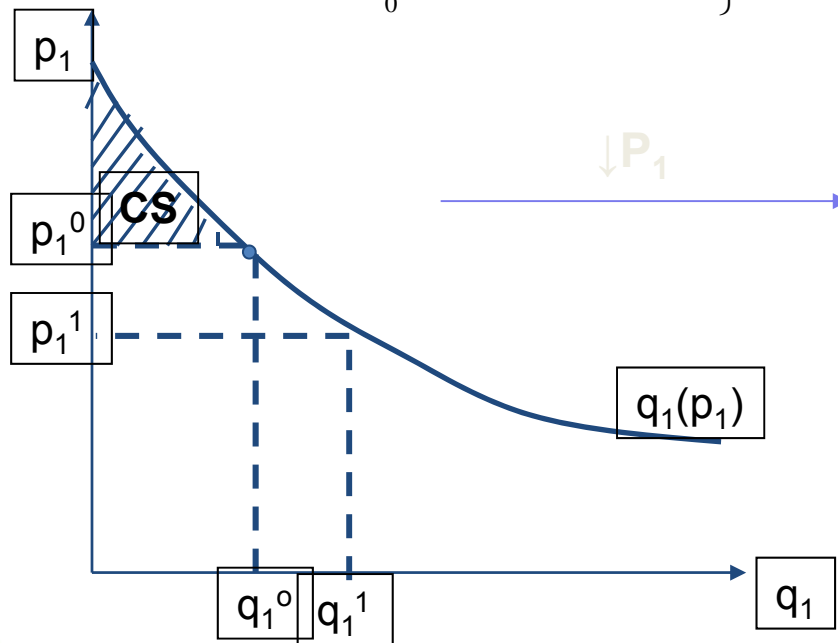
5.2 The consumer surplus

- **Changes in the consumer surplus due to changes in prices:**

  The consumer surplus changes when the price of the good changes, measured in monetary units:

$$CS(q_1^0) = \int_0^{q_1^0} p_1(q_1)dq_1 - p_1^0 q_1^0$$

$$CS(q_1^1) = \int_0^{q_1^1} p_1(q_1)dq_1 - p_1^1 q_1^1$$

$$\Rightarrow \Delta CS = CS(q_1^1) - CS(q_1^0)$$

- If $\downarrow p_1 \Rightarrow \Delta CS > 0$
- If $\uparrow p1 \Rightarrow \Delta CS < 0$

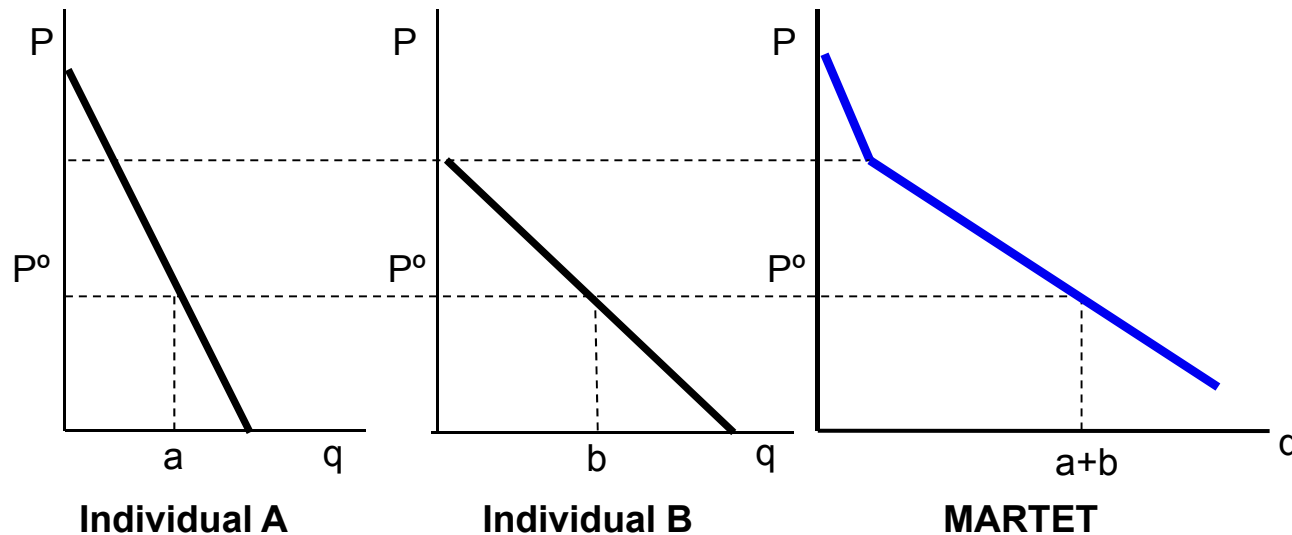# 5.3 From the individual demand functions to the market demand function

- The market demand is the function that provides the total quantity demanded of the good in the market for each possible price. To obtain this market demand, we sum all quantities demanded by all consumers at the same price.

- The market demand curve is the sum of the individual demand curves of all consumers in the market of the good.

5.3 From the individual demand functions to the market demand function

- The market demand curve is graphically obtained by adding for the same price all individual demand curves:

| Individual A | Individual B | MARTET |

- The market demand curve moves to the right as more consumers enter the market.

- All variables affecting the individual demand curves also affect the market demand curve.

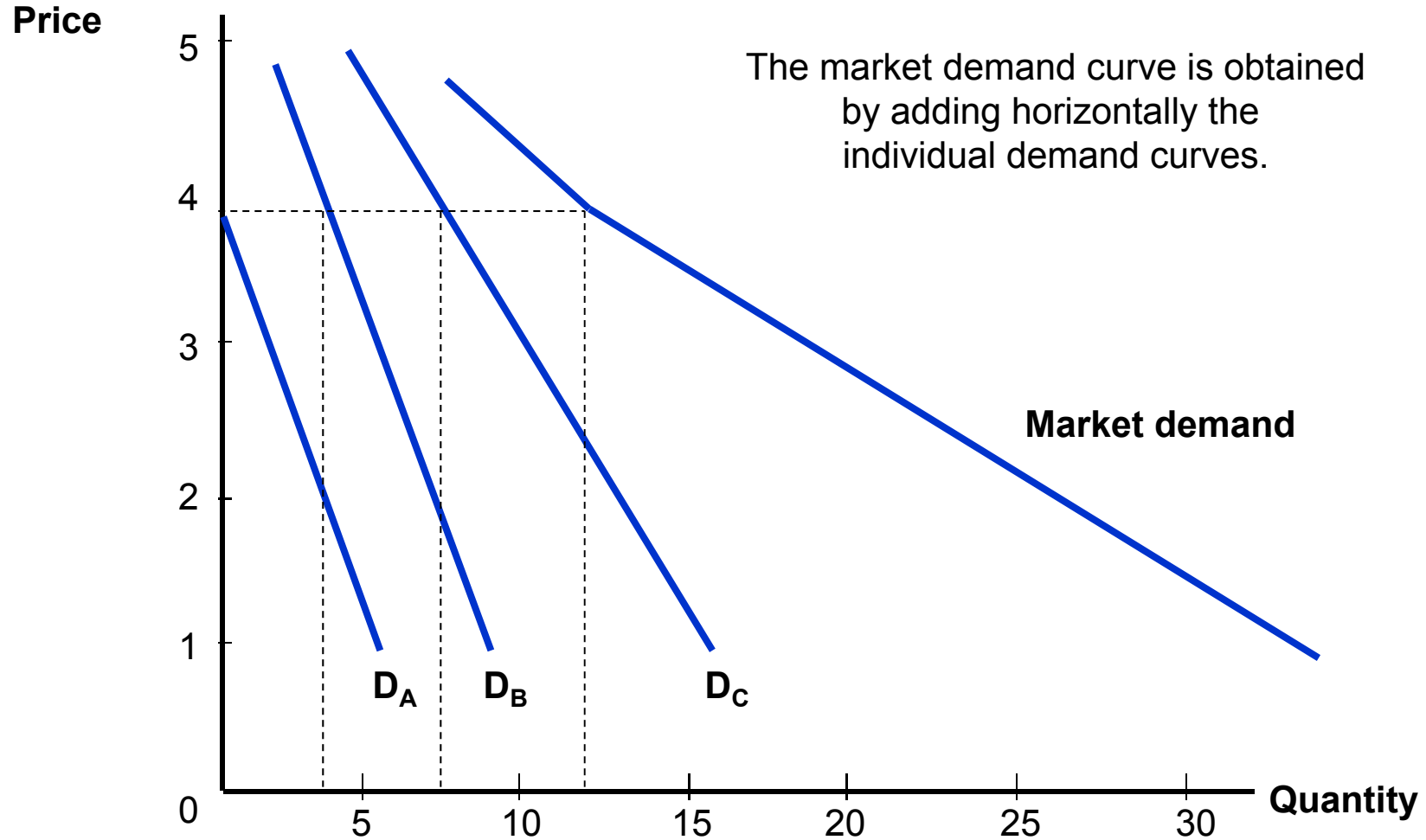# 5.3 From the individual demand functions to the market demand function

- **Example 1:**

Price (euro)	Individual A (units bought)	Individual B (units bought)	Individual C (units bought)	Market (units bought)
1	6	10	16	32
2	4	8	13	25
3	2	6	10	18
4	0	4	7	11
5	0	2	4	6

5.3) From the individual demand functions to the market demand function

- **Example 1:**

The market demand curve is obtained by adding horizontally the individual demand curves.

5.3 From the individual demand functions to the market demand function

Example 2:

The domestic demand of wheat is given by equation:
$$Q_{DD} = 1.700 - 107P$$

The demand of wheat for export is given by the equation:
$$Q_{DE} = 1.544 - 176P$$

$$Q_{total} = Q_{DD} + Q_{ED} = \begin{cases} 3244 - 283P & \text{if } P < 8.77 \rightarrow \text{Segment EF} \\ 1700 - 107P & \text{if } P \geq 8.77 \rightarrow \text{Segment AE} \end{cases}$$

The total demand of wheat is the horizontal sum of the domestic demand *AB* and the demand for export *CD*.
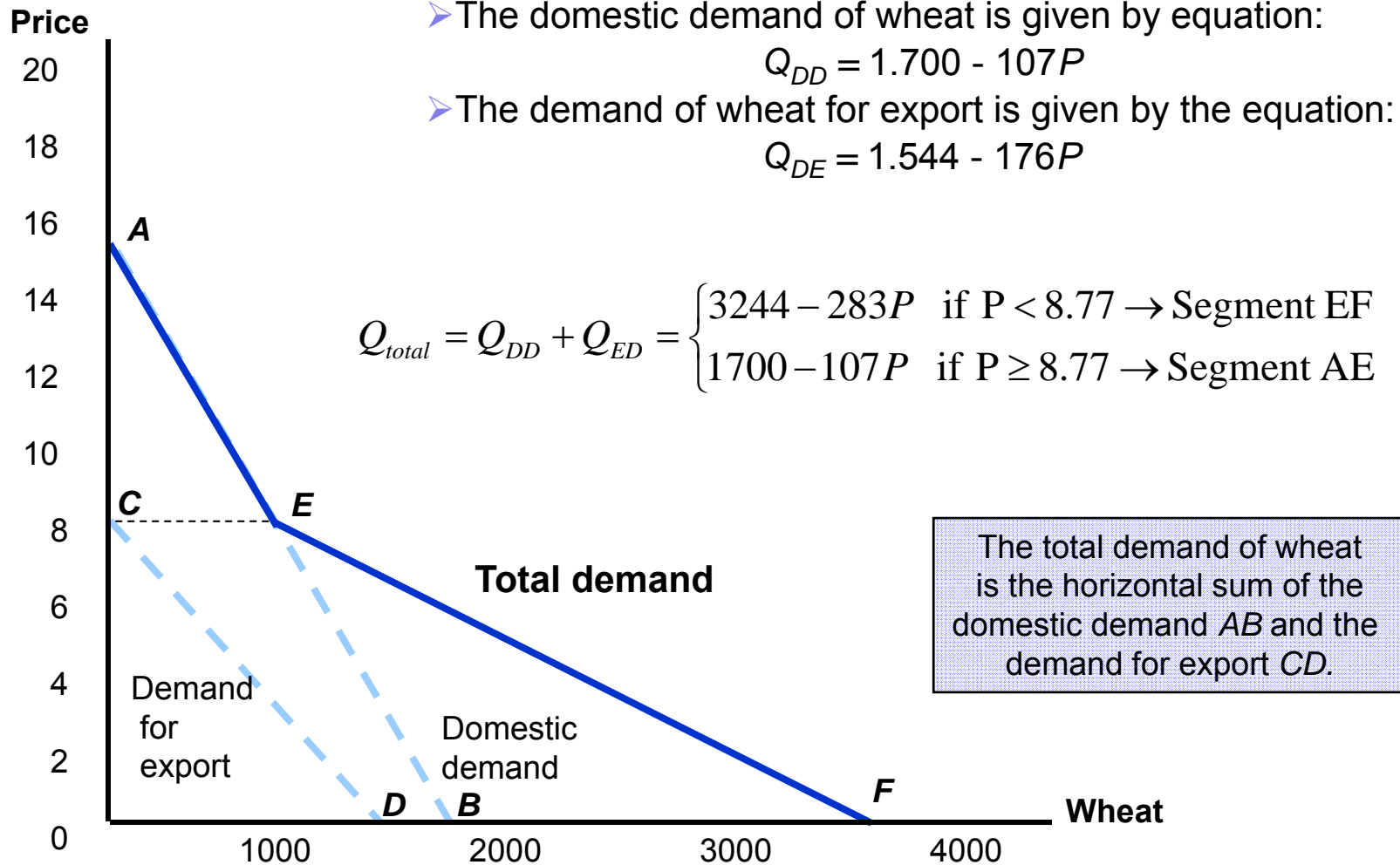
Exercises

1.- The preferences of a family between food (Q_1) and other goods (Q_2) are represented by the utility function: $U = (q_1 - 2)(q_2 + 5)^2$

We know that the family's total income is 22 m.u. and the price of each unit of (Q_2) is 2. Answer the following questions:

  a) Obtain the equilibrium assuming that the unitary price of food is 1.

  b) Assume that the price of food increases to $p_1 = 8$. Obtain the income needed for the consumer's purchasing power to be constant.

  c) Obtain the income and substitution effects and represent the situation graphically.

  d) Classify the goods in gross and net terms.

2.- The preferences can be represented by the utility function: $U = 16 \cdot q_1^2 \cdot q_2^2$

Answer the following questions:

  a) Calculate the quantities at equilibrium when the consumer income is 24., the price of good Q_1 is 2 and the price of good Q_2 is 1

  b) The government establishes a unitary tax on consumption of good Q_1 of 6. Obtain the income and substitution effects under the Slutsky decomposition criterion and represent the situation graphically.

  c) Classify the goods in gross and net terms.

3.- The preferences of a consumer are represented by the function: $U = \dfrac{2}{3} \cdot q_1 \cdot q_2^3$
Answer the following questions:

   a) Calculate the quantities demanded of both goods when the consumer's income is 600, the price of good Q_1 is 2 and the price of good Q_2 is 4.

   b) As a consequences of the liberalization of imports in the market of good Q_2 the price falls to $P_2=3$. Obtain the income and substitution effects and represent the situation graphically. Classify the good in gross and net terms.

   c) Starting from the situation in point b) above, the government is planning to tax the consumption of Q_2 and there are two possibilities:

      I. A tax of 2 per unit of Q_2.

      II. An income tax designed to achieve exactly the same revenue as the tax in (i).

   Determine the appropriate income tax in (ii) and argue which tax will be preferred by the consumer.

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MICROECONOMICS I

J. Alberto Molina -J. I. Giménez Nadal

Unit 5 – Pg. 30

4.- Given the utility function: $U = 3q_1^{1/2}q_2^{1/3}$, answer the following questions:

    a) Calculate the quantities demanded at equilibrium if the income is 400, the price of good Q_1 is 4 and the price of good Q_2 is 8.

    b) Calculate the change in consumer surplus if the price of good Q_1 rises to 8

5.- Considering the utility function: $U = (q_1 + 2)(q_2 + 4)$ answer the following:

    a) Decompose the total effect (direct and crossed) from an initial situation (Y=8; P$_1$=2; P$_2$=1) and a final where P$_1$'=1, and represent graphically.

    b) Explain the relation between the goods in both gross and net terms.

6.- Consider the utility function: $U = q_1^{1/2}q_2^{1/3}$ and answer the following:

    a) Decompose the total effect (direct and crossed) from an initial situation (Y=50; P$_1$=3; P$_2$=4) and a final where P$_2$'=2, and represent graphically.

    b) Explain the relation between the goods in both gross and net terms.

7.- We know that Y=400, P_1=2 and P_2=4. Decompose the total direct effect following the Slutsky criterion when P_2 falls to P_2'=2 in the case of a consumer with the following general demand functions:

$$q_1 = \frac{Y}{2p_1} \qquad q_2 = \frac{Y}{2p_2}$$

8.- George uses red and green pens that give him exactly the same satisfaction, although the red ones are more expensive. Represent graphically the decomposition of the total effect when the price of the green ones rises until it is higher than the price of the red ones.

9.- A consumer of DVDs and players is always willing to exchange two DVDs for a player. The price of the DVD is 5, the price of the player is 15, the income is 900:
   a) Represent the map of indifference curves and the consumer equilibrium.
   b) Decompose the effect if the price of players changes to 9 and represent it.

10.- A rational consumer reduces the quantity of the two goods when P_1 rises:

   a) Determine the relation between the two goods in gross terms.

   b) Determine whether good 2 is normal, inferior or neutral. Answer the same question for good 1.

11.- Considering goods 1 and 2 are perfect substitutes, with $P_1=6$; $P_2=8$; $Y=48$. Decompose the total effect when P_1 falls to $P_1'=4$ and represent it graphically.

12.- You are in charge of opening a bridge with variable use Q in which the user fee is denoted by P. The (inverse) total demand of the use of the bridge is P= 15 – (1/2)Q. Answer the following questions:
   a) Represent graphically the demand curve of the bridge.
   b) How many people will cross the bridge if its use is free.
   c) Calculate the loss in consumer surplus associated with a user fee of 5 €.
   d) You are studying the possibility of increasing the fee to 7. How many people will now cross the bridge? Will revenue rise? Is the answer to the previous question related to the price elasticity of the market demand? Explain.
   e) Find the loss in consumer surplus when the user fee rises from 5 to 7 .

13.- The "Internacional" restaurant (1) in Manhattan, restaurant "Shabeen" in Miami (2) and restaurant "Chon" in Mexico (3) are the only restaurants in the world that buy a delicious species of insect for cooking the dish called "crunchy bees larvae covered with hot chocolate". The three restaurant's demand curves are, respectively:

$$p_1 = 50 - \frac{1}{2}q_1 \qquad p_1 = 40 - \frac{1}{4}q_1 \qquad p_1 = 30 - \frac{1}{5}q_1$$

Find the total demand for the market of bees larvae and represent it graphically.