MICROECONOMICS I
PART II: DEMAND THEORY

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PART II: Demand theory

Demand theory deals with studying consumer behavior, when deciding which goods to buy and how much to buy. It tries to answer the questions: How do consumers distribute their income among each good or service? Which are the explanations of the consumers with respect to their demands?

UNIT 3: Preferences, utility and the budget constraint

Purpose: Understanding why consumers prefer some goods to others, and how purchasing behavior is restricted by current prices and income.

UNIT 4: Consumer choice

Purpose: Analyzing how consumers make their decisions among available goods and services, and how the quantity bought of each good is determined by the utility maximization subject to the budget constraint.

UNIT 5: Individual demand function and market demand function

Purpose: Understanding how the consumer choice changes when prices and/or income change.

UNIT 3
PREFERENCES, UTILITY AND THE BUDGET CONSTRAINT

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UNIT 3
PREFERENCES, UTILITY AND THE BUDGET CONSTRAINT

1. **Choice set and budget constraint (Pindyck → 3.1)**
   - ❑ Choice set.
   - ❑ Preference relation.
   - ❑ Indifference set.
2. **Utility function (Pindyck → 3.1)**
   - ❑ Existence and properties of the utility function
   - ❑ The marginal utility
   - ❑ Correspondence between utility function and indifference curves
3. **The marginal rate of substitution (Pindyck → 3.1)**
   - ❑ Definition
   - ❑ Important issues
4. **The budget constraint (Pindyck → 3.2)**
   - ❑ Budget set and budget constraint
   - ❑ Effective market exchange rate
   - ❑ Shifts of the budget constraint

3.1) Choice set and preference relation

CHOICE SET:

- Consumers typically face a choice problem, since they want to satisfy potentially unlimited needs, but the resources they possess to satisfy them are indeed limited:

Which is the best choice to satisfy consumers' needs?

- In order to answer this question, we need to establish some simplifying assumptions that will allow us to represent the consumers' preferences in a realistic way.

- These assumptions will guarantee that the consumers will be able to compare different baskets of goods and services and, therefore, make rational purchasing decisions:

Will the consumer prefer certail baskets of goods to others?

Or maybe he/she is indifferent among them?

3.1) Choice set and preference relation

THE CHOICE SET:

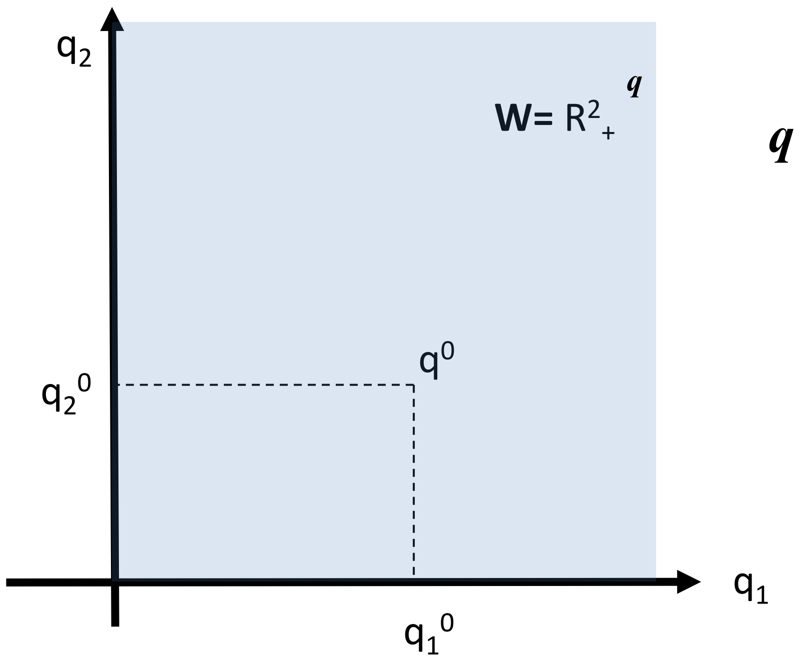
- **The choice set**: is the space in which consumers may choose different possible baskets of goods and services for consumption purposes (**W**).
- **A market basket** (or "consumption bundle"): is a list of quantities of all goods and services present in the market that are potentially available for consumption in the choice set, that is, a particular element of the choice set:
  - The choice set contains "n" possible consumption goods $Q_1, Q_2, \ldots, Q_n$
  - The corresponding quantities of the goods are denoted as $q_1, q_2, \ldots, q_n$
  - The market basket is represented by the vector: $q^0 = (q_1^0, q_2^0, \ldots, q_n^0)$.

- **We assume that:**
  - All market baskets are non-negative
  - It is possible to consume nothing from some, or all, goods
  - All goods are perfectly divisible
  - The choice set is not bounded above, that is, given any market basket, there will always exist another with more quantity of all goods

3.1) Choice set and preference relation

CHOICE SET:

- In order to simplify the analysis, we assume from now on that there are only two goods in the economy (n=2), Q_1 and Q_2.
- Therefore, the choice set consists in the whole non-negative upper-right quadrant of the plane: **W**=R^2.



$$q^0 = (q_1^0, q_2^0)$$ is a specific market basket within the choice set

3.1) Choice set and preference relation

The preference relation: Definitions

- The **preference relation** summarizes the consumer's choosing procedure among all possible market baskets within the choice set. It is the ordering criteria of all possible market baskets, and represent the individual tastes (likes and dislikes) of the consumer.

- Given q^0, $q^1 \in$ **W** :

  – **Weak preference relation**: $q^0 \succsim q^1$ means that "basket q^0 is preferred or indifferent to basket q^1" for the consumer"

  – **Strict preference relation:** $q^0 \succ q^1$ means that "basket q^0 is preferred to basket q^1 for the consumer"

  – **Indifference relation**: $q^0 \sim q^1$ means that ""basket q^0 is indifferent to basket q^1" for the consumer"

- **Indifference set**: set of market baskets that are indifferent among them: $I(q^0) = \{q \in$ **W** $/ q \sim q^0\}$ where $I(q^0)$ is the indifference set of q^0

  The graphical representation for two goods of the indifference set is known as the ***indifference curve,*** and the graphical representation of all indifference curves is the ***map of indifference curves.***

3.1) Choice set and preference relation

THE PREFERENCE RELATION: Axioms

- The preference relation is **complete and transitive**: any pair of market baskets can be compared in terms of preference, and there can be no logical contradictions in the resulting ordering.

$\Rightarrow$ **AXIOMS:** *They determine the shape of the indifference curves*

- **A1. Completeness** $\rightarrow$ Any pair of baskets can be compared (it is always possible to determine if $q^0 \succ q^1$, $q^1 \succ q^0$ or $q^0 \sim q^1$)
- **A2. Transitivity** $\rightarrow$ It guarantees the logical consistency of the ordering among any set of three market baskets (if $q^0 \succ q^1$ and $q^1 \succ q^2 \Rightarrow q^0 \succ q^2$)

$\Rightarrow$ These two assumptions imply that the consumer behavior will be "rational"

- **A3. Non-satiation** $\rightarrow$ Market baskets with more quantity of some goods and no less of any good are always preferred
- **A4. Continuity** $\rightarrow$ Market baskets in the same indifference set are distributed continuously
- **A5. Strict convexity** $\rightarrow$ Indifference curves are always strictly convex with respect to the origin of coordinates
- **A6. Smoothness** $\rightarrow$ Indifference curves are smooth, lacking any angular point

3.1) Choice set and preference relation

THE PREFERENCE RELATION:

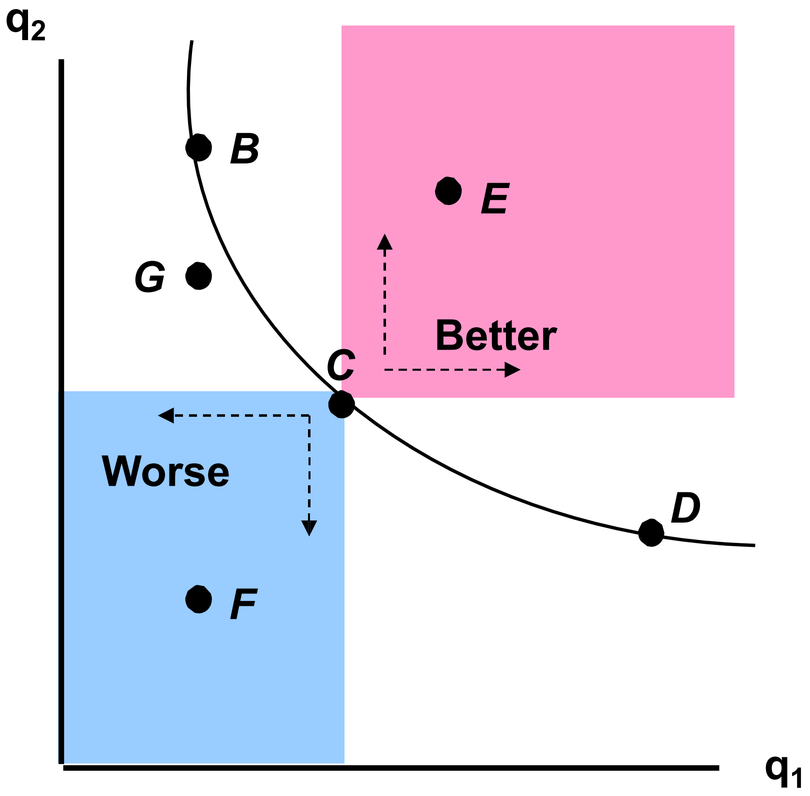
- **Implications of AXIOM 3: Non-satiation** Indifference sets are decreasing

1. Indifference sets located further from the origin are more preferred
2. All indifference sets can be represented by a continuous line, never by a "dense" area
3. There does not exist any "satiation point" or "satiation basket" that is preferred to all others in the choice set
4. We can only consider in the analysis "goods in the strict sense": goods, or services, such that more is better than less. "Bads" like rubbish, for example, are excluded

3.1) Choice set and preference relation

THE PREFERENCE RELATION : Implications of axiom 3→ Non-satiation:

(1) Indifference sets are decreasing:



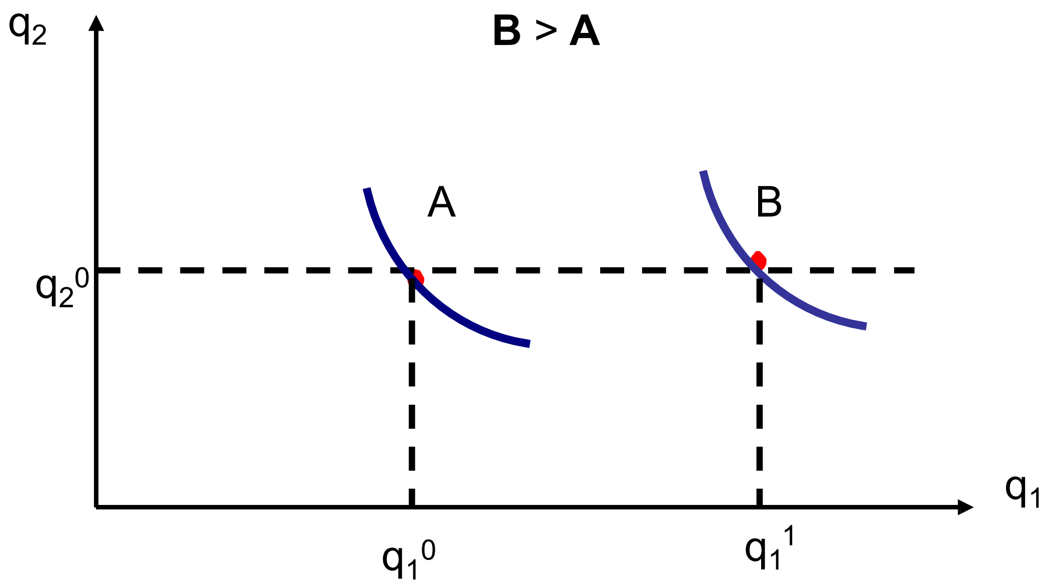
(1)Consumers prefer market basket *C* to any other basket in the blue area.
(2)When comparing market baskets in the pink area with basket *C* , all of them are preferred to C.
(3)Therefore, the baskets belonging to the indifference set of basket C (that is, all baskets considered indifferent to C) must be found in the remaining two sectors of the plane with respect to C: the upper-left and the lower-right non-colored sectors.

Therefore, in order to move across baskets along the same indifference set, we must always proceed by substituiting one good for another

3.1) Choice set and preference relation

THE PREFERENCE RELATION : Implications of Axiom 3→ Non-satiation

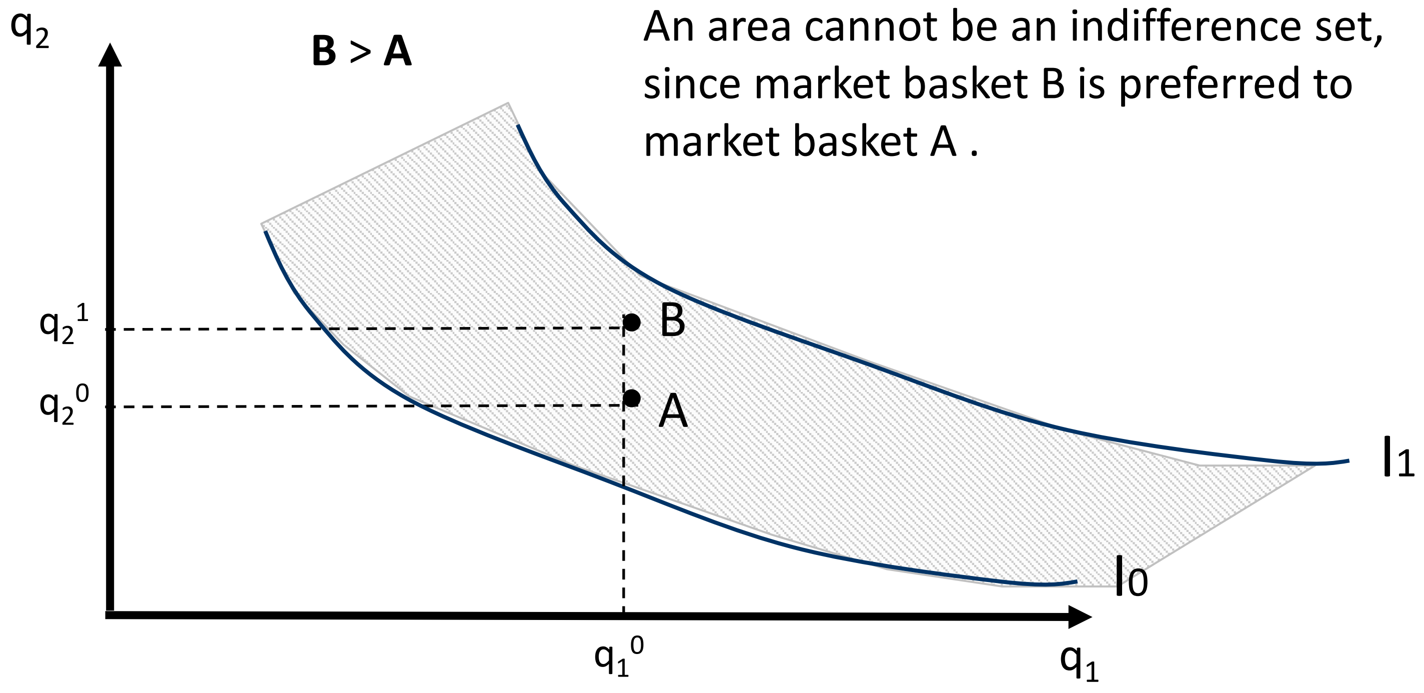
(2) Indifference sets located further from the origin are preferred to those located closer to it:



3.1) Choice set and preference relation

THE PREFERENCE RELATION : Implications of Axiom 3→ Non-satiation

(3) Indifference sets are represented by single lines: never by a broad area:

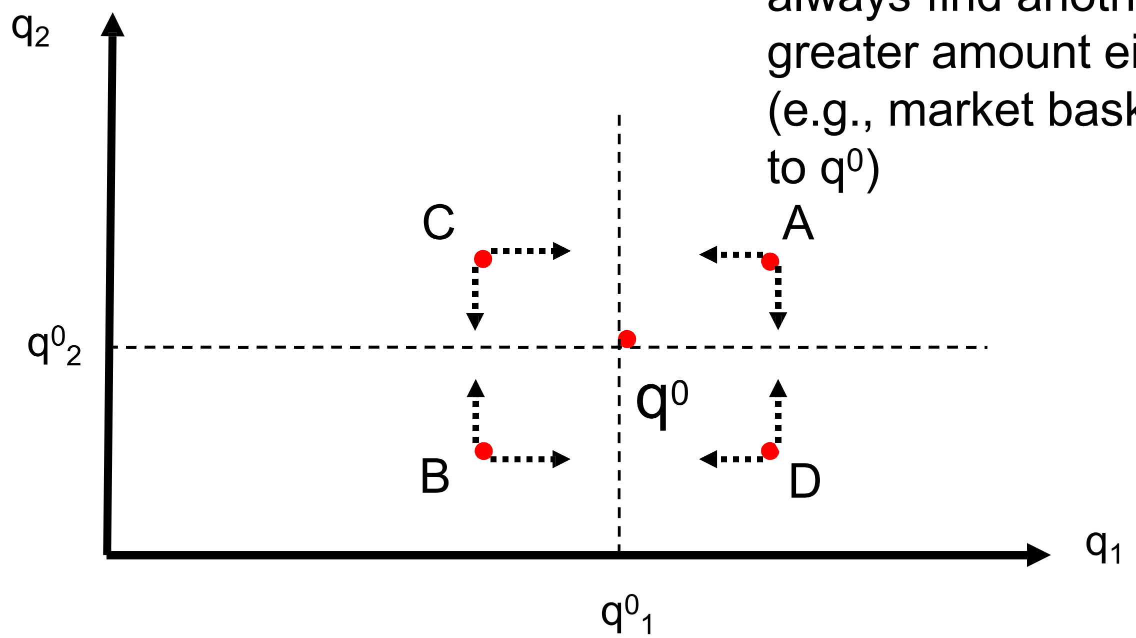


An area cannot be an indifference set, since market basket B is preferred to market basket A .

3.1) Choice set and preference relation

THE PREFERENCE RELATION : Implications of Axiom 3→ Non-satiation

(4) There cannot exist any satiation point/basket: there cannot be a market basket preferred to all others in the choice set:



The market basket q^0 cannot be preferred to all others, since we can always find another one with a greater amount either of q_1 or q_2 (e.g., market basket "A" is preferred to q^0)

3.1) Choice set and preference relation

THE PREFERENCE RELATION : Implications of Axiom 3→ Non-satiation

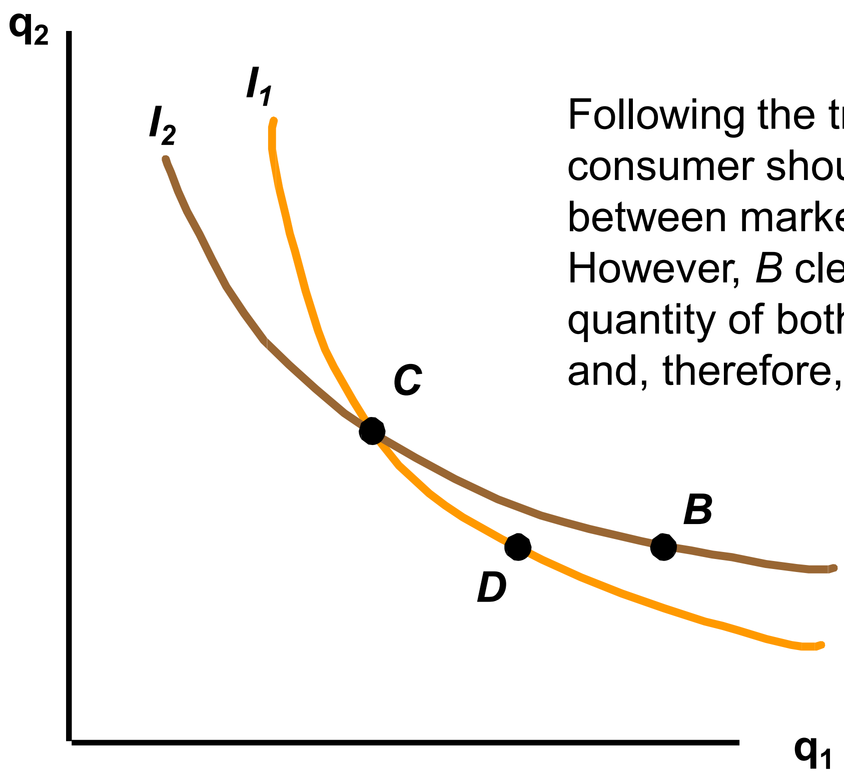
(5) We only allow in the analysis "goods in the strict sense":

- ***Good*** (in the strict sense): commodity such that the consumer prefers more rather than less amount
- ***Bad***: commodity such that the consumer prefers less rather than more amount
- ***Neutral good***: commodity such that the consumer is indifferent between having more or less amount, that is, it does not contribute to her utility
- ***Perfect complements***: commodities that must always be consumed in fixed proportions to obtain utility (e.g. shoes, gloves)

3.1) Choice set and preference relation

THE PREFERENCE RELATION : Implications of Axiom 3→ Non-satiation

(6) Indifference curves cannot intersect each other:



Following the transitivity axiom, the consumer should feel indifferent between market baskets C, *B* and *D*. However, *B* clearly contains more quantity of both goods with respect to *D* and, therefore, will be preferred.

3.1) Choice set and preference relation

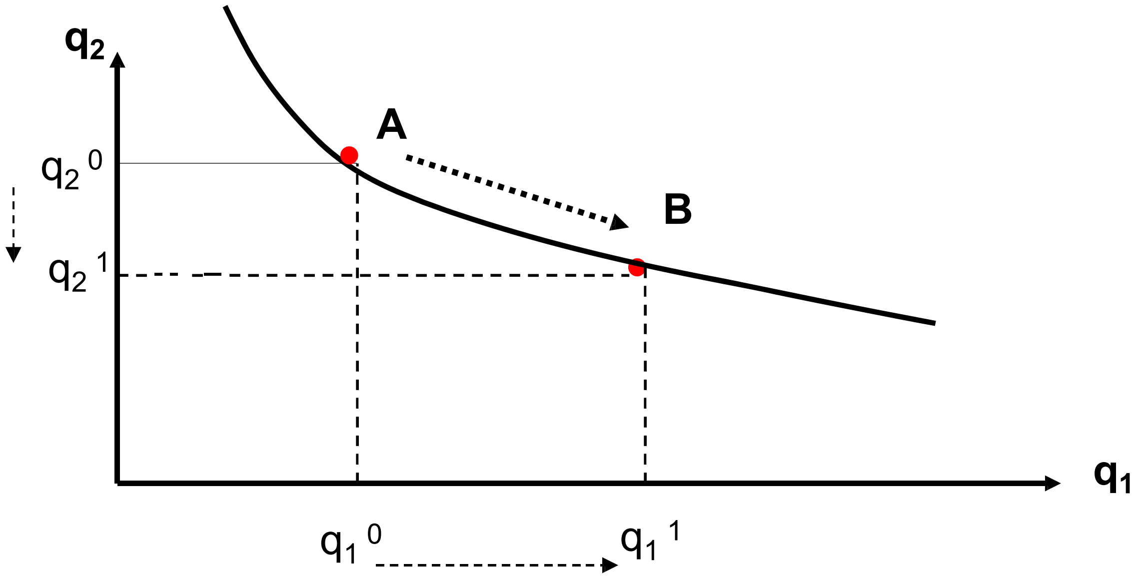
THE PREFERENCE RELATION:

- **Implications of AXIOM 4 → Continuity**

  In between two indifferent market baskets, it is always possible to find a third basket belonging to the same indifference set

  – Indifference lines do not show "jumps": they are continuous

  – We can always decrease the quantity of a good and increase the quantity of the other in sufficient amount to find a basket indifferent to the former one:



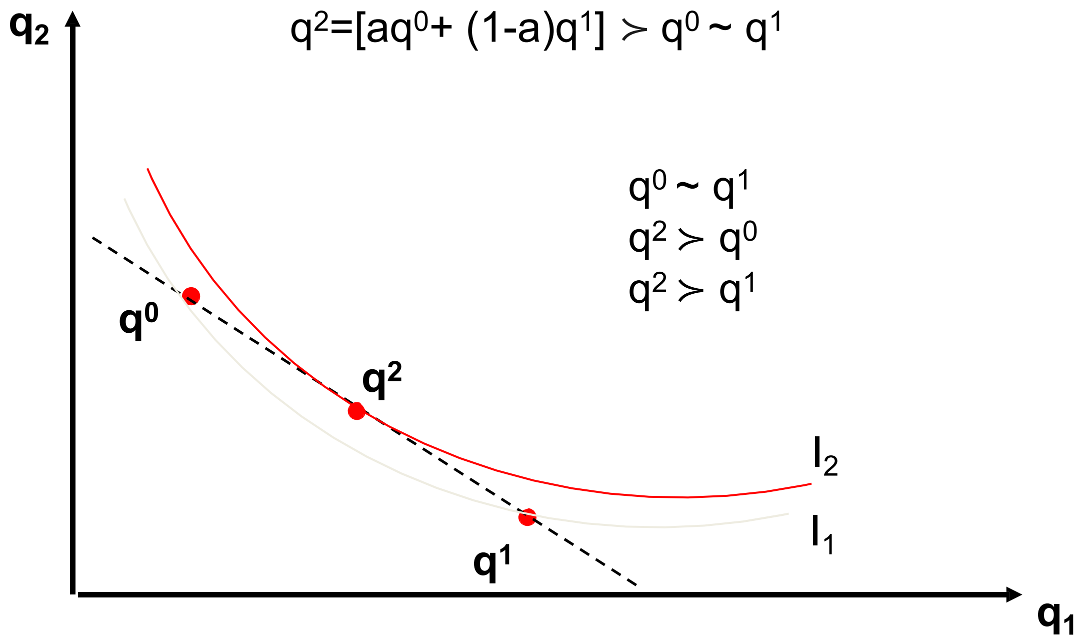
3.1) Choice set and preference relation

THE PREFERENCE RELATION:

- **Implications of AXIOM 5 → Strict convexity**
  – The weighted average of two indifferent market baskets must be strictly preferred to both. More “balanced” or mixed market baskets are preferred to “extreme” ones:

For all $q^0, q^1 \in \mathbf{W}$ such that $q^0 \sim q^1$, it holds that

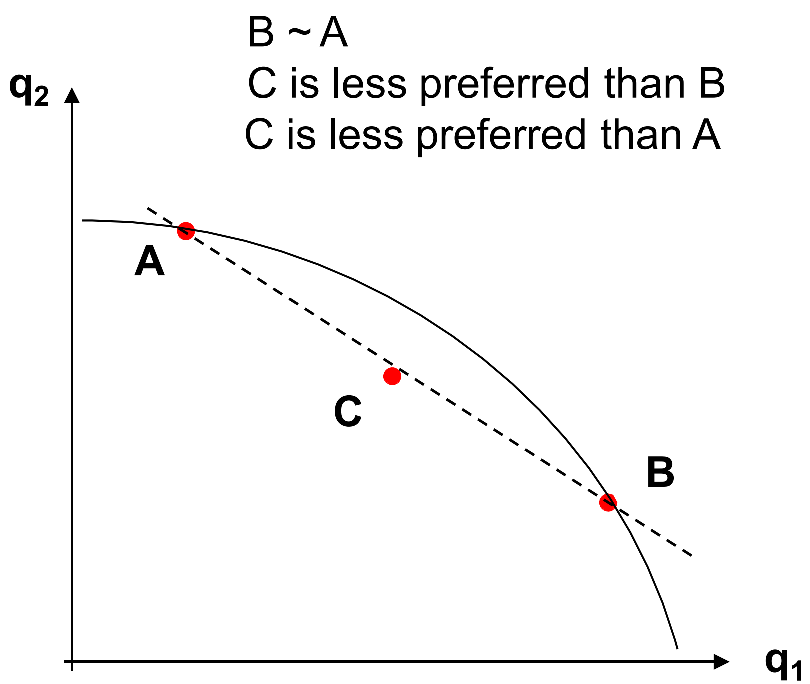
$q^2=[aq^0+(1-a)q^1] \succ q^0 \sim q^1$

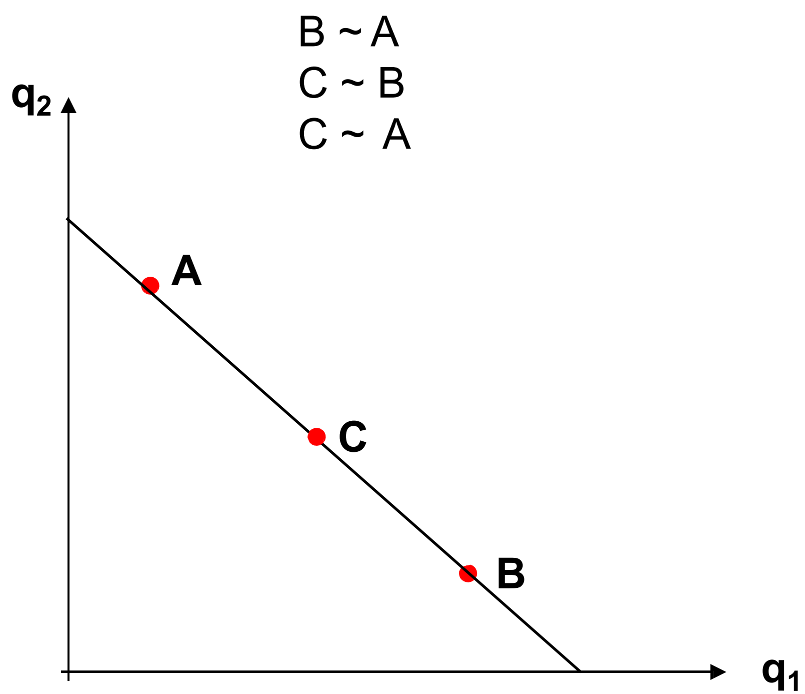


3.1) Choice set and preference relation

THE PREFERENCE RELATION:

- **Implicactions of AXIOM 5 → Strict convexity**
  - Indifference lines cannot have straight segments and cannot be either concave or wave-shaped:



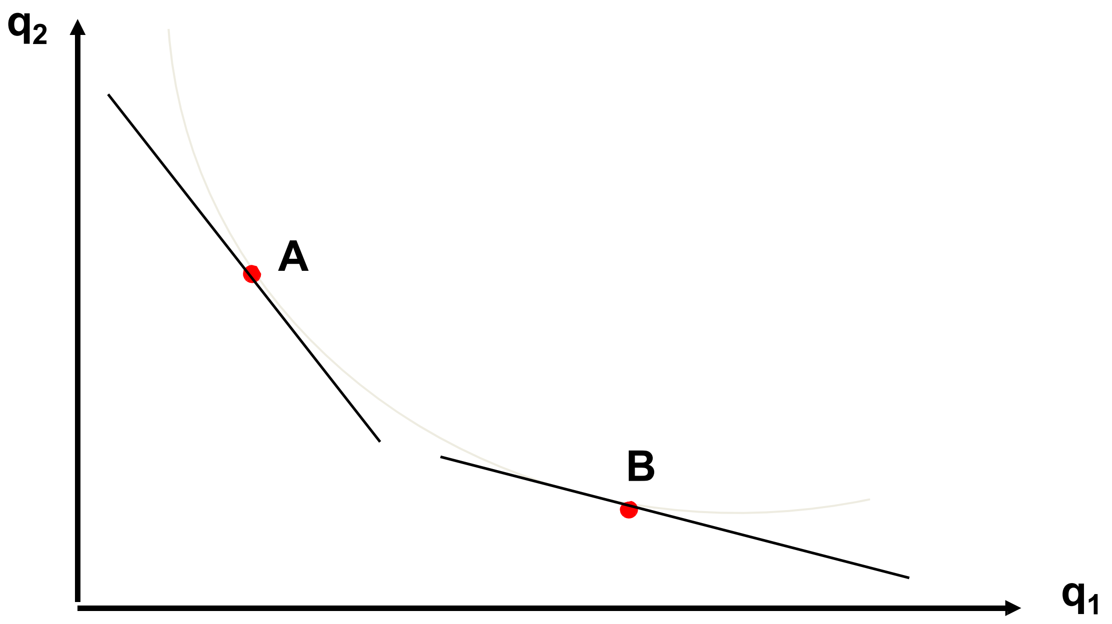


3.1) Choice set and preference relation

THE PREFERENCE RELATION:

- **Implications of AXIOM 6 → Smoothness**
  - Indifference lines are smooth, lacking any angular points, that is,

  the slope at any point along the indifference line is different with respect to any other point:



3.2) The utility function

Construction of the utility function:

- Consumer preferences can always be summarized and represented in a graph with the help of "indifference curves" according to the above properties/axioms. It is generally sufficient to define consumer choice (our next step), but …..
- It could be useful to establish a numerical criterion to rank the consumption baskets by assigning a single number to each indifference set in such a way that:
  - Baskets within the same indifference set will have the same number
  - Baskets of preferred indifference sets will have larger numbers
  - It is a fully **ORDINAL** assignment: it has no quantitative meaning
- **The utility function** represents a numerical assignment criterion which assigns a real number to each market basket, in such a way that the number represents the exact place of the basket in the consumer preference ordering.

3.2) The utility function

Construction of the utility function:

- ***Def:*** The **<u>utility function</u>** is a function U: $R^n \rightarrow R$ that assigns a real number to every market basket of the choice set respecting the underlying consumer preference ordering, in the following sense:
  - If $q^0 \sim q^1 \Rightarrow U(q^0) = U(q^1)$
  - If $q^0 \succ q^1 \Rightarrow U(q^0) > U(q^1)$
  - If $q^0 \succcurlyeq q^1 \Rightarrow U(q^0) \geq U(q^1)$
- The numerical values of the utility function are arbitrary, in such a way that **they only produce a ranking**:
  - Their specific numerical value has no other meaning or interpretation and, therefore, neither has the difference between any two values
  - Only the sign of the difference is important ("+" or "-").
- The utility function which represents some consumer preferences **is not unique**: any monotonically increasing transformation of the utility function can be considered a new utility function representing exactly the same preferences.

3.2) The utility function

Construction of the utility function:

Example 1: The utility function, defined on the choice set, where the quantities of both goods are denoted by q_1 and q_2 is the following:

$$U(q_1,q_2) = q_1 + 2q_2$$

Basket	q_1	q_2	Utility
A	8	3	8 + 2x3 = 14
B	6	4	6 + 2x4 = 14
C	4	4	4 + 2x4 = 12

The consumer is indifferent between baskets A and B.

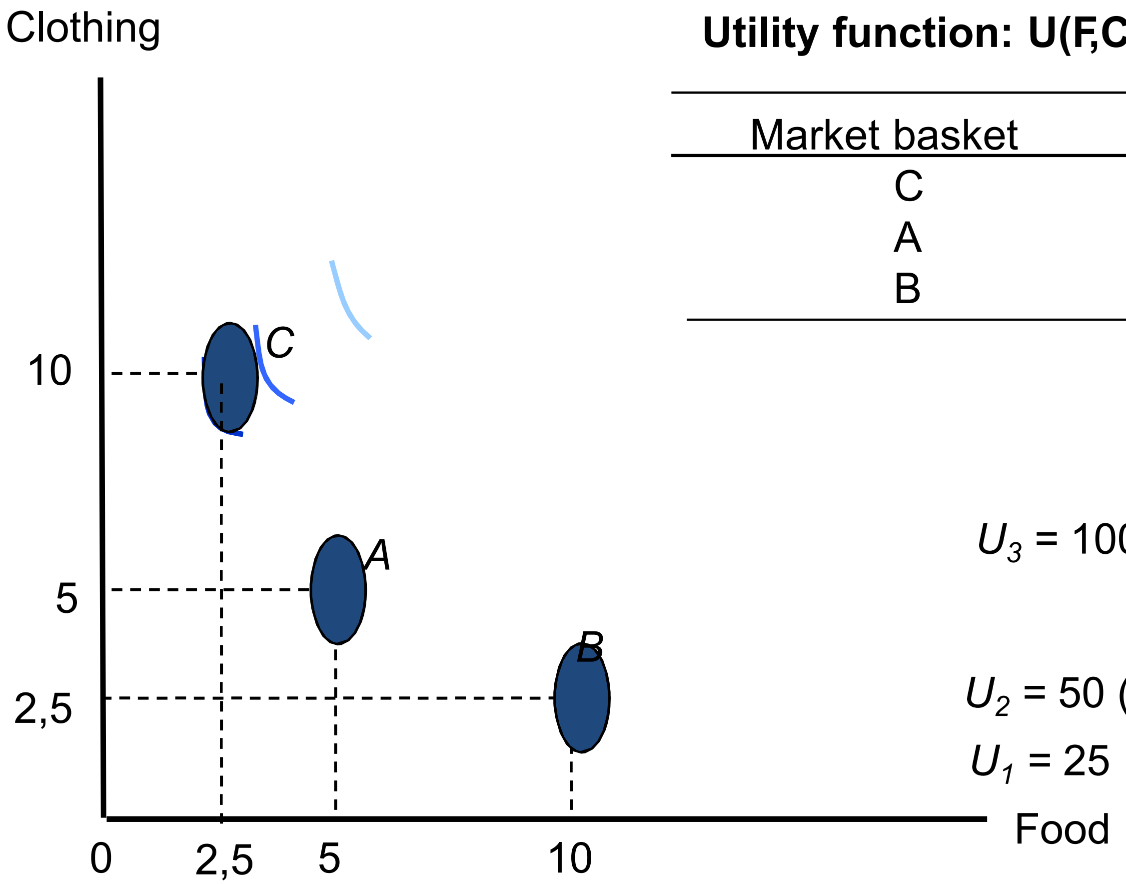
The consumer prefers A or B to C.

3.2) The utility function

Construction of a utility function: Example 2

Utility function: U(F,C) = FxC

Market basket	U = FxC
C	25 = 2,5x10
A	25 = 5x5
B	25 = 10x2,5



U_3 = 100 (better than U_2)

U_2 = 50 (better than U_1)

U_1 = 25

3.2) The utility function

Properties of the utility function:

When all the axioms imposed on the preference relation hold, there will always exist an OUF (ordinal utility function) "U" which represents the preference relation, and with the following properties:

1) U is continuous and twice differentiable, so we will be able to use differential calculus (derivates and integrals, optimization theory..)
2) U is monotonic and increasing (the first derivate is positive), with this being a direct consequence of the non-satiation axiom
3) U is strictly quasi-concave, with this being a direct consequence of the axiom of strict convexity of preferences

$U^0 = U^1$ if and only if $q^0 \sim q^1$

$U^0 > U^1$ if and only if $q^0 \succ q^1$

3.2) The utility function

Marginal Utility:

- ***Def*: The <u>Marginal utility</u>** of a good is the variation in utility due to the infinitesimal variation in the consumption level of a good when the quantity consumed of the other good remains constant:
  - Marginal utility of q_1:

$$MU_1 = \frac{\partial U}{\partial q_1} > 0$$

  - Marginal utility of q_2:

$$MU_2 = \frac{\partial U}{\partial q_2} > 0$$

- Marginal utilities are always positive, given the non-satiation Axiom.
- There exist first and second derivatives of the OUF, although their sign is not determined:

$$U_{ii} = \frac{\partial^2 U}{\partial q_i \partial q_i} \qquad U_{ij} = \frac{\partial^2 U}{\partial q_i \partial q_j} = \frac{\partial^2 U}{\partial q_j \partial q_i} = U_{ji} (symmetry)$$

3.2) The utility function

Relation between the utility function and the indifference curves:

- The indifference curves are the "contour level curves" of the utility function → set of consumption baskets such that the utility level remains constant
- If we previously determine the utility level ($U^0=U(q_1, q_2)$), we then obtain the implicit mathematical function that locates those baskets with the same utility → The indifference curve with level U^0
- From the 1st derivative of the utility function and under the restriction of a constant level of utility, we obtain that the slope of the indifference curve coincides with the ratio of the marginal utilities of both goods:

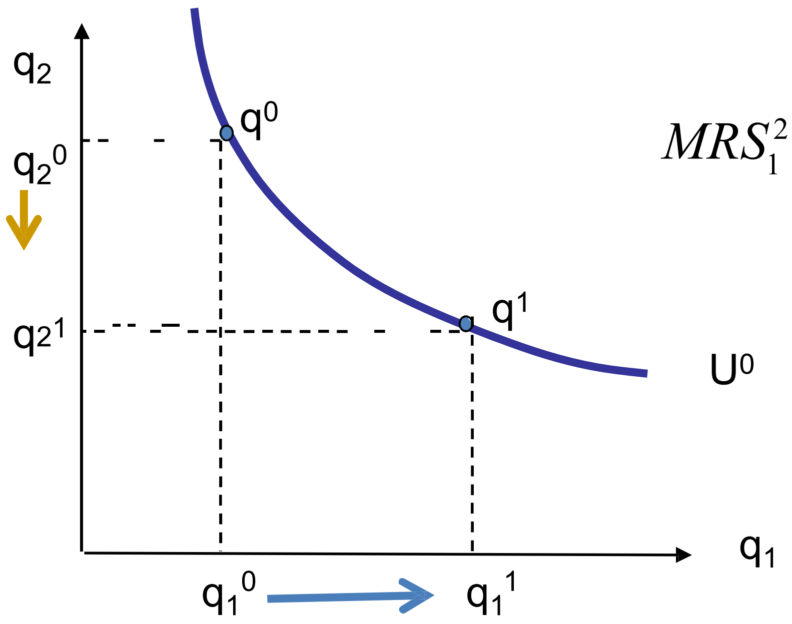
$$dU^0 = 0 = \frac{\partial U}{\partial q_1} dq_1 + \frac{\partial U}{\partial q_2} dq_2 = MU_1 dq_1 + MU_2 dq_2 \longrightarrow \underbrace{\frac{dq_2}{dq_1}}_{\text{Slope of the indifference curve}} = -\underbrace{\frac{MU_1}{MU_2}}_{\text{Ratio of marginal utilities}} < 0$$

- The 2nd derivative is positive given the strict convexity axiom: $\frac{d^2 q_2}{dq_1^{\,2}} > 0$

3.3) The marginal rate of substitution

Marginal rate of substitution:

- It answers the question of how much are we willing to give up from the consumption of a good, in exchange for one additional unit of the other good, with the utility being constant:
- We change the sign of the derivative to define the MRS as positive



$$MRS_1^2 = -\left.\frac{\Delta q_2}{\Delta q_1}\right|_{U=cte}$$

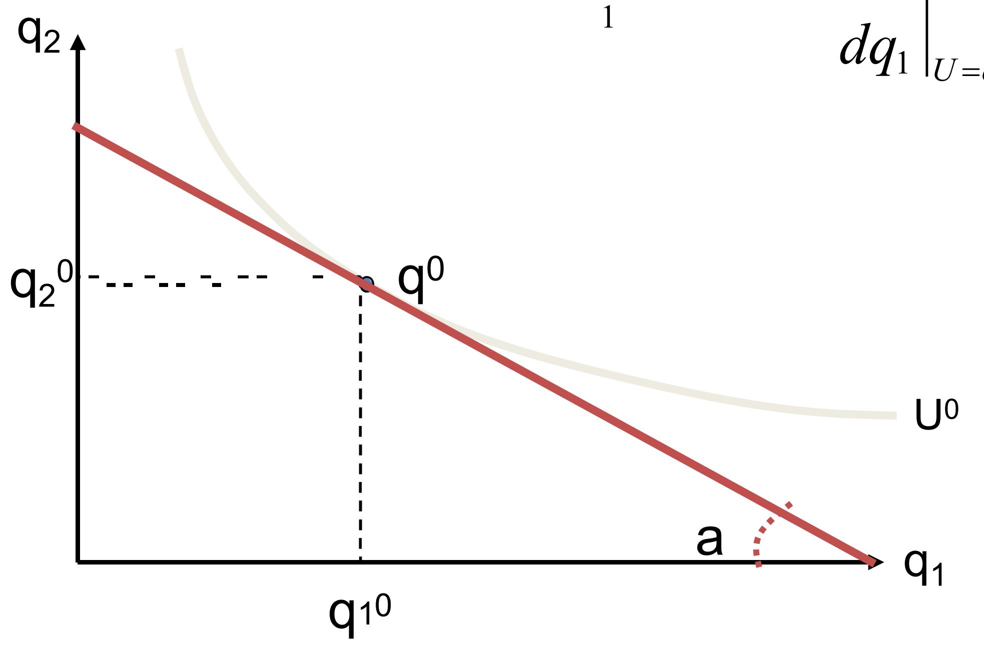
Maximum number of units of good 2 that the consumer is willing to give up in exchange for one additional unit of good 1, holding constant the utility level.

3.3) The marginal rate of substitution

Marginal rate of substitution:

- The MRS of good 2 for good 1 coincides with the slope (internal slope) of the indifference curve when we consider infinitesimal variations:

$$MRS_1^2(q^0) = -\left.\frac{dq_2}{dq_1}\right|_{U=cte} = tg\,\alpha$$



3.3) The marginal rate of substitution

Marginal rate of substitution: Important issues

(1) The MRS is the ratio of marginal utilities:

$$MRS_1^2 = -\frac{dq_2}{dq_1} = \frac{MU_1}{MU_2}$$

(2) The MRS is unique, that is, any utility function representing the same preferences will generate exactly the same MRS

(3) It always hold that:

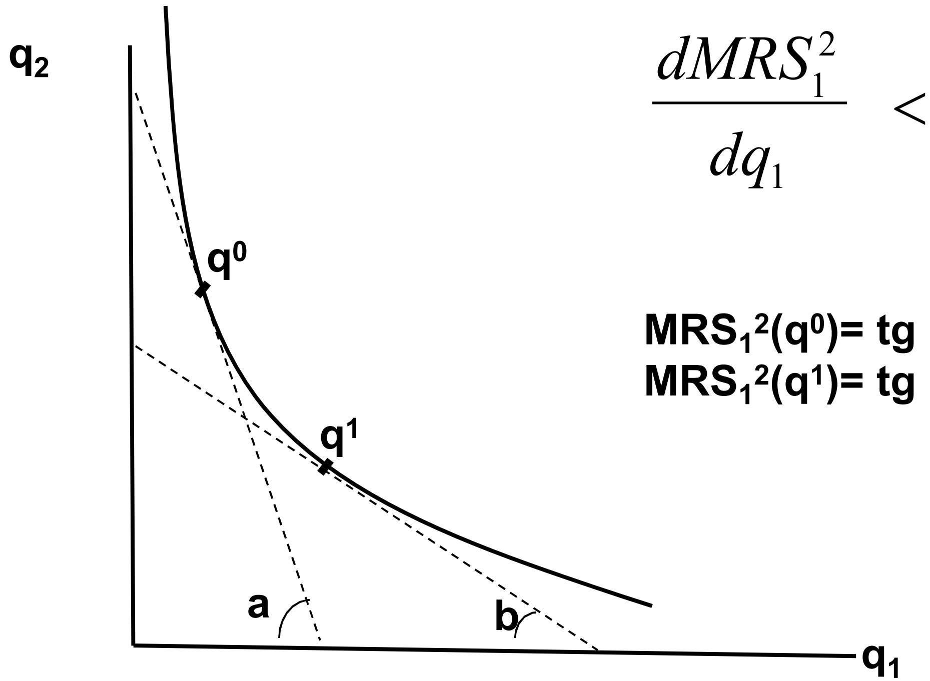
$$MRS_2^1 = -\frac{dq_1}{dq_2} = \frac{MU_2}{MU_1} = \frac{1}{MRS_1^2}$$

3.3) The marginal rate of substitution

Marginal rate of substitution: Important issues

(4) Each point of a particular indifference curve has a different particular slope. Therefore, each consumption basket in the choice set has an associated particular value of the MRS.

(5) Principle of diminishing MRS: The MRS_1^2 decreases across the same indifference curve when we increase the quantity of good 1:

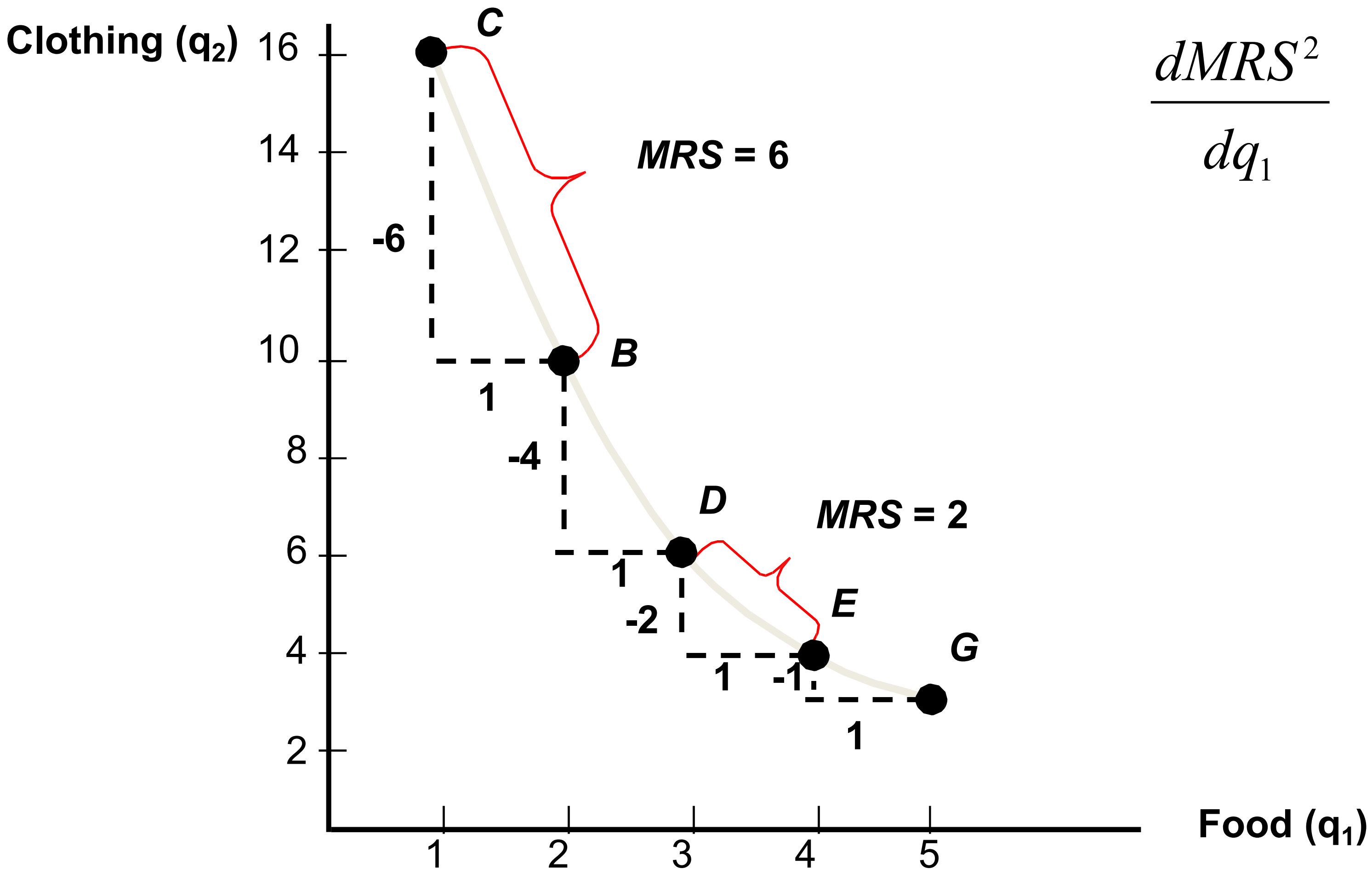


$$\frac{dMRS_1^2}{dq_1} < 0 \qquad \frac{dMRS_2^1}{dq_2} < 0$$

$MRS_1^2(q^0)$= tg a
$MRS_1^2(q^1)$= tg b < tg a

3.3) The marginal rate of substitution

Marginal rate of substitution: Principle of diminishing MRS



$$\frac{dMRS^2}{dq_1}$$

3.4) The budget constraint

Budget constraint:

- **Budget constraint**: Consumers cannot buy all goods, and in all amounts, in their choice sets because they have limited income:

$$p_1q_1 + p_2q_2 \leq Y$$

- **Budget set**: The set of market baskets that are available to a consumer given her income and the market prices of the goods:

$$B = \{q = (q_1, q_2) \in \Omega \mid p_1q_1 + p_2q_2 \leq Y\}$$

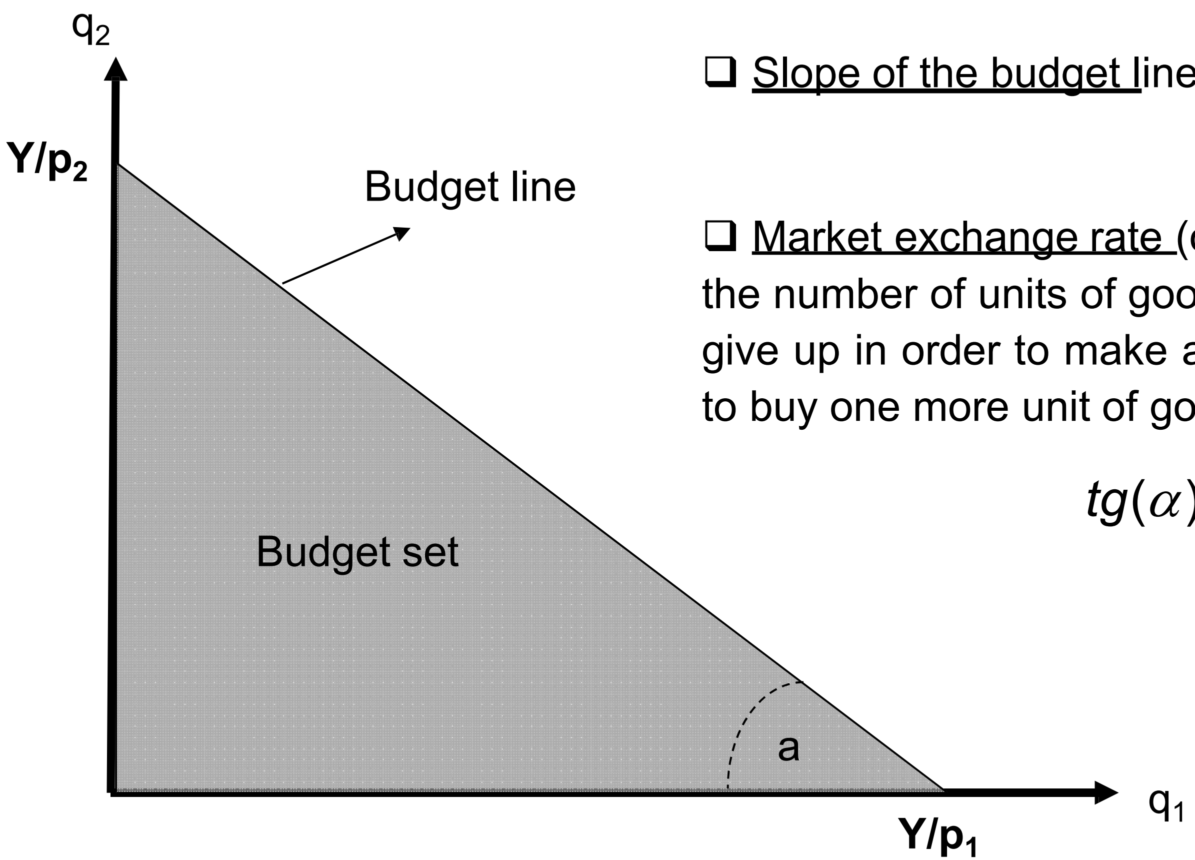
- **Budget line**: The set of market baskets on which the consumer spends all his/her available income:

$$p_1q_1 + p_2q_2 = Y$$

3.4) The budget constraint

Budget constraint:

Solving for q_2 in the budget line: $q_2 = Y/p_2 - p_1/p_2 \cdot q_1$



- Slope of the budget line: $\frac{dq_2}{dq_1} = -\frac{p_1}{p_2} < 0$

- Market exchange rate (or "relative price"): p_1/p_2 is the number of units of good 2 that any consumer must give up in order to make available the income needed to buy one more unit of good 1:

$$tg(\alpha) = \frac{p_1}{p_2}$$

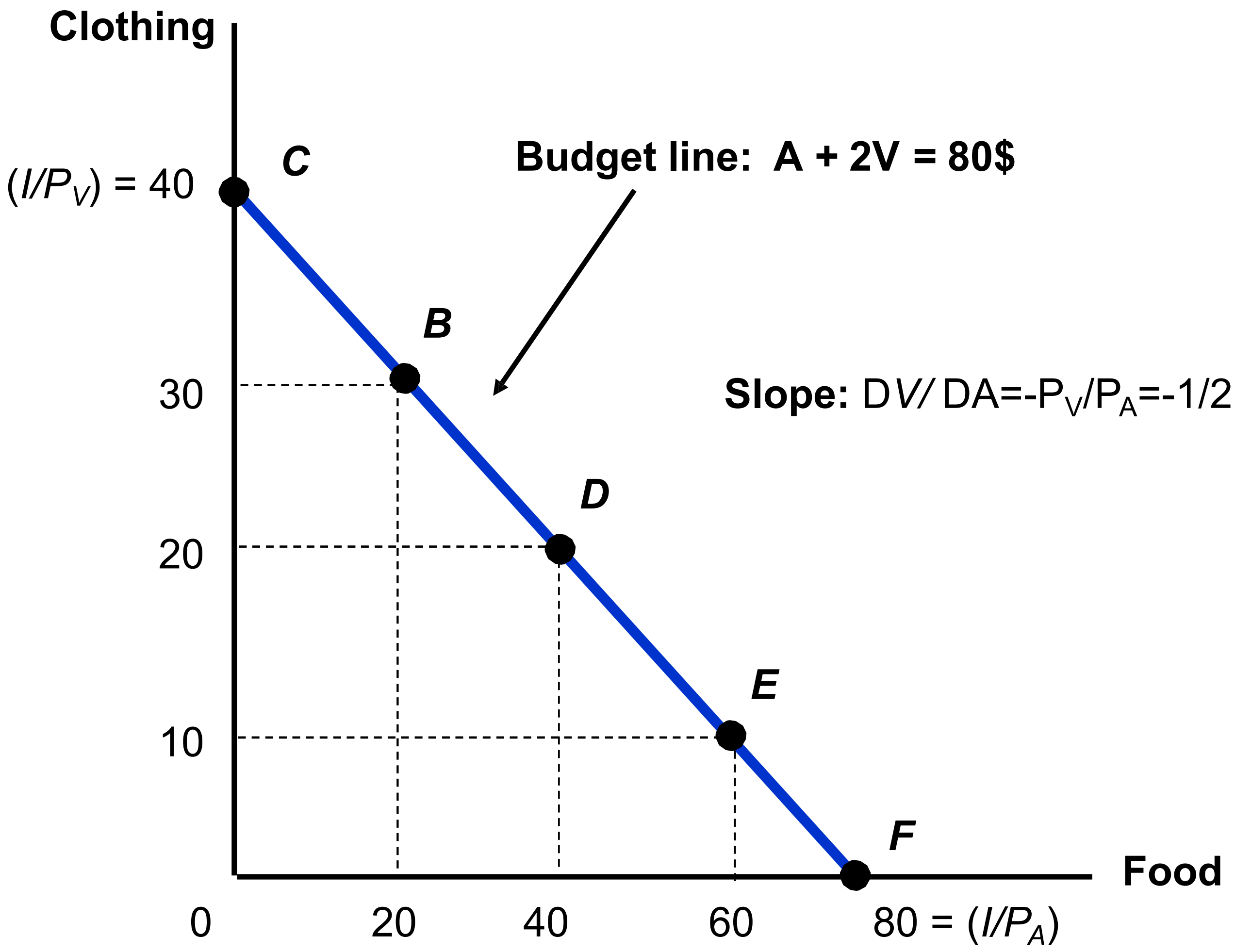
3.4) The budget constraint

Budget constraint:

Market basket	**Food (A)** **P_A = (\$1)**	**Clothing (V)** **P_V = (\$2)**	**Total expenditure** **$P_AA + P_VV = I$**
C	0	40	80$
B	20	30	80$
D	40	20	80$
E	60	10	80$
F	80	0	80$

3.4) The budget constraint

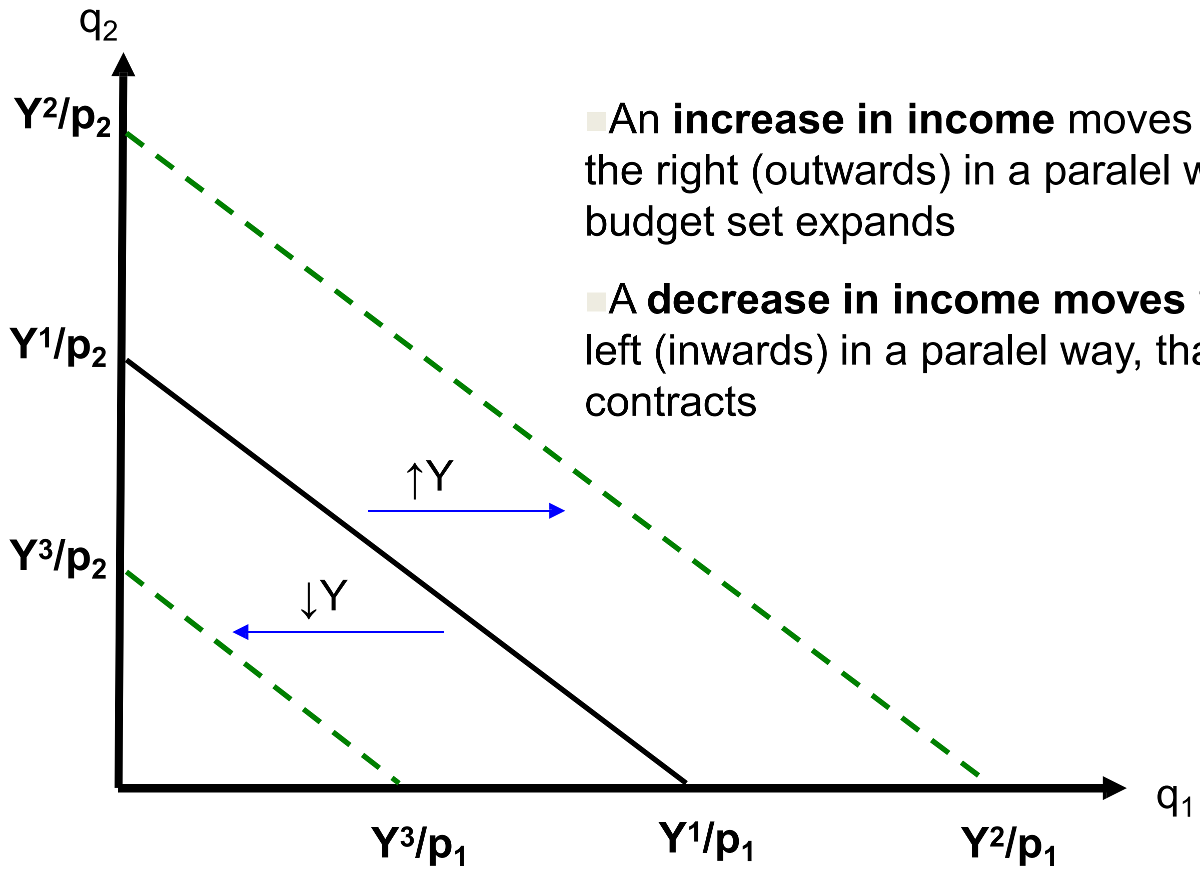
Budget constraint:



3.4) The budget constraint

Shifts of the budget line:

a) Changes in consumer's income

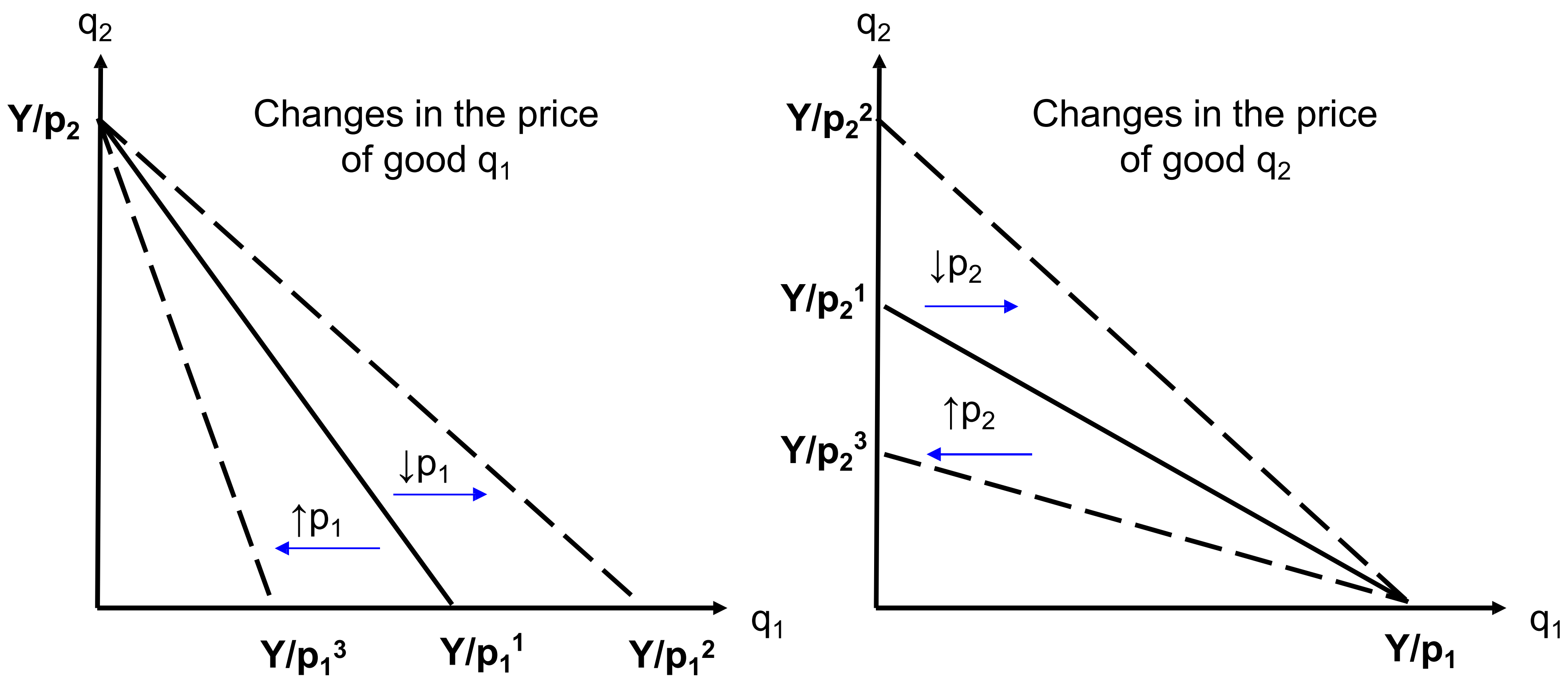


- An **increase in income** moves the budget line to the right (outwards) in a paralel way, that is, the budget set expands
- A **decrease in income moves** the budget line to the left (inwards) in a paralel way, that is, the budget set contracts

3.4) The budget constraint

Shifts of the budget line:

b) Changes in the price of a good

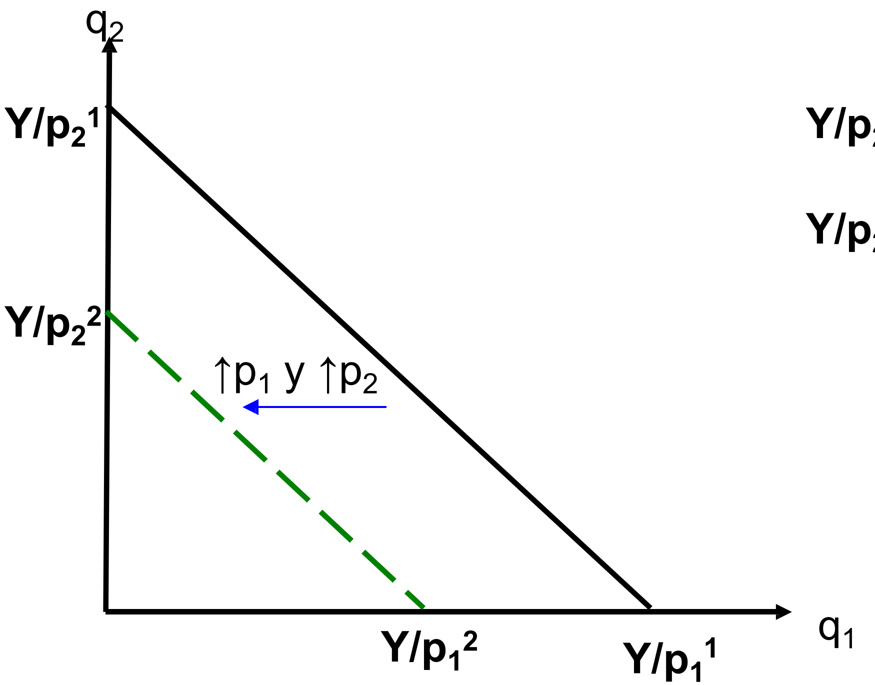


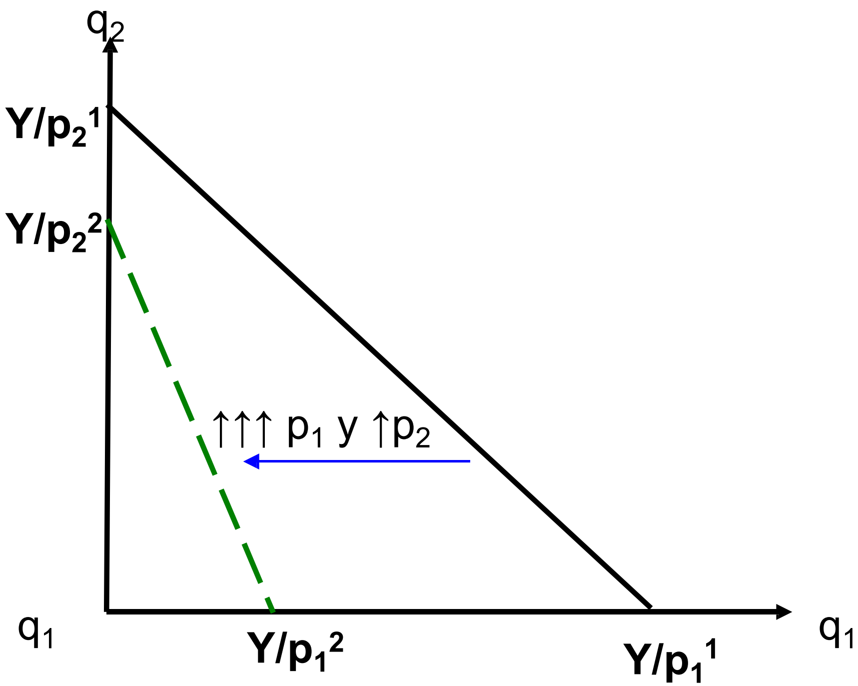
3.4) The budget constraint

Shifts of the budget line:

c) Changes in the price of both goods in the same direction

- When both prices rise in the same proportion, the budget line moves towards the corner in paralel and relative prices (slope of the budget line) do not change:
- When both prices increase in different proportions, the consumer also loses purchasing power and, moreover, the relative prices (slope) change:



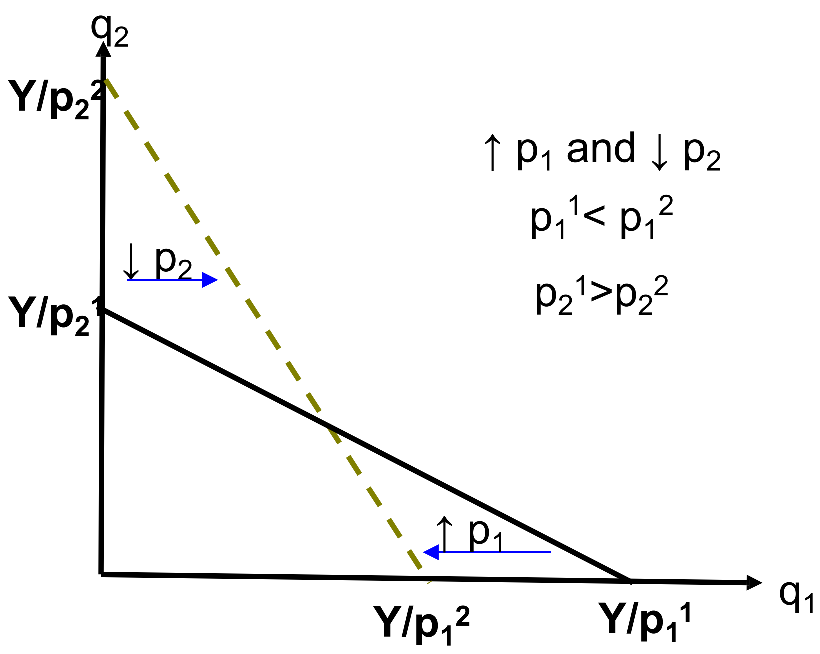


3.4) The budget constraint

Shifts in the budget line:

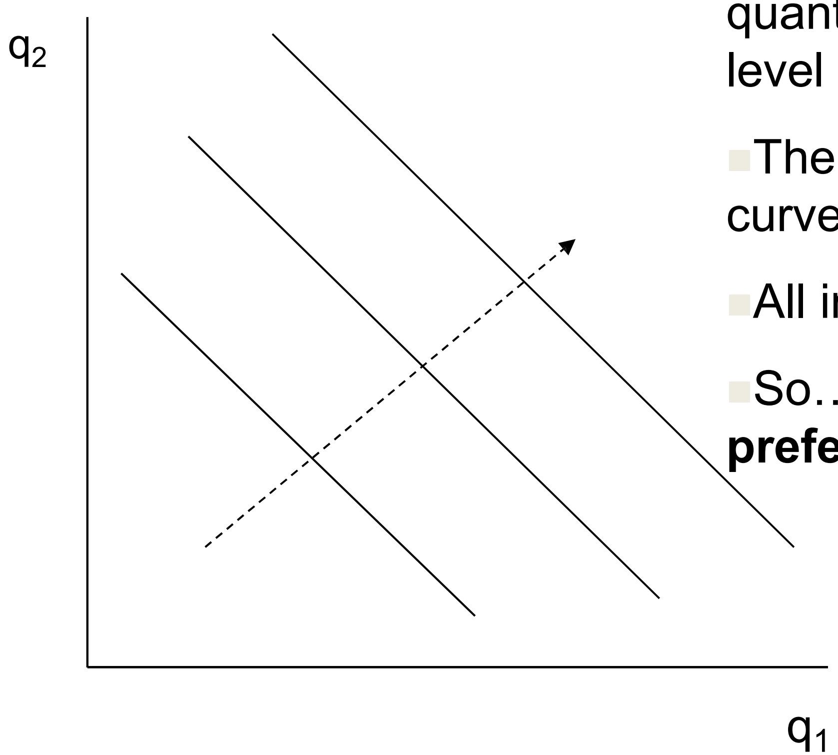
d) Changes in the price of both goods: one rises and the other falls

- When one price rises and the other falls, the slope of the budget line changes but the overall effect on the consumer purchasing power (="size" of the budget set) is undetermined:



Special cases of indifference curves

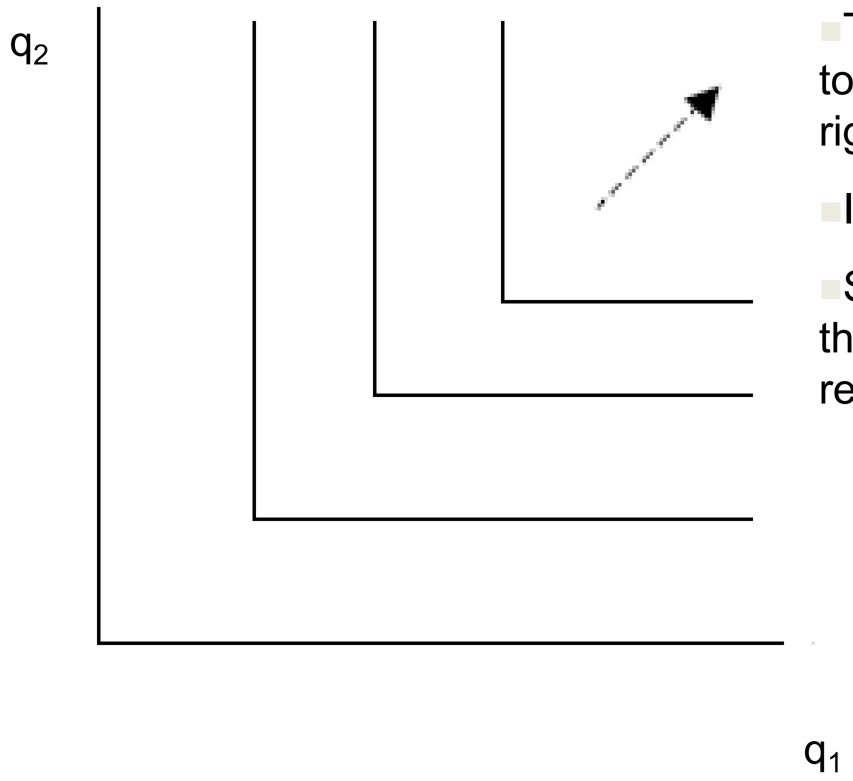
- **Perfect substitute goods**



- They can be interchanged in the same fixed quantities without affecting the total satisfaction level (coffe/tea)
- The MRS holds constant across the indifference curve
- All indifference curves are straight lines
- So… they do not satisfy the **strict convexity of preferences** axiom.

Special cases of indifference curves

- **Perfect complement goods**



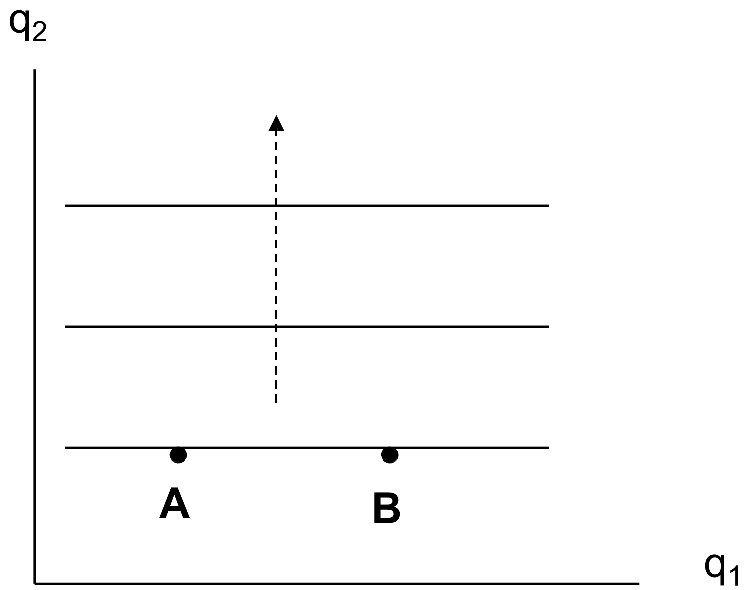
- These are goods that are optimally consumed together in some fixed proportions (left shoes and right shoes)
- Indifference curves are L-shaped
- So…they do not satisfy the **smoothness** axiom: the left and right derivatives taken in the baskets representing the optimal proportions are different

Special cases of indifference curves

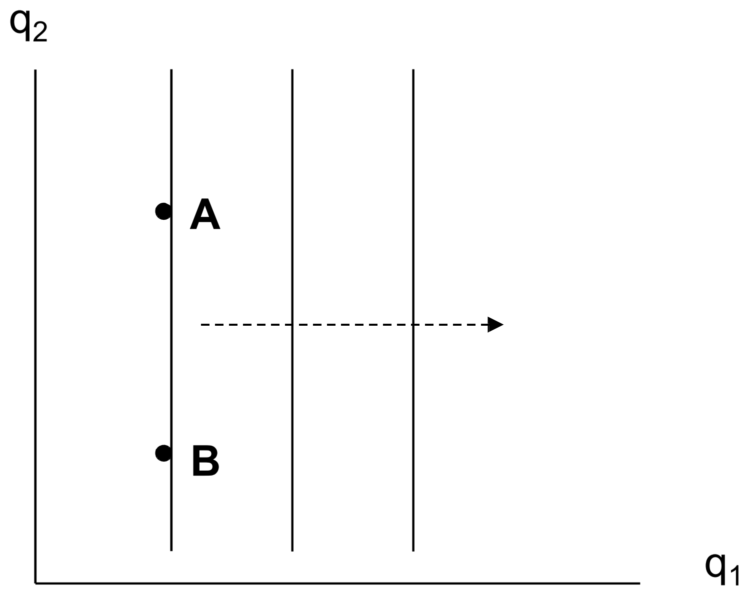
- **Neutral goods:**

  They do not provide higher or lower utility, so they do not satisfy the **non-satiation** axiom:

q_1 is a neutral good



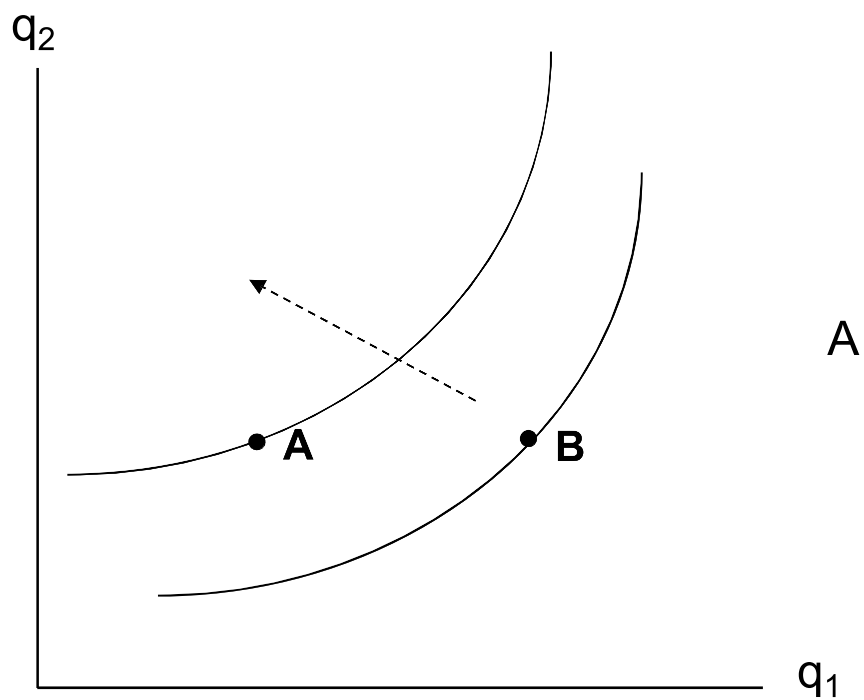
q_2 is a neutral good



Special cases of indifference curves

- **Bads:**

  Q_1 is a bad when its consumption actually reduces satisfaction, so again it does not satisfy the **non-satiation** axiom:

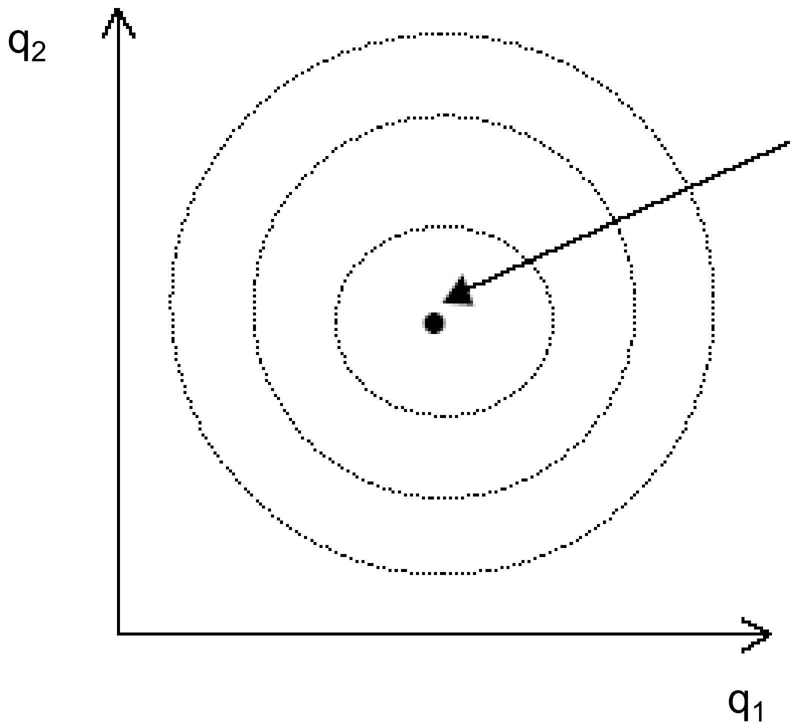


A > B

Special cases of indifference curves

- **Satiable goods:**

  Consuming further from some given satiation point diminishes consumer's satisfaction, so they do not satisfyl the **non-satiation** axiom:



Exercises

1.- Consider the following goods (beverages): J&B (Q_1) and Coca Cola (Q_2), draw the map of indifference curves derived from the preferences of the following individuals:

a) The teetotaler, a person who hates alcohol but likes soft-drinks.

b) The alcoholic, a person who is insensitive to the consumption of soft-drinks.

c) The "cocktail king", who dislikes to consume both alcohol and soft-drinks separately.

2.- Mr Jones enjoys driving cars, measured in miles, but is also concerned about the pollution cars generate, measured in tons of CO2 per mile. Answer the following questions:

a) Draw the overall shape of Mr Jone's indifference curves.

b) Which axiom(s) on consumer preferences are being violated?

Give reasons for your answers.

3.- Anne has just begun the BA degree. She is very happy because she likes Microeconomics a lot, measured in hours of study, although she is neutral with respect to the other subjects, measured in hours spent on them. Answer the questions:

a) Draw Anne's map of indifference curves.

b) Which assumption(s) or axiom(s) are being violated? Give reasons justifying your answers.

4.- Peter uses either, as washing powder, Ariel or Colón. He knows that the more washing powder he uses, the cleaner his clothes will be. Answer the following questions:

a) Draw indifference curves from Peter's preferences between both goods.

b) Which assumption(s) or axiom(s) are being violated? Give reasons justifying your answers.

c) Find a functional form of a possible utility function representing Peter's preferences.

d) Peter has noticed recently that Ariel provides his clothes with double the brightness of Colón. Show the change in his utility function.

5.- Diane always takes each cup of coffee with 2 spoons of sugar. Answer the following questions:

a) Draw Diane's map of indifference curves.

b) Which assumption(s) or axiom(s) are being violated? Give reasons justifying your answers.

c) Show a functional form of a possible utility function representing Diane's preferences.

6.- Alfred obtains utility from drinking wine and listening to music. Assuming that his preferences can be represented by the following utility function (from the family called Cobb-Douglas utility functions):

$$U = q_1^{\frac{1}{2}} q_2^{\frac{1}{2}}$$

where q_1 represents his consumption of wine (number of glasses) and q_2 represents his consumption of music (number of CDs). Answer the following questions:

a)Show Alfred's indifference curve with utility level U=10 associated with the utility function.

b)If Alfred consumes 5 glasses of wine, how many CDs should he listen to in order to obtain a utility level U=10? Find the MRS between both goods evaluated in this market basket. Find the MRS in the case that he is consuming 10 units of each good, and explain the meaning of the MRS.

c)Consider now the logarithmic transformation of Alfred's utility function. Find the mathematical expression of the MRS for this utility function. Does this new utility function represent the same preferences of Peter as the original function? Give reasons for your answers.

7.- Given the following utility functions:

$$U = q_1 q_2$$

$$U = q_1^2 q_2^2$$

$$U = \ln q_1 + \ln q_2$$

Show that all of them exhibit the same decreasing MRS, but that their marginal utilities are different. What do you conclude from this?

8.-Consider a consumer whose Marginal Relation of Substitution of good 2 for good 1 is constant and equal to 1 for the whole consumption set:

a) Draw the map of the consumer's indifference curves.

b) Give an example of two goods following the above preferences.

9.- Assume that Paul has income Y = 100 monetary units. Answer the following questions:

a) Draw the budget lines corresponding to the following cases:

	Price of Q_1	Price of Q_2
i)	10	20
ii)	5	20
iii)	10	10
iv)	10	25

b) Analyze, for each case, the change of the budget line from the initial position (i).

10.- Represent the budget lines corresponding to the following combinations of income (Y) and prices of two goods (p_1, p_2):

	Y	p_1	p_2
(i)	300	2	6
(ii)	900	6	18

What conclusions can you find from comparing both budget sets?