UNIT 4. CONSUMER BEHAVIOR

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UNIT 4. CONSUMER BEHAVIOR

4.1 Consumer equilibrium (Pindyck → 3.3, 3.5 and T.4)
- ❑ Graphical analysis.
- ❑ Analytical solution.

4.2 Individual demand function (Pindyck → 4.1)
- ❑ Derivation of the individual Marshallian demand
- ❑ Properties of the individual Marshallian demand

4.3 Individual demand curves and Engel curves (Pindyck → 4.1)
- ❑ Ordinary demand curves
- ❑ Crossed demand curves
- ❑ Engel curves

4.4 Price and income elasticities (Pindyck → 2.4, 4.1 and 4.3)
- ❑ Price elasticity of demand
- ❑ Crossed price elasticity
- ❑ Income elasticity

4.5 Classification of goods and demands (Pindyck → 2.4, 4.1 and 4.3)

APPENDIX: Relation between expenditure and elasticities

4.1 Consumer equilibrium

Consumer equilibrium:

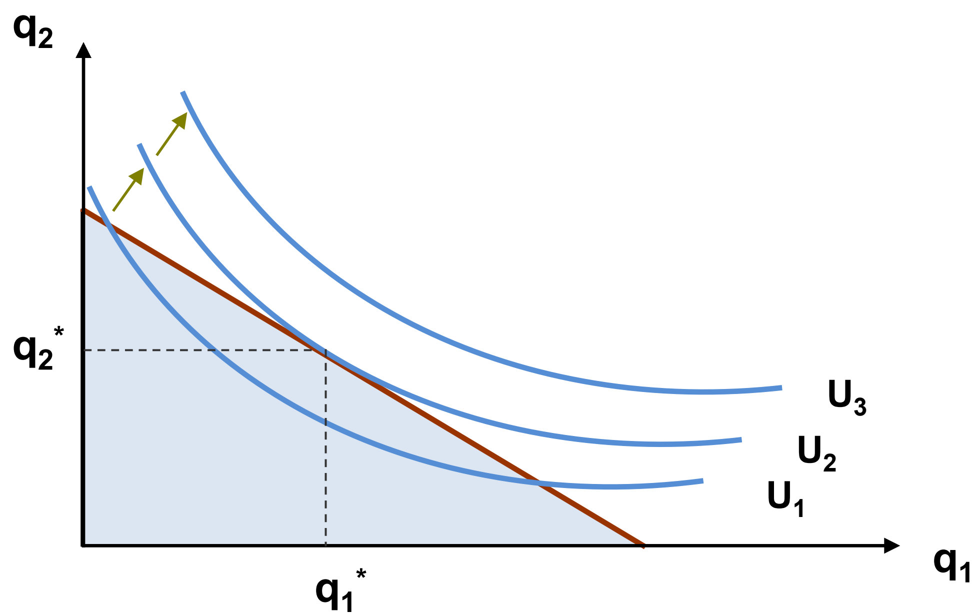
- We proceed to analyze how the consumer chooses the quantity to buy of each good or service (market basket), given his/her:
  - Preferences
  - Budget constraint
- We shall assume that the decision is made **rationally**:

  Select the quantities of goods to purchase in order to maximize the satisfaction from consumption given the available budget
- We shall conclude that this market basket maximizes the utility function:
  - The chosen market basket must be the preferred combination of goods or services from all the available baskets and, particularly,
  - It is on the budget line since we do not consider the possibility of saving money for future consumption and due to the non-satiation axiom

4.1 Consumer equilibrium

Graphical analysis

- The **equilibrium** is the point where an indifference curve intersects the budget line, with this being the upper frontier of the budget set, which gives the highest utility, that is to say, where the indifference curve is tangent to the budget line



4.1 Consumer equilibrium

Consumer problem

- The consumer will assign limited resources (i.e., income "Y") among different goods or services (q_1, q_2) in order to maximize utility:

$$\max_{q_1, q_2} U = U(q_1, q_2)$$

$$\text{subject to:}$$

$$Y \geq p_1 q_1 + p_2 q_2$$

$$q_1, q_2 \geq 0$$

- Assumptions:
  - There is always some positive consumption of all goods or services
  - The consumer spends all income

- Therefore, the optimization problem can be written as follows:

$$\max_{q_1, q_2} U = U(q_1, q_2)$$

$$\text{s.t. } Y = p_1 q_1 + p_2 q_2$$

4.1 Consumer equilibrium

Mathematical resolution of the consumer problem:

- **Lagrange Multipliers Method (LMM)**: We use an auxiliary function called " *The Lagrangian function*" (λ: Lagrange multiplier):

$$L = L(q_1, q_2, \lambda) = U(q_1, q_2) + \lambda[Y - p_1 q_1 - p_2 q_2]$$

which is maximized: $\max\limits_{q_1, q_2, \lambda} L = L(q_1, q_2, \lambda)$

The First Order Conditions (F.O.C) are:

$$\left.\begin{array}{lll}
L_1 = \dfrac{\partial L}{\partial q_1} = U_1 - \lambda p_1 = 0 & \Rightarrow \quad U_1 = \lambda p_1 & (1) \\
L_2 = \dfrac{\partial L}{\partial q_2} = U_2 - \lambda p_2 = 0 & \Rightarrow \quad U_2 = \lambda p_2 & (2) \\
L_\lambda = \dfrac{\partial L}{\partial \lambda} = Y - p_1 q_1 - p_2 q_2 = 0 & \Rightarrow \quad Y = p_1 q_1 + p_2 q_2 & (3)
\end{array}\right\}$$

Given particular values for the exogenous variables (income and prices), we solve the system of three equations for the three endogenous variables (q_1, q_2, λ)

4.1 Consumer equilibrium

Mathematical resolution of the problem:

- From equations (1) and (2) we derive:

$$\lambda = \frac{U_1}{p_1} = \frac{U_2}{p_2}$$

Equal marginal principle (EMP): The consumer must, in equilibrium, obtain the same utility from the last monetary unit spent on either good 1 or good 2.

- In equilibrium, we obtain:

$$\frac{U_1}{p_1} = \frac{U_2}{p_2} \iff MRS^2_1 = \frac{U_1}{U_2} = \frac{p_1}{p_2}$$

That is to say, the consumer is in equilibrium when the following two magnitudes are equated:

- The **subjective** exchange rate between goods ($MRS^2{}_1$): the rate at which the consumer wants to exchange one good for the other with constant utility
- The objective exchange **rate between goods** offered in the market (P_1/P_2)

4.1 Consumer equilibrium

Mathematical resolution of the problem:

- **SUMMARIZING:** the consumer problem is solved as follows:

Problem

$$\max_{q_1,q_2} U = U(q_1, q_2)$$
$$s.t. \; Y = p_1 q_1 + p_2 q_2$$

⇒

From F.O.C.s:

$$\frac{U_1}{p_1} = \frac{U_2}{p_2}$$
$$Y = p_1 q_1 + p_2 q_2$$

⇒

Solution:

Given p_1^0, p_2^0 and Y^0

(q_1^*, q_2^*)

- Endogenous variables: q_1 and q_2
- Exogenous variables: p_1, p_2 and Y

In equilibrium : $MRS_1^2 = \frac{U_1}{U_2} = \frac{p_1}{p_2}$

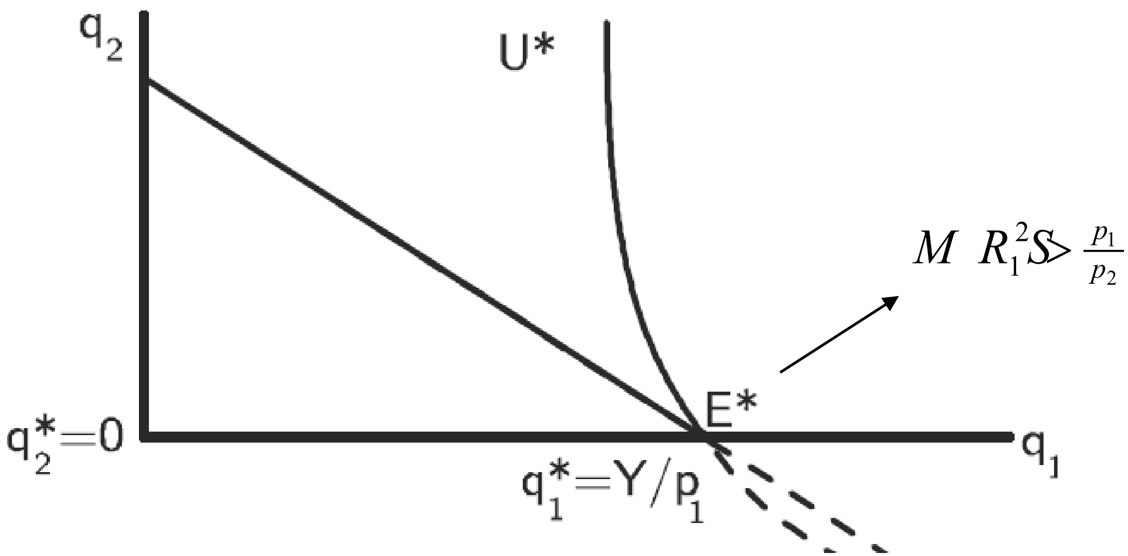
The marginal rate of substitution must be equal to the relative price

4.1 Consumer equilibrium

Mathematical resolution of the problem:

- **SPECIAL CASE:** Corner solution $MRS_1^2 \neq \frac{p_1}{p_2}$

  The consumer spends the total income on just one good or service and decides not to buy any of the other good:



4.1 Consumer equilibrium

Economic interpretation of the Lagrange Multiplier

- In equilibrium, the Lagrange Multiplier is equal to the marginal utility of income:

(1) First we differentiate the utility function: $dU = U_1 dq_1 + U_2 dq_2$

(2) Also the budget line equation (constant prices): $dY = P_1 dq_1 + P_2 dq_2$

(3) We know that in equilibrium: $U_1 = \lambda P_1$ and $U_2 = \lambda P_2$

(4) Finally, substituting (3) into (1): $dU = \lambda P_1 dq_1 + \lambda P_2 dq_2 = \lambda (P_1 dq_1 + P_2 dq_2) = \lambda dY$

And using (2)

$$\lambda = \frac{dU}{dY}$$

The Lagrange Multiplier is equal to the marginal utility of income

4.2 Individual demand function

Derivation of the the Marshallian individual demand function

Def: The **general demand function (or Marshallian demand function)** is the mathematical relationship, in equilibrium, between prices and income. and the corresponding quantity of the good:

$q_1 = q_1(p_1, p_2, Y)$	Demand function of good 1
$q_2 = q_2(p_1, p_2, Y)$	Demand function of good 2

These functions are obtained from the F.O.C.s of the consumer maximization problem, considering p_1, p_2 and Y as parameters/variables in place of of specific numerical values.

4.2 Individual demand function

Properties of the Marshallian individual demand functions

- They are functions such that:
  - For each three values (p_1, p_2, Y) there is only one optimal quantity demanded of each good or service, that is to say, the optimal equilibrium basket is unique
- They describe optimal behavioral relations:
  - They incorporate the objectives and restrictions of the consumer problem
  - The consumer tastes or preferences fully determine the functional form
- They are homogeneous degree "0" functions in prices and income: If all prices and income are multiplied by the same positive constant "k", the equilibrium remains:

$$q_i(kp_1, kp_2, kY) = k^0 q_i(p_1, p_2, Y) = q_i(p_1, p_2, Y); \quad i = 1, 2$$

4.3 Individual demand curves and Engel curves

Once we have the individual demand functions, we can define several behavioral curves according to the particular values of specific exogenous variables (p_1, p_2, Y):

- **(1) Ordinary demand curve:** Shows the quantity demanded of a good as a function of its price (the income and the price of the other good are given):

$$q_1 = q_1(p_1, p_2{}^0, Y^0) = q_1(p_1) \quad \text{O d r ced o ugmi 1f}$$
$$q_2 = q_2(p_1^0, p_2, Y^0) = q_2(p_2) \quad \text{O d r ced o ugmi 2f}$$

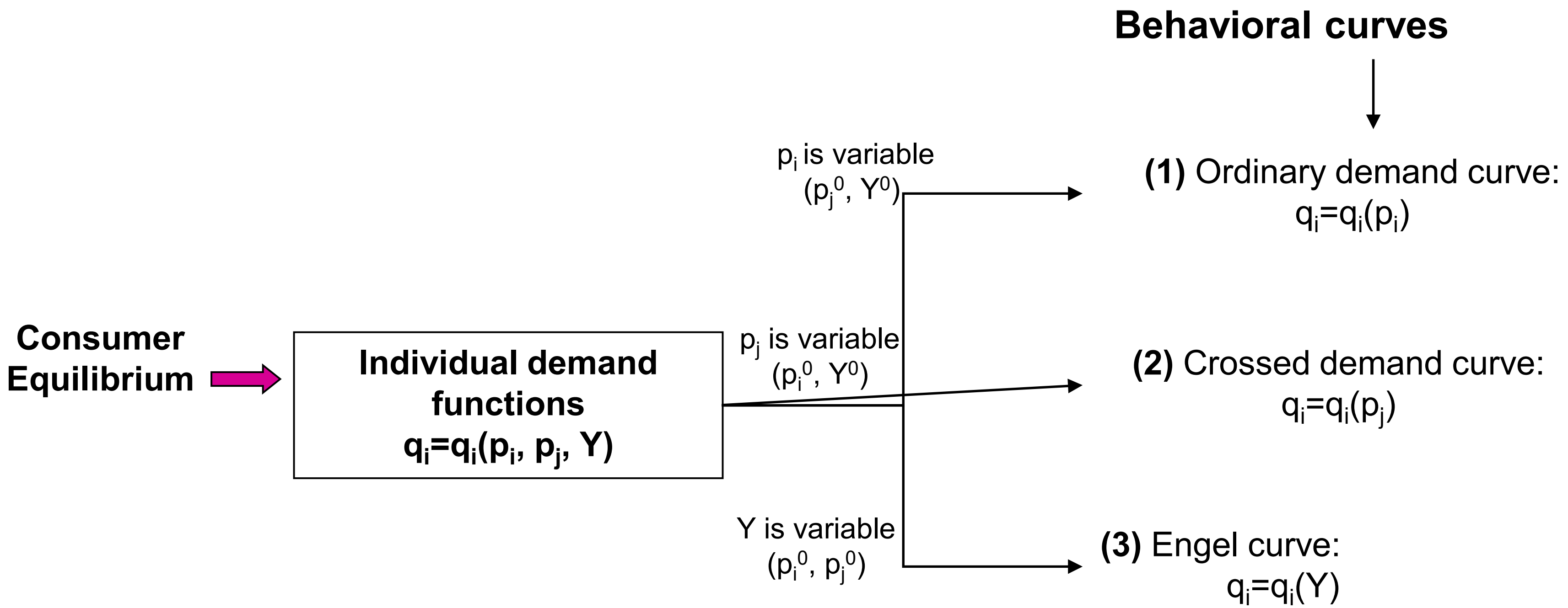
- **(2) Crossed demand curve:** Shows the quantity demanded of a good as a function of the price of the other good (the income and the price of the same good are given):

$$q_1 = q_1(p_1^0, p_2, Y^0) = q_1(p_2) \quad \text{C d r f e o u o ngs n1 1}$$
$$q_2 = q_2(p_1, p_2^0, Y^0) = q_2(p_1) \quad \text{C d r f e o u o ngs n2}$$

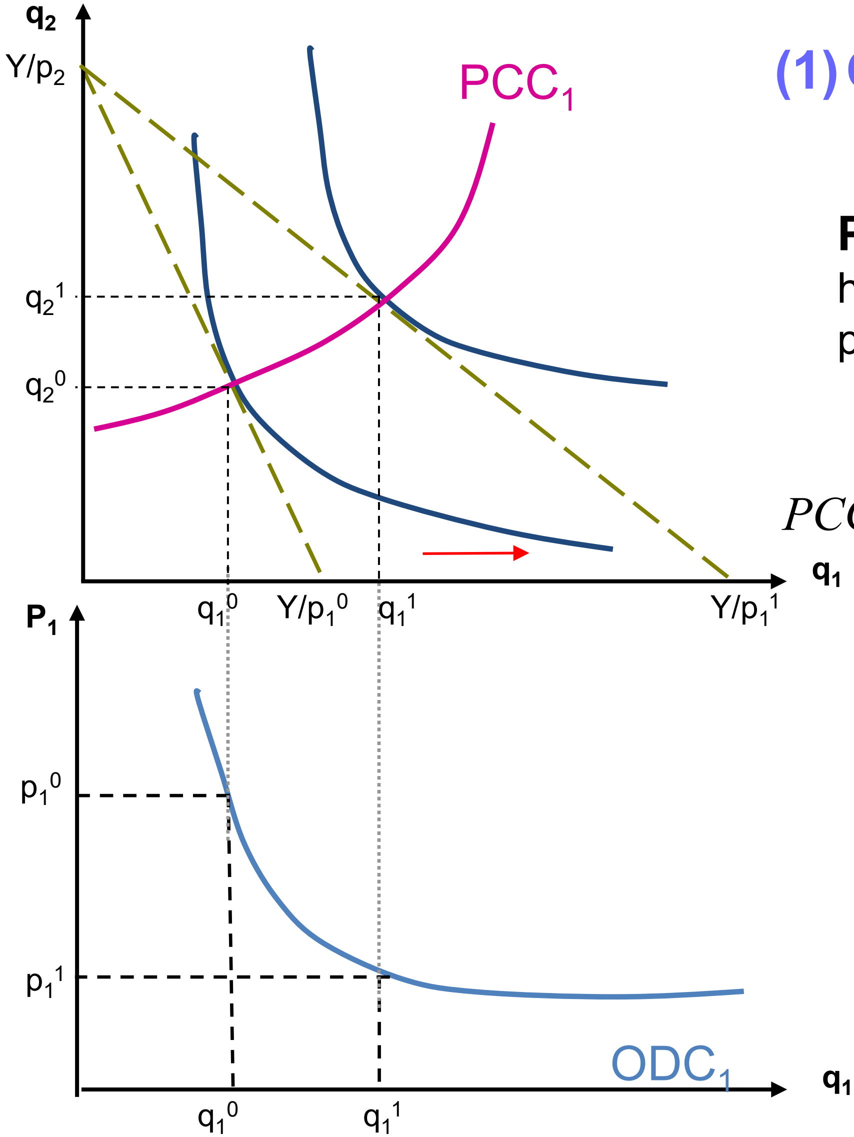
- **(3) Engel curve:** Shows the quantity demanded of a good as a function of the consumer's income (both prices are given):

$$q_1 = q_1(p_1{}^0, p_2{}^0, Y) = q_1(Y) \quad \text{E c n ou g g f1}$$
$$q_2 = q_2(p_1{}^0, p_2{}^0, Y) = q_2(Y) \quad \text{E c n ou g g f2}$$

4.3 Individual demand curves and Engel curves



4.3 Individual demand curves and Engel curves



(1) Ordinary demand curve of good 1:

Price-consumption curve (PCC): Shows how the quantity demanded changes when prices change. First we change p_1:

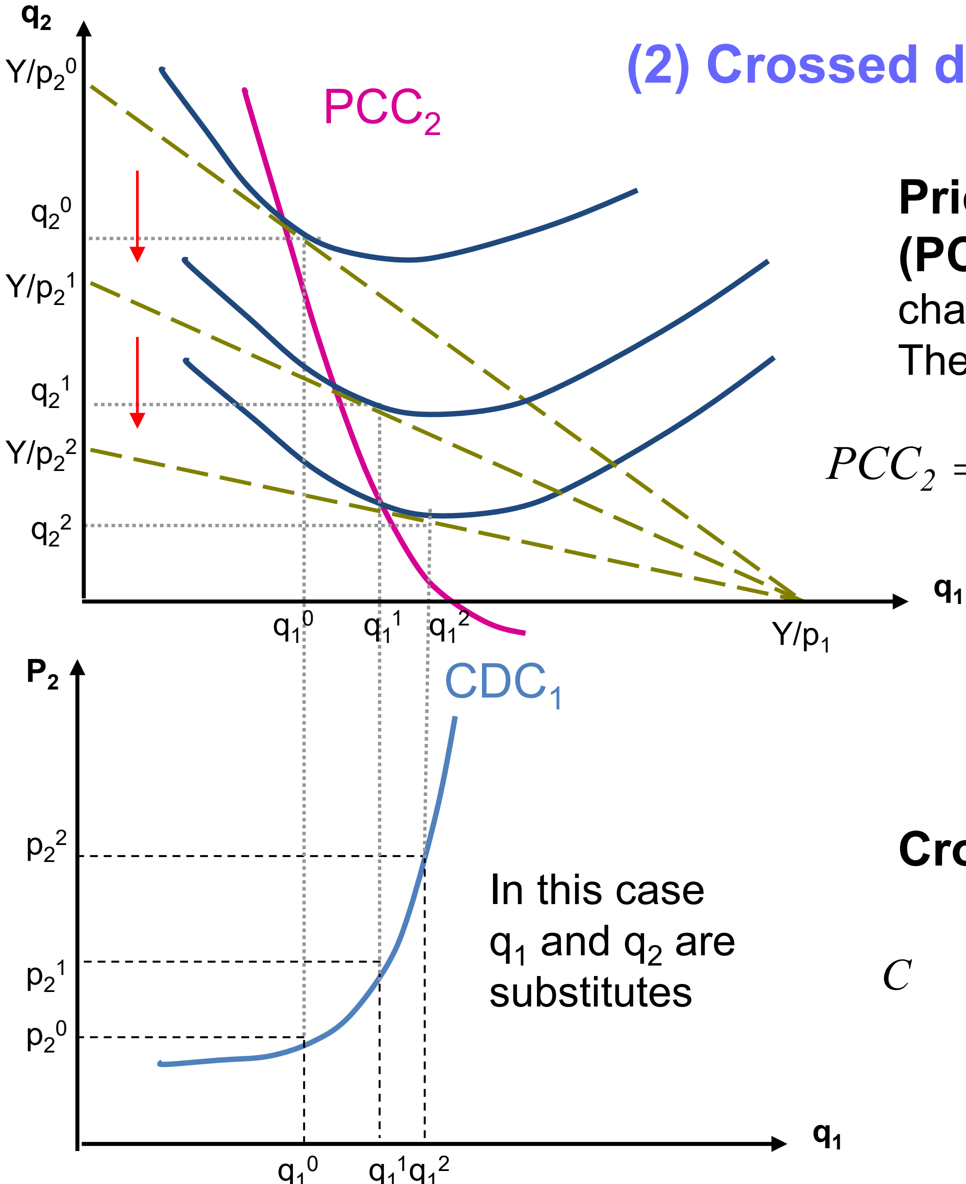
$$PCC_1 = \left\{ (q_1, q_2) \,\middle|\, MRS_1^2 = \frac{p_1}{p_2};\ p_2, Y \text{ c} \quad S \text{ o} \quad p_1 \text{ f} \quad \text{n} \right\}$$

Ordinary demand curve (ODC):

$$O \quad _1 = \left\{ (q_1, p_1) \,\middle|\, MRS_1^2 = \frac{p_1}{p_2};\ p_2, Y \text{ c } S \quad \text{o} \right\}$$

4.3 Individual demand curves and Engel curves

(2) Crossed demand function of good 1:



In this case q_1 and q_2 are substitutes

Price-consumption curve of good 1 (PCC): Shows how the quantity demanded changes when the price of good 2 changes. Therefore, we allow p_2 to change:

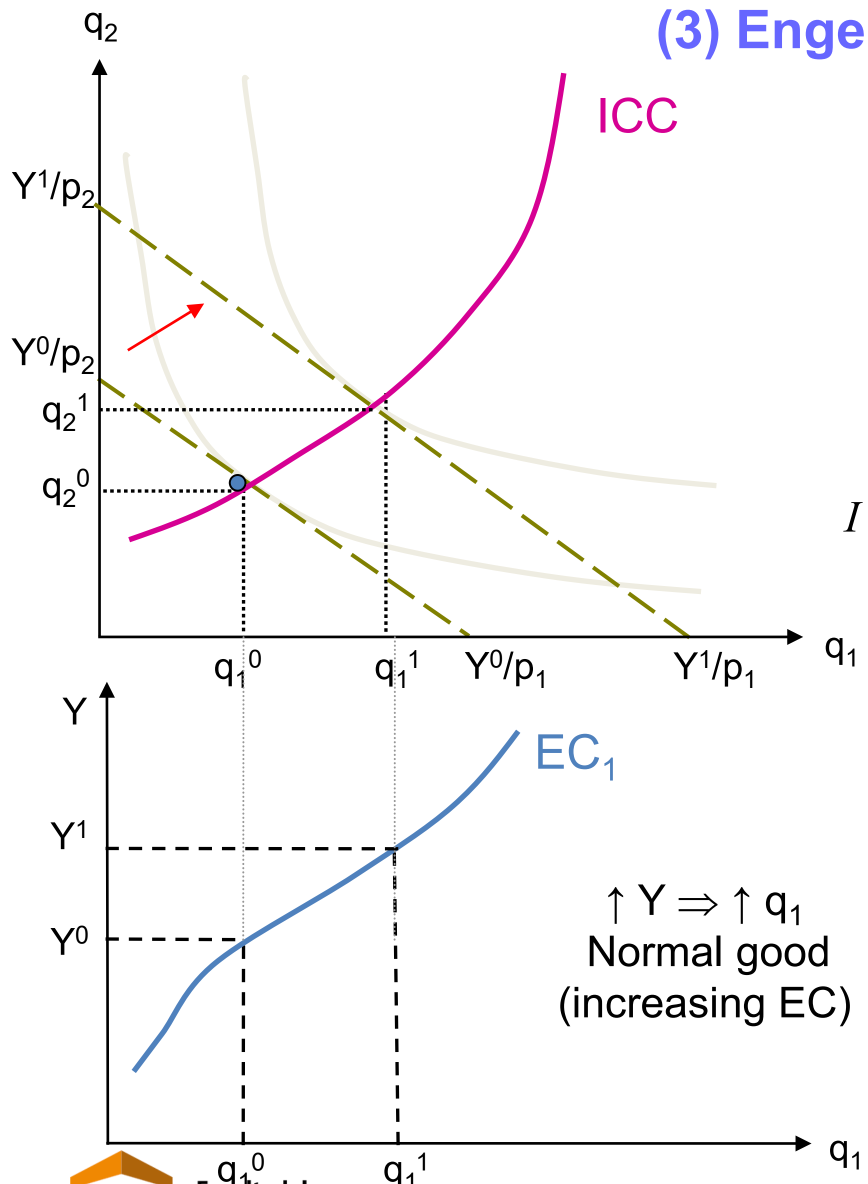
$$PCC_2=\left\{(q_1,q_2)\,\middle|\, M\ C_1^2=\frac{p_1}{p_2};\ p_1, Y \text{ c} \quad S \text{ o} \quad p_2 \text{ f n}\right\}$$

Crossed demand curve (CDC):

$$C_1=\left\{(q_1,p_2)\,\middle|\, MC_1^2=\frac{p_1}{p_2};\ R\ p_1, Y \text{ c } S \quad \text{o}\right\}$$

4.3 Individual demand curves and Engel curves

(3) Engel curve of good 1: NORMAL GOOD



Income-consumption curve (ICC):
Shows how the quantity demanded changes when the consumer's income changes:

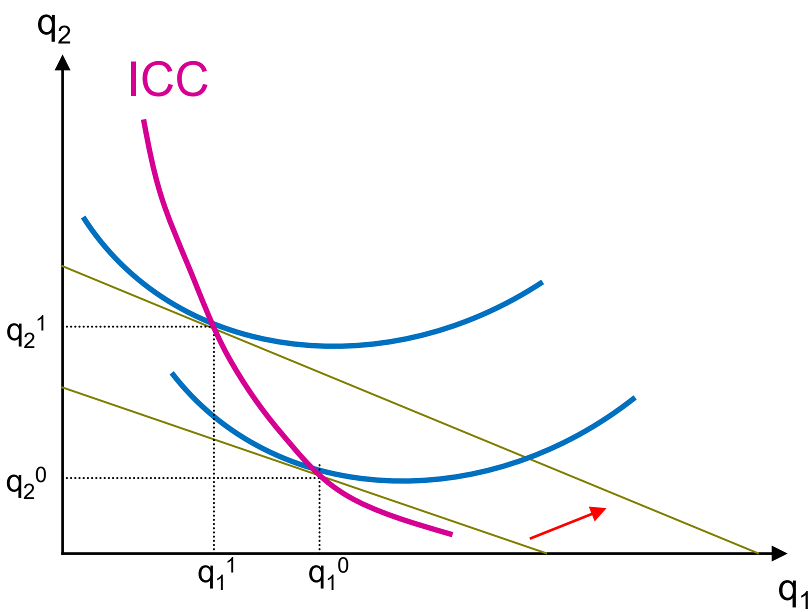
$$I = C\left\{(q_1, q_2) \;\middle|\; M\ {}^{2}_{1} = \frac{p_1}{p_2};\ R\ p_1, p_2\ c\quad S\ o\ Y\ f\ n\right\}$$

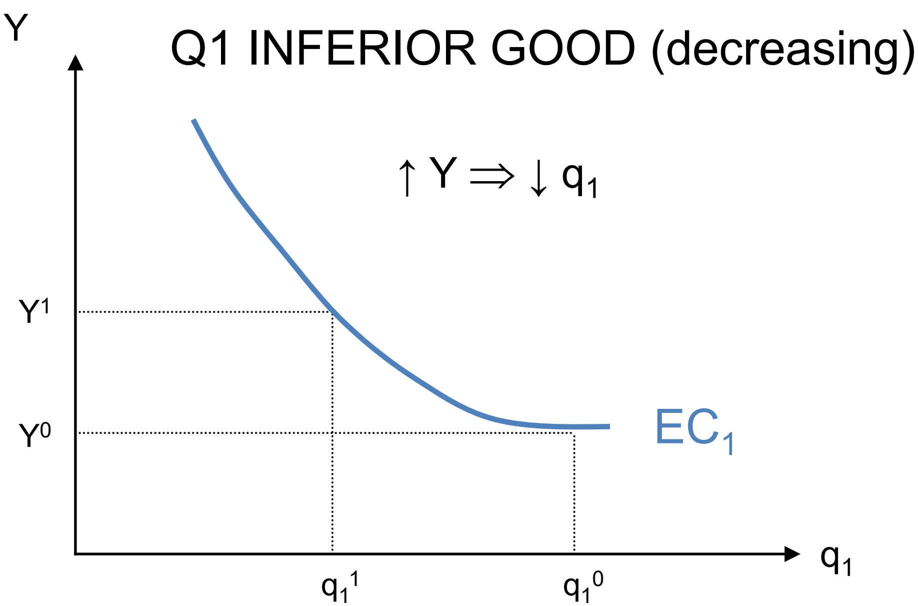
Engel curve (EC):

$$E_1 = \left\{(C q_1, Y) \;\middle|\; M\ {}^{2}_{1} = \frac{p_1}{p_2} R\ p_1, p_2\ S\quad o\right\}$$

(3) Engel curve of good 1 : INFERIOR GOOD

- The Engel curve of an inferior good is decreasing
- ↑ consumer's income ⇒ ↓ quantity demanded

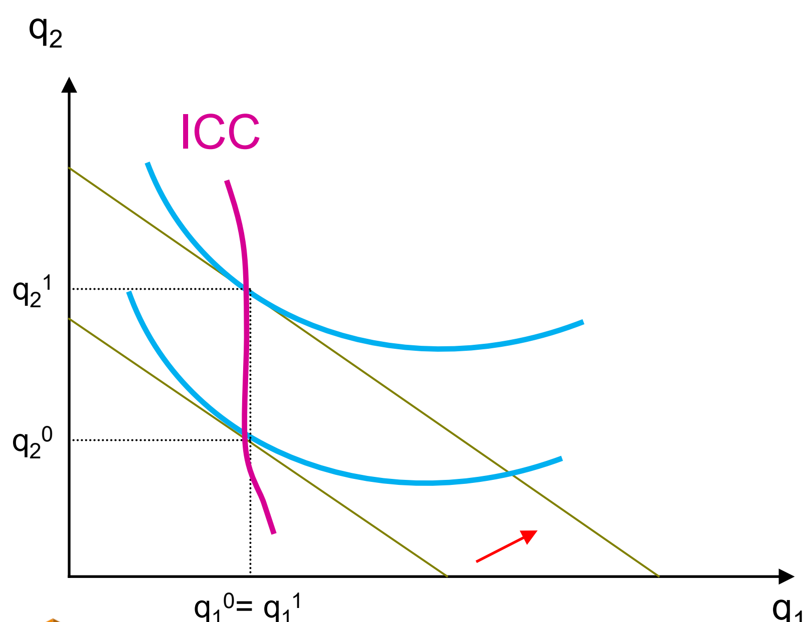




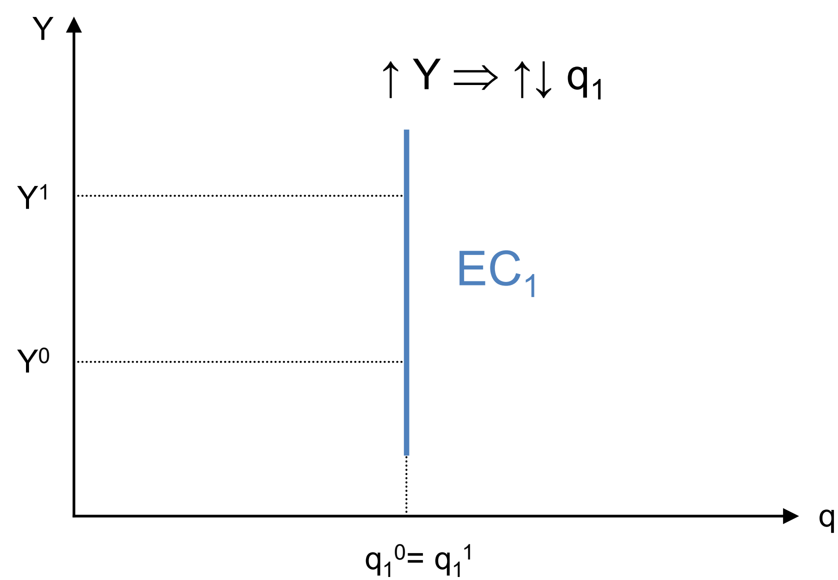
4.3 Individual demand curves and Engel curves

(3) Engel curve of good 1 : NEUTRAL GOOD

- The Engel curve of a neutral good is vertical
- $\uparrow$ consumer's income $\Rightarrow$ the quantity demanded neither $\uparrow$ nor $\downarrow$



Q1 NEUTRAL GOOD



4.4 Price and income elasticities

Elasticity:

- Measures the sensitivity of one variable to changes in another variable. It is defined as the percentual variation produced in variable B due to a percentual change in variable A:

$$e_{AB} = \frac{\Delta \% B}{\Delta \% A} = \frac{\frac{\Delta B}{B}}{\frac{\Delta A}{A}} = \frac{A}{B}\frac{\Delta B}{\Delta A}$$

- Elasticities are quantitative measures associated with each point on a behavioral curve:

Individual demand functions
$q_i=q_i(p_i, p_j, Y)$

→ Ordinary demand curve $q_i=q_i(p_i)$ → Price elasticity

→ Crossed demand curve $q_i=q_i(p_j)$ → Crossed price elasticity

→ Engel curve $q_i=q_i(Y)$ → Income elasticity

4.4 Price and income elasticities

(1) PRICE ELASTICITY:

Price elasticity measures the variation in the quantity demanded of a good due to a change in its own price:

$$e_{p_1q_1} = -\frac{\text{Variation (\%) in quantity demanded of good 1}}{\text{Variation (\%) in price of good 1}} = -\frac{p_1}{q_1}\frac{\Delta q_1}{\Delta p_1}$$

- As a general rule, the demand of the good decreases when its price rises (normal demand), so we typically change the sign of the elasticity to positive (it is the only elasticity defined with the opposite sign). The price elasticity allows us to **CLASSIFY DEMANDS**.
- In infinitessimal terms:

$$e_{p_1q_1} = -\frac{p_1}{q_1}\frac{dq_1}{dp_1}$$

4.4 Price and income elasticities

(2) CROSSED PRICE ELASTICITY:

The crossed price elasticity measures the variation in the quantity demanded of a good due to a change in the price of the other good:

$$e_{p_2 q_1} = \frac{\text{Variation (\%) in quantity demanded of good 1}}{\text{Variation (\%) in the price of good 2}} = \frac{p_2}{q_1}\frac{\Delta q_1}{\Delta p_2}$$

- The sign of this elasticity does not change, since it reveals information about the **RELATIONSHIP BETWEEN THE GOODS.**
- In infinitessimal terms:

$$e_{p_2 q_1} = \frac{p_2}{q_1}\frac{d\,q_1}{d\,p_2}$$

4.4 Price and income elasticities

(3) INCOME ELASTICITY:

The income elasticity measures the variation in the quantity demanded of a good due to a change in the consumer's income:

$$e_{Y q_1} = \frac{\text{Variation (\%) in quantity demanded of good 1}}{\text{Variation (\%) in consumer's income}} = \frac{Y}{q_1}\frac{\Delta q_1}{\Delta Y}$$

- The sign does not change, since it is informative about the **NATURE OF THE GOOD**.
- In infinitesimal terms: $e_{Y q_1} = \dfrac{Y}{q_1}\dfrac{d\, q_1}{d\, Y}$

4.5 Classification of goods and demand

- The elasticities associated with behavioral curves allow us to classify the individual demand of a good:

(1) Price elasticity $\Rightarrow$ Classification of the demand:

Price elasticity → **Classification of demand** → Normal / Abnormal

(2) Crossed price elasticity $\Rightarrow$ Relationship between goods:

Crossed price elasticity → **Relationships between goods** → Gross complements / Gross independent / Gross substitutes

(3) Consumer's income elasticity $\Rightarrow$ Classification of goods:

Consumer's income elasticity → **Classification of goods** → Normal / Neutral / Inferior

4.5 Classification of goods and demand

(1) Price-elasticity $\Rightarrow$ **Classification of demand:**

$$e_{p_i q_i} = -\frac{p_i}{q_i}\frac{dq_i}{dp_i}
\begin{cases}
\text{Normal demand: } \downarrow p_i \rightarrow \uparrow q_i \Rightarrow \dfrac{dq_i}{dp_i} < 0 \Rightarrow e > 0 \begin{cases} e > 1 \Rightarrow \text{Elast} \\ e = 1 \Rightarrow \text{Elast} \\ e < 1 \Rightarrow \text{Inela} \end{cases} \\
\text{Abnormal demand: } \downarrow p_i \rightarrow \downarrow q_i \Rightarrow \dfrac{dq_i}{dp_i} > 0 \Rightarrow e < 0 \\
\text{Extreme cases} \begin{cases} \text{Totally inelastic demand: } \dfrac{dq_i}{dp_i} = 0 \Rightarrow e = 0 \\ \text{Totally elastic demand: } \dfrac{dq_i}{dp_i} = \infty \Rightarrow e = \infty \end{cases}
\end{cases}$$

4.5 Classification of goods and demand

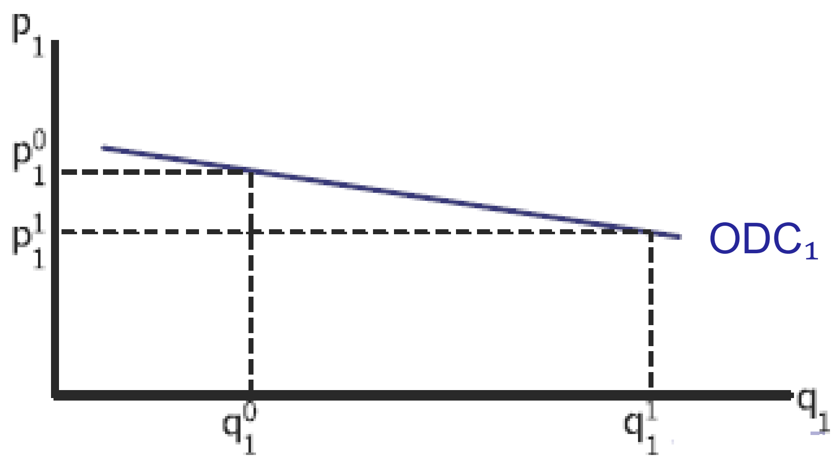
(1) Price-elasticity $\Rightarrow$ **Classification of demand:**

Normal demand: Different variations in price and quantity:

$$\downarrow p_i \rightarrow \uparrow q_i \Rightarrow \frac{dq_i}{dp_i} < 0 \Rightarrow e_{p_i q_i} > 0$$

i. Elastic: Quantity demanded changes in a higher proportion than price:

$$\downarrow p_1 (= 1\%) \rightarrow \uparrow q_1 (> 1\%) \Rightarrow e_{p_1 q_1} > 1$$



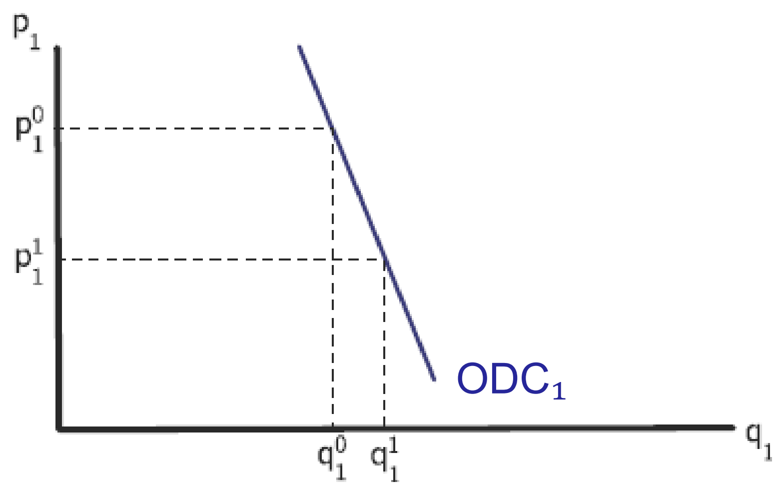
4.5 Classification of goods and demand

(1) Price-elasticity $\Rightarrow$ **Classification of demand**

Normal demand:

ii. Inelastic: Quantity demanded changes in a lower proportion than price:

$$\downarrow p_1 (= 1\%) \rightarrow \uparrow q_1 (< 1\%) \Rightarrow 0 < e_{p_1 q_1} < 1$$



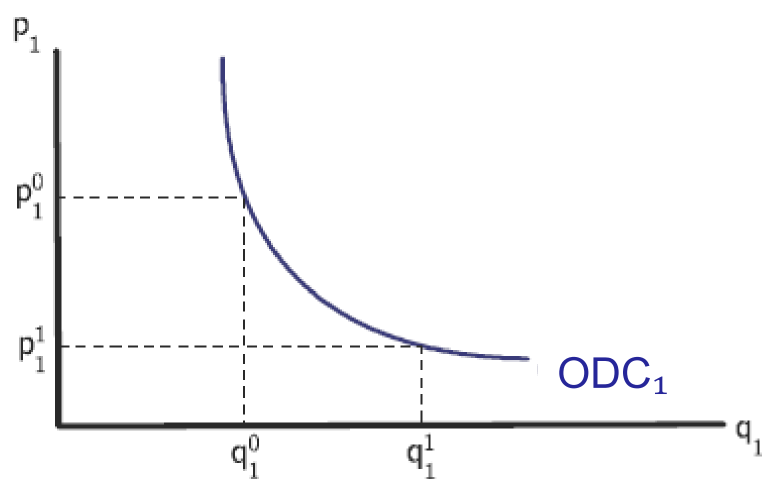
4.5 Classification of goods and demand

(1) Price-elasticity $\Rightarrow$ **Classification of demand**

Normal demand:

iii. Unitary demand: Quantity demanded changes in the same proportion as price:

$$\downarrow p_1 (= 1\%) \rightarrow \uparrow q_1 (= 1\%) \Rightarrow e_{p_1 q_1} = 1$$

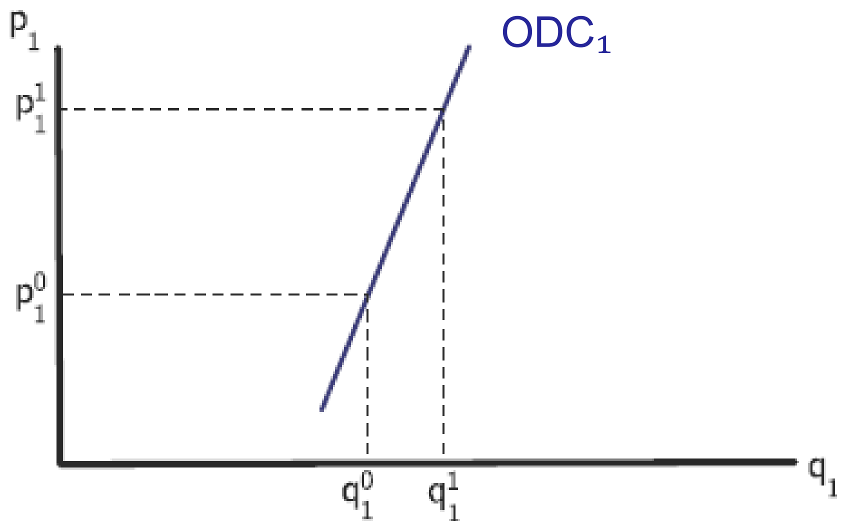


4.5 Classification of goods and demand

(1) Price-elasticity $\Rightarrow$ **Classification of demand:**

Abnormal demand: Variations in price and quantity demanded move in the same direction (also known as “Giffen goods”):

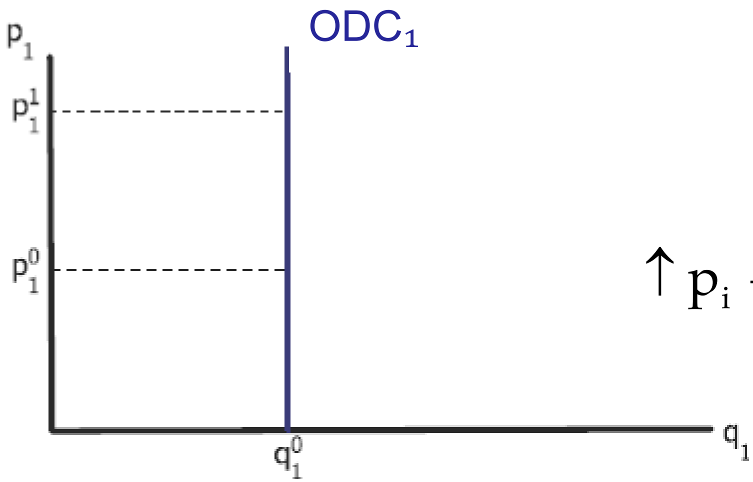
$$\uparrow p_i \rightarrow \uparrow q_i \Rightarrow \frac{dq_i}{dp_i} > 0 \Rightarrow e_{p_i q_i} < 0$$



4.5 Classification of goods and demand

(1) Price-elasticity $\Rightarrow$ **Classification of demand**

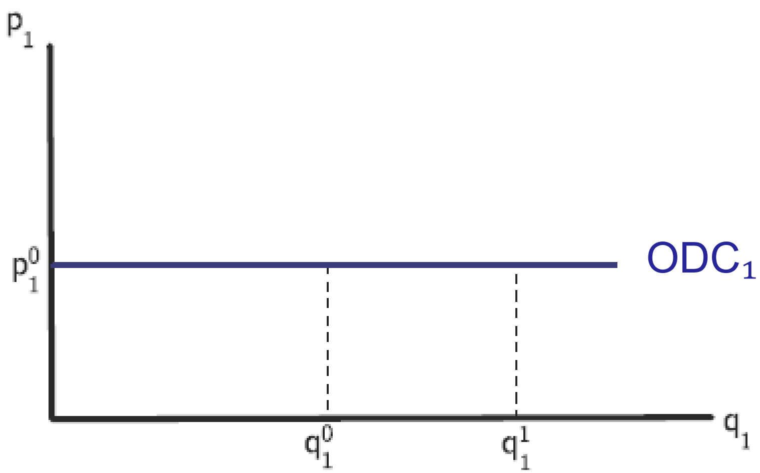
Extreme cases:



➢ **Perfectly inelastic (rigid) demand:**

The quantity demanded is not sensitive to changes in price:

$$\uparrow p_i \rightarrow q_i \text{ does not change} \Rightarrow \frac{dq_i}{dp_i} = 0 \Rightarrow e_{p_i q_i} = 0$$



➢ **Perfectly elastic demand:**

Any quantity is demanded, but only to a single fixed price:

$$\frac{dq_i}{dp_i} = \infty \Rightarrow e_{p_i q_i} = \infty$$

4.5 Classification of goods and demand

(2) Crossed price-elasticity $\Rightarrow$ **Relationship between goods:**

$$e_{p_i q_j} = \frac{p_i}{q_j}\frac{dq_j}{dp_i} \begin{cases} \text{Complements: } \uparrow p_i \left(\downarrow q_i\right) \downarrow q_j \Rightarrow \dfrac{dq_j}{dp_i} < 0 \Rightarrow e_{p_i q_j} < 0 \\ \text{Substitutes: } \uparrow p_i \left(\downarrow q_i\right) \uparrow q_j \Rightarrow \dfrac{dq_j}{dp_i} > 0 \Rightarrow e_{p_i q_j} > 0 \\ \text{Independent: } \uparrow p_i \left(\downarrow q_i\right) \text{ unchanged } q_j \Rightarrow \dfrac{dq_j}{dp_i} = 0 \Rightarrow e_{p_i q_j} = 0 \end{cases}$$

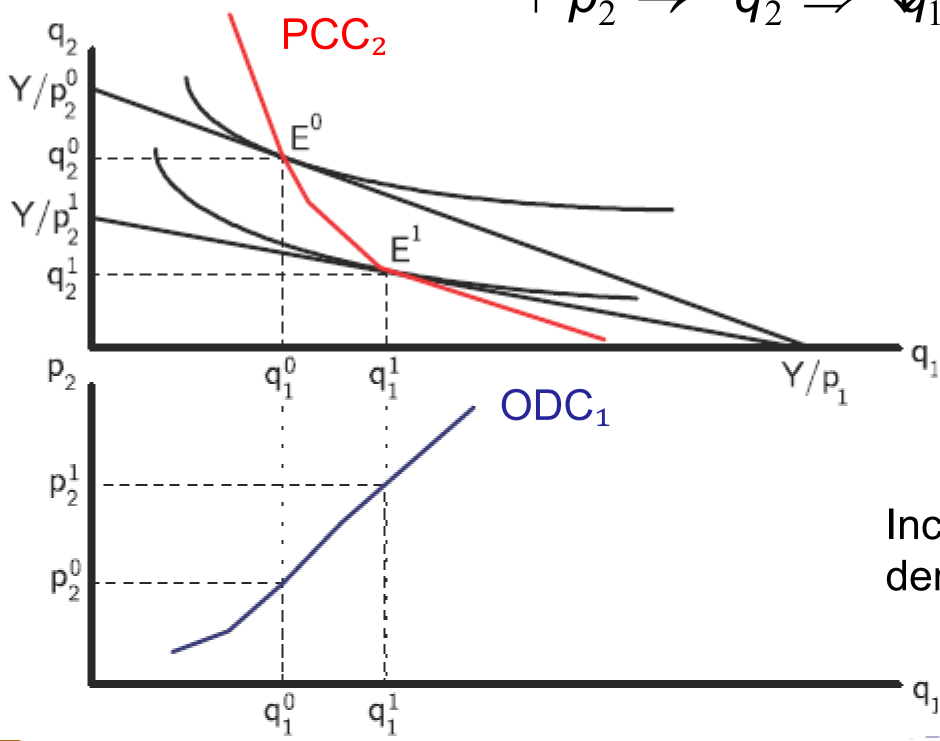
4.5 Classification of goods and demand

(2) Crossed price-elasticity $\Rightarrow$ **Relationship between goods:**

Substitutes (gross):

Satisfy the same need or desire. When the price of one good changes, the quantity demanded of the other good changes in the same direction:

$$\uparrow p_2 \rightarrow q_2 \Rightarrow \downarrow q_1 \Rightarrow e_{p_2 q_1} = \frac{p_2}{q_1}\frac{dq_1}{dp_2} > 0$$



Increasing crossed demand curve

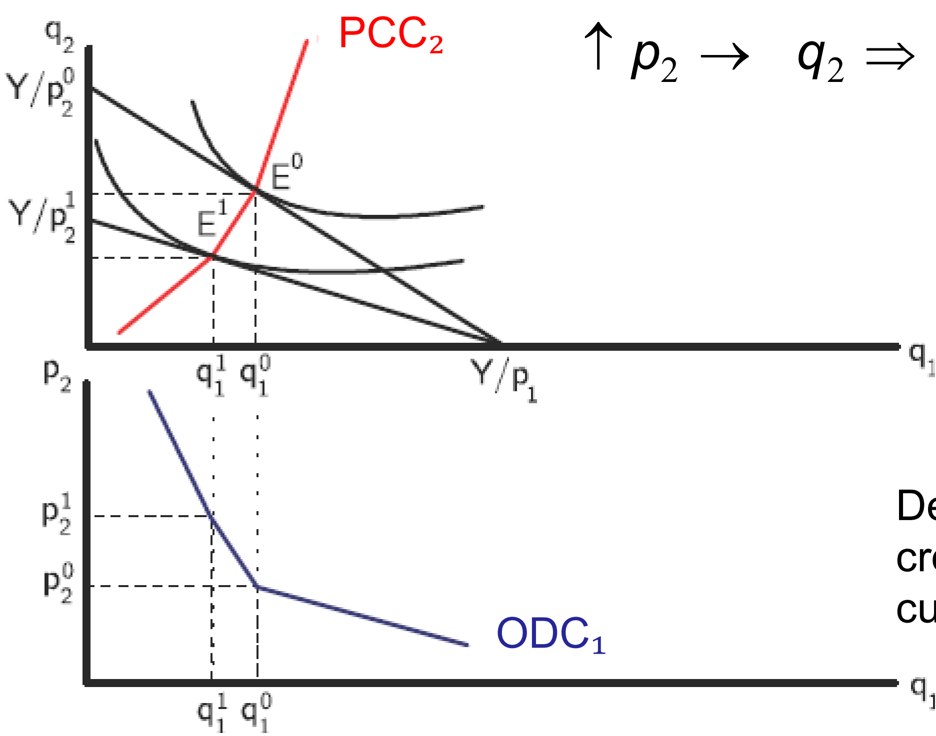
4.5 Classification of goods and demand

(2) Crossed price-elasticity $\Rightarrow$ **Relationship between goods:**

Complements (gross):

Satisfy jointly the same need. When the price of one good changes, the quantity demanded of the other good changes in the same direction:

$$\uparrow p_2 \rightarrow q_2 \Rightarrow \downarrow q_1 \Rightarrow e_{p_2 q_1} = \frac{p_2}{q_1}\frac{dq_1}{dp_2} < 0$$



Decreasing crossed demand curve

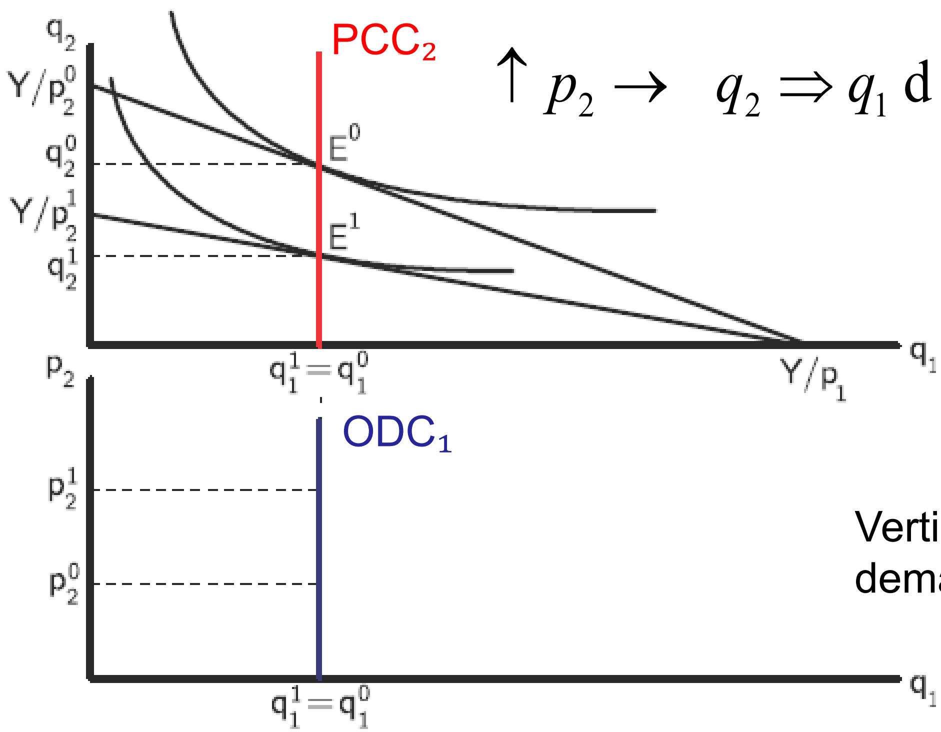
4.5 Classification of goods and demand

(2) Crossed price-elasticity $\Rightarrow$ **Relationship between goods:**

Independent (gross):

The needs or desires satisfied by both goods are unrelated:

$$\uparrow p_2 \rightarrow \downarrow q_2 \Rightarrow q_1 \text{ does not change} \Rightarrow e_{p_2 q_1} = \frac{p_2}{q_1}\frac{dq_1}{dp_2} = 0$$



4.5 Classification of goods and demand

(3) Income elasticity $\Rightarrow$ **Classification of goods:**

$$e_{Yq_i} = \frac{Y}{q_i}\frac{dq_i}{dY}$$

- Normal good: $\uparrow dY \rightarrow \uparrow q_i \Rightarrow \frac{dq_i}{dY} > 0 \Rightarrow e_{Yq_i} > 0 \Rightarrow$
  - $e_{Yq_i} > 1 \rightarrow \frac{dq_i}{q_i} > \frac{dY}{Y} \Rightarrow$ Luxury good
  - $e_{Yq_i} = 1 \rightarrow \frac{dq_i}{q_i} = \frac{dY}{Y} \Rightarrow$ Unit elasticity good
  - $e_{Yq_i} < 1 \rightarrow \frac{dq_i}{q_i} < \frac{dY}{Y} \Rightarrow$ Basic need good
- Neutral good: $\uparrow dY \rightarrow q_i$ does not change $\Rightarrow \frac{dq_i}{dY} = 0 \Rightarrow e_{Yq_i} = 0$
- Inferior good: $\uparrow dY \rightarrow \downarrow q_i \Rightarrow \frac{dq_i}{dY} < 0 \Rightarrow e_{Yq_i} < 0$

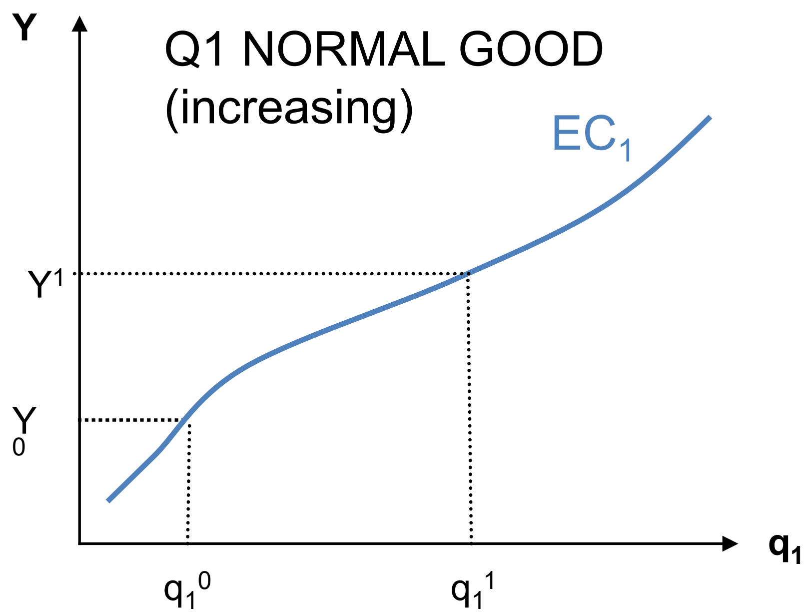
4.5 Classification of goods and demand

(3) Income elasticity $\Rightarrow$ **Classification of goods:**

Normal good:

When income rises, quantity demanded rises (increasing EC):

$$\uparrow Y \rightarrow q_1 \Rightarrow \uparrow \frac{d q_1}{d Y} > 0 \Rightarrow e_{Yq_i} > 0$$



4.5 Classification of goods and demand

(3) Income elasticity $\Rightarrow$ **Classification of goods:**

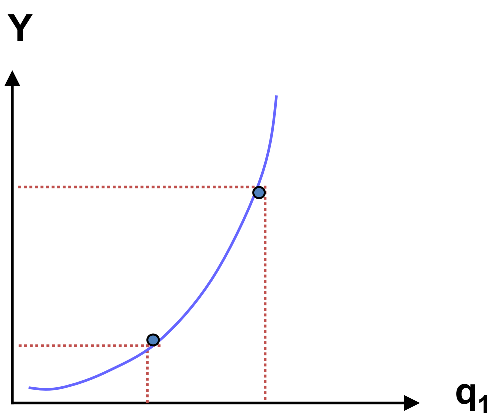
Normal good:

Basic need good: quantity demanded increases in a lower proportion than income

Luxury good: quantity demanded increases in a higher proportion than income

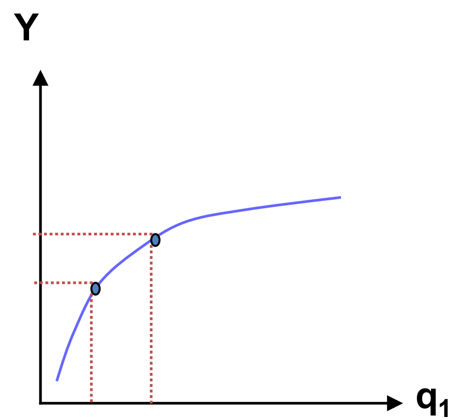
Unit income-elasticity: Both variables change in the same proportion

Basic need good



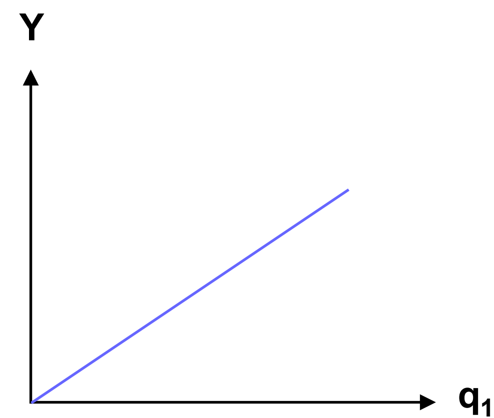
$$0 < e_{Yq_i} < 1$$

Luxury good



$$e_{Yq_i} > 1$$

Unit income-elasticity



$$e_{Yq_i} = 1$$

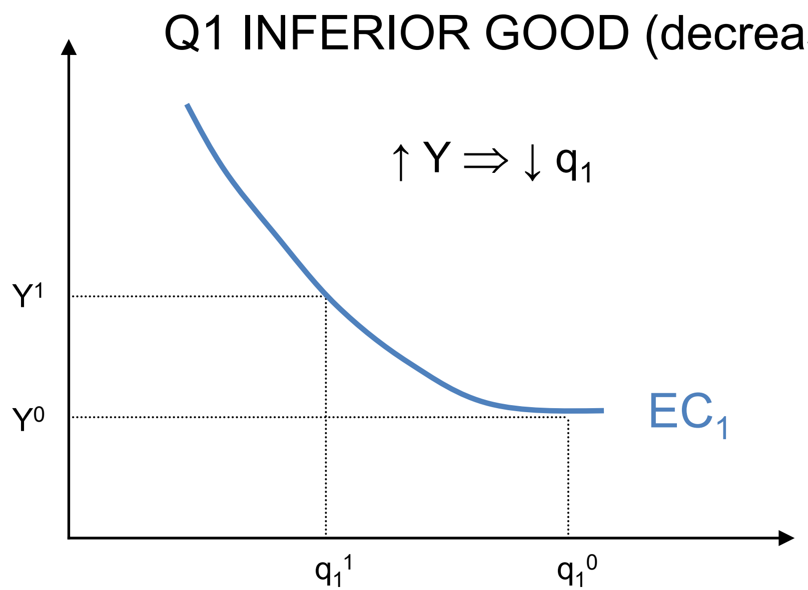
4.5 Classification of goods and demand

(3) Income elasticity $\Rightarrow$ **Classification of goods:**

Inferior good:

When income decreases the quantity demanded increases (decreasing EC):

$$\uparrow Y \rightarrow q_1 \Rightarrow \downarrow \frac{dq_1}{dY} < 0 \Rightarrow e_{Yq_i} < 0$$



4.5 Classification of goods and demand

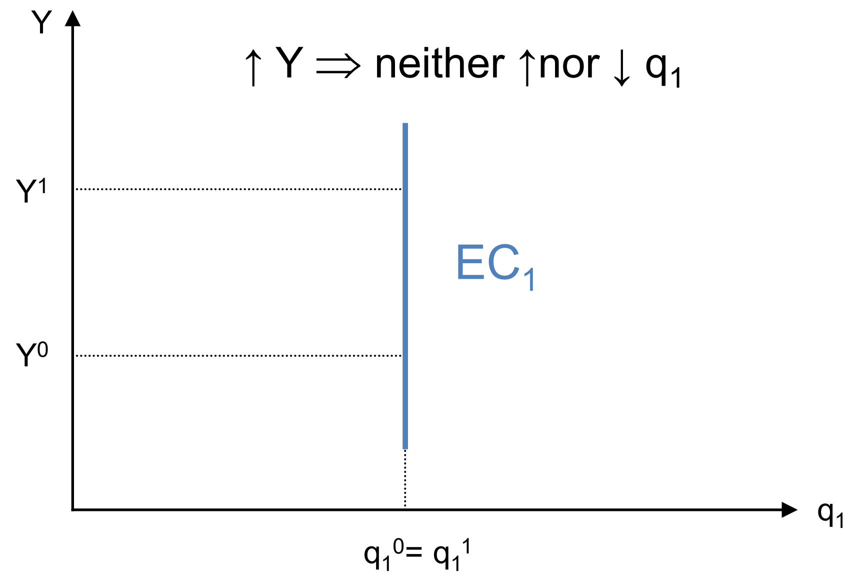
(3) Income-elasticity $\Rightarrow$ **Classification of goods:**

Neutral good:

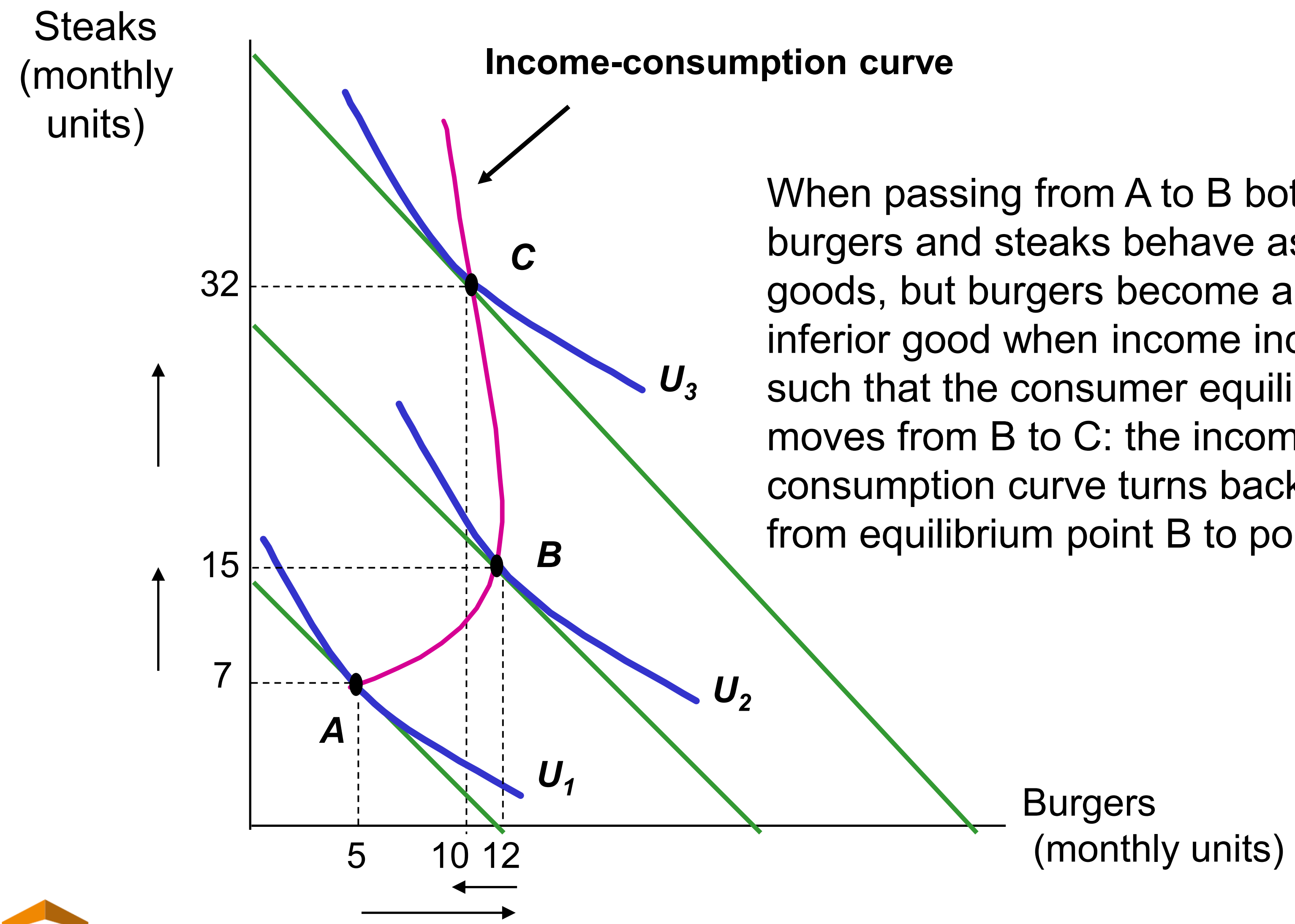
When income changes the quantity demanded does not change (inelastic EC):

$$\uparrow Y \rightarrow q_1 \text{ does not change} \Rightarrow \frac{dq_1}{dY} = 0 \Rightarrow e_{Yq_i} = 0$$

Q1 NEUTRAL GOOD

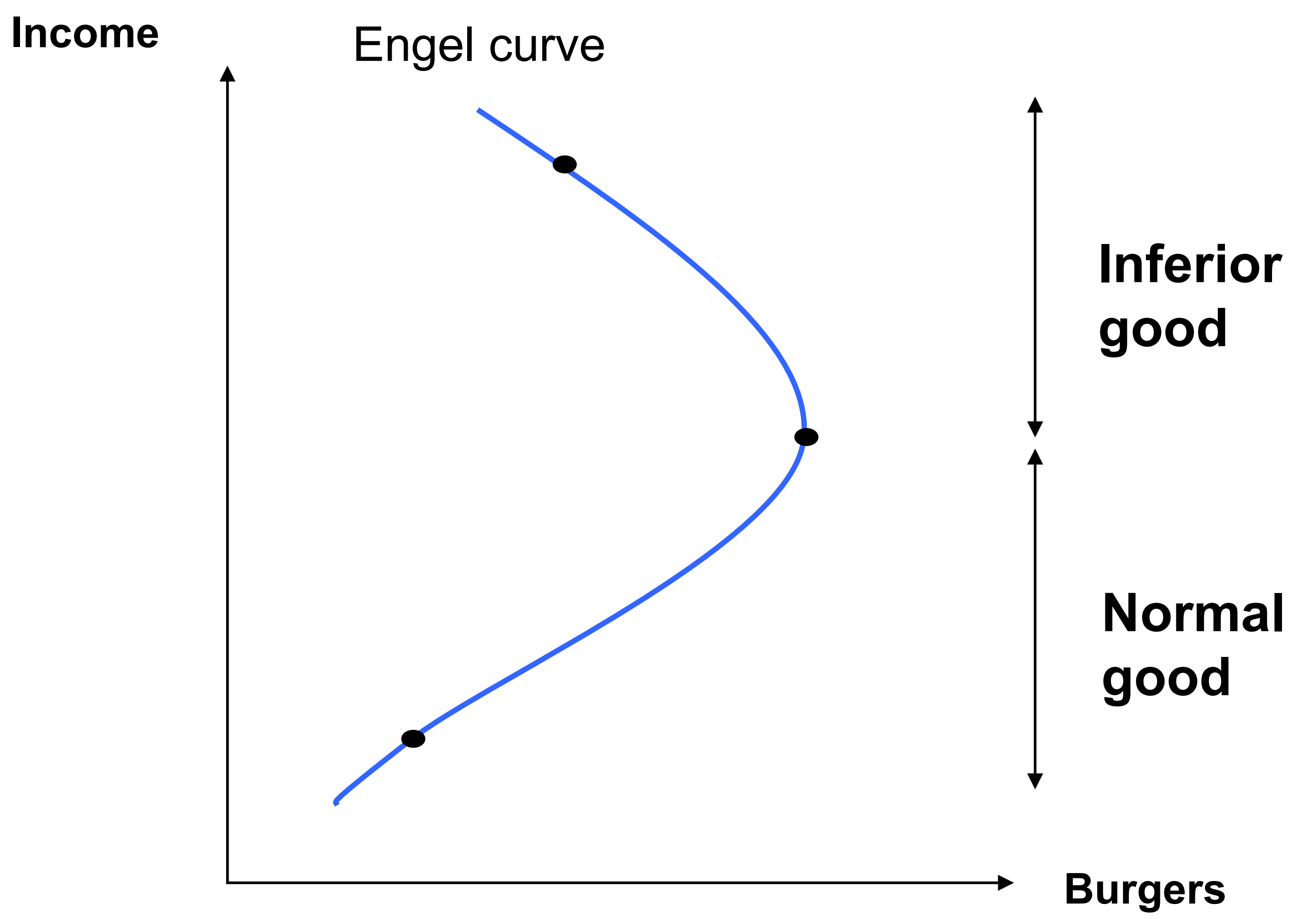


Examples: Inferior good

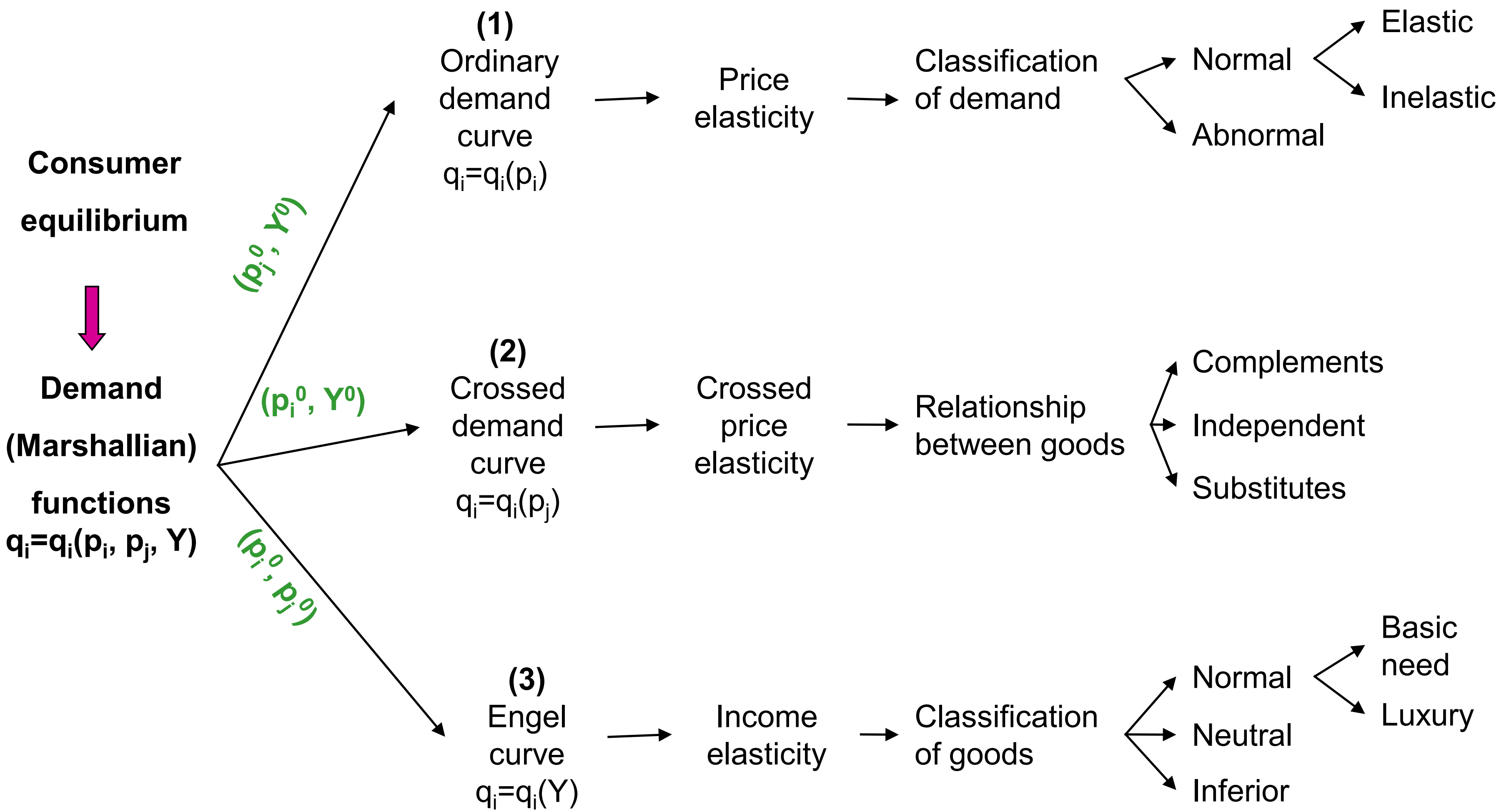


When passing from A to B both burgers and steaks behave as normal goods, but burgers become an inferior good when income increases, such that the consumer equilibrium moves from B to C: the income-consumption curve turns backwards from equilibrium point B to point C:

Examples: Inferior good



SUMMARY OF THE UNIT:



APPENDIX: Relation between expenditure and elasticities

(1) Price elasticity and its relation with expenditure

- **Expenditure on good: 1:** $G_1 = p_1 q_1 = \left[O \quad {}_1 : q_1 Ð\, q_1(p_1) \right] \in p_1 q_q(p_1)$
- Variation in expenditure when price changes:

$$\frac{d \quad {}_1}{d \quad {}_1} = q_1 \overset{G}{\underset{p}{+}} p_1 \frac{d \quad {}_1}{d \quad {}_1} = q_1 \overset{q}{\underset{p}{+}} p_1 \frac{q_1}{q_1} \frac{d \quad {}_1}{d \quad {}_1} = q_1 \left[\overset{q}{\underset{p}{1}} + \frac{p_1}{q_1} \frac{d \quad {}_1}{d \quad {}_1} \right] = q_1 \overset{q}{\underset{p}{[}} 1 - e_{p_1 q_1}]$$

- Therefore:

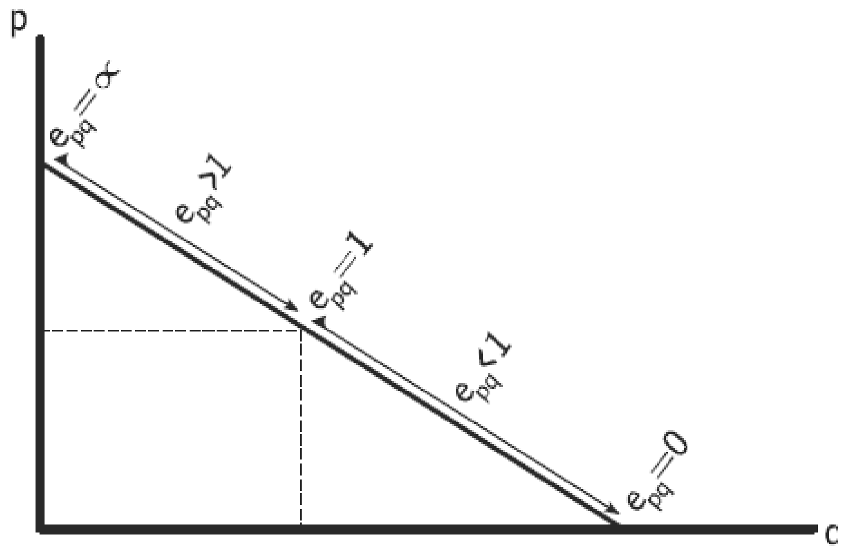
$$\frac{d \quad {}_1}{d \quad {}_1} \begin{cases} \overset{}{<0} & \text{d w ei e sm 1 ea } (e_{p_1 q_1} \text{ a ns l}) \Leftrightarrow \text{ t } p_1 \text{i d} \rightarrow \text{c } G_1 \uparrow \quad \downarrow \\ \overset{G}{\underset{p}{=0}} & \text{e w l ia lhs s } (e_{p_1 q_1} \text{ t} = 1) \Leftrightarrow \text{ c } p_1 \text{ i} \rightarrow G_1 \text{ d} \uparrow \text{y n o c o eh} \\ >0 & \text{d w ei i sm } (e_{p_1 q_1} \text{ a} < 1) \Leftrightarrow \text{ a } p_1 \text{ d} \rightarrow G_1 \uparrow \text{ i d} \uparrow \end{cases}$$

APPENDIX: Relation between expenditure and elasticities

(1) Price elasticity and its relation with expenditure:

Particular case → Linear demand

The same demand curve can behave as elastic and inelastic when evaluated at different points along the curve:



APPENDIX: Relation between expenditure and elasticities

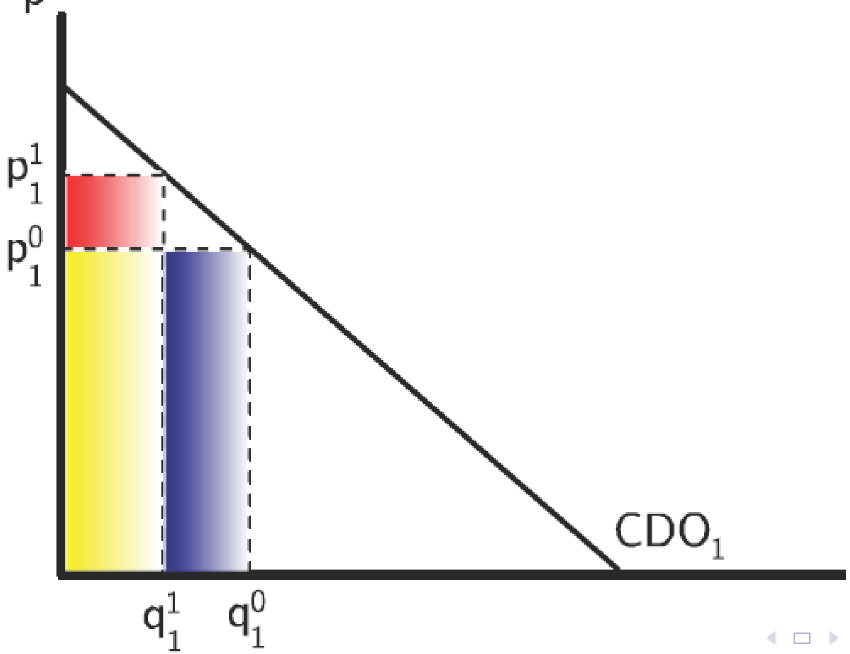
(1) Price elasticity and its relation with expenditure:

The variation in consumer expenditure ($G_1=p_1q_1$) when the price changes depends on the price elasticity of demand:

- Elastic range:

$$e_{p_1q_1} > 1$$

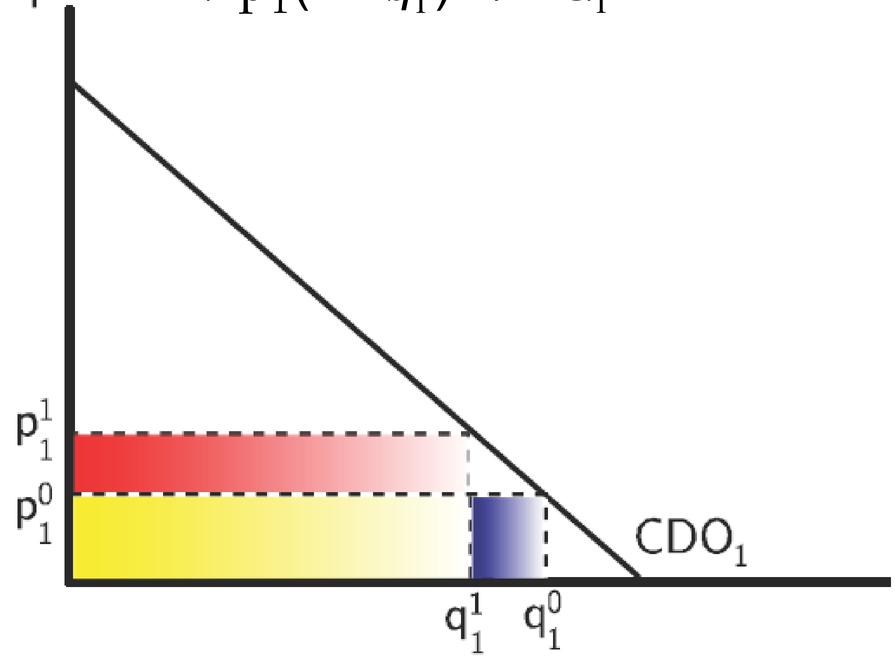
$$\uparrow p_1 (< \downarrow q_1) \Rightarrow \downarrow G_1$$



- Inelastic range:

$$0 < e_{p_1q_1} < 1$$

$$\uparrow p_1 (> \downarrow q_1) \Rightarrow \uparrow G_1$$



APPENDIX: Relation between expenditure and elasticities

(2) Income and its relation with the % share of income spent on the good:

- **% Share of income spent on good 1:** $g_1 = \dfrac{p_1 q_1}{Y}$

- Variation of the % share of income spent on good 1 when income changes:

$$\frac{dg_1}{dY} = \frac{d\left(\frac{p_1 q_1}{Y}\right)}{dY} = \frac{Y p_1 \frac{dq_1}{dY} - p_1 q_1}{Y^2} = \frac{p_1 q_1}{Y^2}\frac{Y}{q_1}\frac{dq_1}{dY} - \frac{p_1 q_1}{Y^2} =$$

$$= \frac{p_1 q_1}{Y^2} e_{Yq_i} - \frac{p_1 q_1}{Y^2} = \frac{p_1 q_1}{Y}\frac{1}{Y}(e_{Yq_i} - 1) = \frac{g_1}{Y}(e_{Yq_i} - 1)$$

- Therefore:

$$\frac{dg_1}{dY} = \frac{g_1}{Y}(e_{Yq_i} - 1) \begin{cases} < 0 \text{ i } & e_{Yq_i} < 1 \Leftrightarrow Y \rightarrow \downarrow g_1 \uparrow \quad \uparrow \\ = 0 \text{ i } & e_{Yq_i} = 1 \Leftrightarrow Y \rightarrow g_1 \text{ d} \uparrow \quad \text{no c oe h} \\ > 0 \text{ i } & e_{Yq_i} > 1 \Leftrightarrow Y \rightarrow \uparrow g_1 \uparrow \quad \downarrow \end{cases}$$

Exercises

1.- Victor is 12 years old and has lunch at school every day. His mother gives him 1,20 euros every day for the lunch. Victor only likes "Bollycaos" (Q_1) and orange juice (Q_2), and consumption yields him a utility given by the following utility function: $U=U(q_1, q_2)= (q_1 \cdot q_2)^{1/2}$

Answer the following questions:

a) Let's suppose that both "Bollycaos" and orange juice cost 60 cens per unit, how do you think that Victor should spend the Money that his mother gives him for lunch?

b) Compute the marginal rate of substitution in equilibrium and explain its economic meaning.

2.- Marisol's utility function is: $U=U(q_1, q_2)= 2q_1^2 \cdot q_2^2$, where Q_1 y Q_2 are the only goods available, with market prices $p_1=2$ y $p_2=1$ respectively. The consumer's income is $Y = 12$.

a) What will the consumed quantities of both goods be in equilibrium (Find first the individual marshallian demand functions).

b) Compute the MRS in equilibrium and explain its economic meaning.

c) Suppose that there is a change in the price of good 2, such that now $p_2=2$. Find the new equilibrium. Are the individual marshallian demand functions useful? Why?

3.- Helen and Rachel have identical tastes regarding goods Q_1 and Q_2. Both earn the same income (Y).

a. Will they demand the same quantity of both goods? Justify your answers using graphs.

b. If the prices of both goods double at the same time as Helen's income doubles and Rachel's income triples, then, can we conclude with certainty that both Helen and Rachel will demand a larger quantity of Q_1? Justify your answers using graphs.

c. Consider the initial situation and assume that income is constant while the price of Q_1 doubles and the price of Q_2 is reduced by 50%. Do the utilities of both Rachel and Helen increase? Justify your answers using graphs.

4.- Consider an economy with only two goods (Q_1 y Q_2) with prices p_1=6 y p_2=3, respectively. A consumer chooses a market basket from the choice set such that MRS^2_1=1/2. Can we be sure that the chosen market basket is the one that maximizes utility? Justify your answers using graphs.

5.- Natalia and Pablo spent the same income consuming the two goods Q_1 and Q_2. Natalia's preferences can be represented by the following utility function: $U=U(q_1, q_2)= q_1 \cdot q_2$, and Pablo's preferences by utility function $U=U(q_1, q_2)= ½ \cdot (q_1 q_2)$. Can we guarantee that, under these conditions, the quantities Q_1 and Q_2 demanded by Natalia will be twice the quantity chosen by Pablo?

6.- Give reasons for or against the following statements regarding consumer choices in equilibrium:

a) Prices are exogenous variables.
b) Quantities are exogenous variables.
c) Income is endogenous.
d) Income should not be spent in full.
e) The marginal rate of substitution is constant.

7.- Peter is very fond of antiquities and his preferences are given by: $U=(q_1-2)^2(q_2+5)^3$ where Q_1 represents the number of XIX century chairs and Q_2 the number of XVIII century wardrobes. Obtain:

a) The individual marshallian demand functions. Check that they are homogeneous functions with degree zero with respect to prices and income. What's the meaning of this property?
b) The consumer equilibrium when $Y = 95$, $p_1 = 5$, $p_2 = 3$.
c) The analytical expression of the Engel curve of good Q_1.
d) The analytical expression of the income elasticity of good Q_1.
e) The value of the income elasticity of good Q_1 for $Y = 100$.
f) The analytical expression of the Engel curve of good Q_2
g) The analytical expression of the income elasticity of good Q_2
h) The value of the income elasticity of good Q_2 for $Y = 100$.

8.- The Engel curve of good Q_1 is given by the expression: $q_1= q_1(Y)=100Y-2Y^2$. Answer the questions:

a) Draw the graphical representation of the above curve.
b) Obtain the analytic expression of the income elasticity of good Q_1.
c) Classify the good using income.

9.- Consider a utility function representing a given consumer's preferences from the Cobb-Douglas family of functions, with $0 < \alpha < 1$. Income is Y and prices are p_1 and p_2. Answer the questions:

a) Obtain the individual Marshallian demand functions of both goods.
b) Find analytically and graphically the Engel curve of good Q_2 when prices are $p_1=p_2=1$.
c) What's the meaning of parameter α? Describe its influence on the Engel curve and on the income elasticity of demand.

10. The optimal quantity of clothing (q_1) chosen by Clara for a given combination of prices (p_1 and p_2) can be represented by the following individual Marshallian demand function: $q_1 = \frac{3Y - p_1 + 2p_2}{2p_1}$ where Y= 50, p_1 = 10 and p_2 = 5.

Answer the following questions:

a) Find the analytical expression of the ordinary demand curve of good Q_1
b) Find the analytical expression of the direct price elasticity of good Q_1.
c) Classify the ordinary demand curve of Q_1 under different intervals of the price.
d) Find the analytical expression of the crossed demand function of good Q_1
e) Find the analytical expression of the crossed price elasticity of good Q_1.
f) How can we classify the relation between these two goods (in gross terms)? Give reasons for your answer.

11.- Peter considers coffee (q_1) and sugar (q_2) as perfect complements. For each cup of coffee he drinks he always put a single spoon of sugar. A utility function representing his preferences is the following:

$$U = \min\{q_1, q_2\}$$

Peter has a given budget Y for spending on these two goods and market prices are p_1 and p_2. Answer the following questions:

a) Find the individual demand functions of both goods.
b) Find analytically the Engel curve of coffee assuming that both prices are equal to one (unitary).
c) Find the income elasticity of the good. What is the percentage change of the quantity of sugar if the government increases income taxes by 25%.

12.- Discuss whether the following statements are true or false in the light of consumer theory and give reasons: When the consumer's income rises but the prices of the goods are constant, this:

a) Changes the Engel curve of good Q_1.
b) Changes the analytical expression of the income elasticity of any good.
c) Changes the specific value of the income elasticity.
d) Changes the price-consumption curve.

13.- Discuss whether the following statements are true or false in the light of consumer theory and give reasons: A change in the price p_2 ("caeteris paribus"):

a) Changes the marginal rate of substitution of good 1 for good 2 (RMS_2^1).
b) Changes the Engel curve of good Q_2
c) Changes the ordinary demand curve of good Q_2.
d) Changes the crossed demand curve of good Q_1.

14.- Discuss whether the following statements are true or false in the light of consumer theory and give reasons: If the price-consumption curve of good Q_1 (PCC_1) is horizontal:

a) The ordinary demand curve of Q_1 should be horizontal.

b) The ordinary demand curve of Q_1 is vertical.

c) The price elasticity of demand of Q_1 is positive.

Answer the previous questions for the case of the PCC_1 being vertical instead of horizontal.

15.- Discuss whether the following statements are true or false in the light of consumer theory and give reasons: A change in consumer tastes:

a) Changes the marginal rate of substitution.

b) Changes the consumer equilibrium.

c) Changes the specific value of the marginal rate of substitution at equilibrium.

d) Changes the income-consumption curve.

e) Changes the price-consumption curve.

f) The Engel curve does not change.

g) The function of the price elasticity remains the same.

16.- Knowing that the demand of food is inelastic, give reasons for whether consumers will spend more or less on food when, due to a bad crop, the price of food rises.

17. The ordinary demand curve of a good is given by the following expression: $q=100/p$. Can you predict how will the consumer spending change when the price of the good changes?

18. Given an ordinary demand curve, which among the following situations is likely to cause a fall in the total spending on the good by the consumer?

a) A fall in the price from a point at any inelastic section of the curve.

b) A rise in the price from a point at any inelastic section of the curve.

c) A fall in the price from a point at any elastic section of the curve.

d) A fall in the price at a section of unit elasticity.

e) None of the above is true.

19. Given the ordinary demand curve of a good: q = q(p) = 5-p, answer the following questions:

a) Draw a graph of the curve.

b) Calculate the point or point intervals given the values of the price elasticity $e_{pq} = 1$, $e_{pq} < 1$, $e_{pq} > 1$.

c) If you were a buyer with that curve, at which point of the curve would you prefer to be located? Why?